# ENPM 643 Spatial Logic <br> Framework for Building Layouts 

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## Background

In Last semester the aim of the project was to establish a methodology and prepare UML diagrams for building designs representation at a higher level of abstraction to form a basis for tool development so as to automatically check potential building designs against specification quickly, easily and in early phases of design.

Building design representation could be looked upon from multiple viewpoints:

- Architectural viewpoint
- Structural viewpoint
- Plumbing viewpoint
- Electrical viewpoint
- Security viewpoint ....so on

Due to initial project complexity and an abstract concept (absence of explicit system behavior), we first considered only architectural viewpoint and a simple one-bedroom apartment floor plan. We tried to establish the architectural viewpoint for this one bedroom apartment's floor plan as shown in fig 1.0.


Figure 1: One bedroom apartment floor plan

Architectural viewpoint of the building design was concerned with the hierarchical decomposition of spaces within blocks. In this view point, shapes were transformed into "architectural regions" (rooms) during the early phases of the design and it also involved preliminary evaluation of properties (like size, shape, orientation, and adjacency) coupled with the assignment of properties to regions.

So in last semester we completed following tasks:

- Defined and categorized the design requirements of a building from an architectural viewpoint including writing down all the architectural requirements and arranging them in a hierarchical way.
- Prepared the System Structure diagrams (Class Diagram) at a higher level of abstraction including identification of objects and their attributes.
- Defined Validation Parameters like proximity, access type, area, adjacency, orientation so as
- To allow the architect to check potential building designs against the specification
- Quickly
- Easily \&
- In early phases of the design


## Goal

The goal of the project is to achieve the pathway of End to End development from requirements to UML representations and to engineering drawings i.e. how the requirements will be traced from a requirements diagram to structure diagram, behavior diagram and finally on floor plan.


Figure 2:End-to-End development from requirements to UML representations and to engineering drawings

## Requirements Analysis

The architectural requirements of one bedroom apartment shown in Fig 1 can be broadly categorized in two ways:

- Apartment Level
- An apartment should have one bedroom, one living room, one restroom and one kitchen
- An apartment entrance should not be through bedroom
- An apartment should have easy pathway towards exit in case of emergency
- Room Level
- Size of bedroom should be X Sqft.
- Rest room should be close to bedroom
- Rest room should be far from kitchen

However all the requirements are as follows:

## Apartment Level:

1. Area of the apartment should be at least 10000 sq units.
2. The apartment should have 1 bedroom, 1 living room, 1 kitchen, 1 restroom and one passageway.
3. The entrance of the apartment should not be through bedroom.
4. The apartment should have easy access to exit in case of fire.

## Room Level:

5. Occupancy of the bedroom should be two.
6. Area of the bedroom should be 3500 sq. unit.
7. Bedroom should be adjacent to the restroom.
8. Bedroom should have a closet.
9. The closet in the bedroom should be a walk-in closet.
10. Proximity strength between restroom and bedroom is 1 .
11. Bedroom should be properly ventilated.
12. Bedroom should have two windows.
13. Bedroom should be sound proof.
14. Bedroom should have air tight doors.
15. Orientation of the bedroom should be towards the west.
16. Occupancy of the Kitchen should be two.
17. Area of the kitchen should be 900 square units.
18. Kitchen should be far from rest room.
19. Kitchen should be adjacent to living room.
20. Proximity strength between Kitchen and living room is 1.
21. Proximity strength between Kitchen and restroom is greater than eight.
22. A joint in the wall separates living room and kitchen.
23. Occupancy of the rest room should be 2 .
24. Area of rest room is 600 sq units.
25. Occupancy of the passageway should be for two people crossing each other in opposite direction.
26. With of passage way is 30 Sq . units.
27. Living Room occupancy should be 4 .
28. Area of living room is 3500 sq. unit.
29. Living room should be properly ventilated.
30. Living room should have a opening and a window.

## System Structure - Generic Class Diagram

The complete architectural viewpoint was divided in to three sub classes:

- Spaces
- Dividers
- Portals

Spaces consisted of each type of room, which includes bedroom, kitchen, living room, restroom and passageway. Whereas dividers consisted of floor and walls which basically divides the spaces horizontally and vertically respectively. And the third category of portals included doors, windows, joints (opening without door) and vent (for ventilation).

Now in order to show relationship between different rooms (Association_Rooms), between rooms and walls (Association_Rooms_Walls), and between walls and portals (Association_Walls_Portals), we used different association classes. The properties of these association classes addressed the different kind of relationships between rooms, walls and portals.

These properties were
Proximity strength - which address the proximity issue between different rooms i.e. are they close or apart.

Access Type - What type of access is available?
Access Vent - Is it for ventilation purpose
Access Light - Is it allowing light to pass through i.e. is it transparent
Access admit - Is it allowing people to enter or exit
Access audible - Is it sound proof or not

## Validation and Verification:

This semester we will try to verify if the requirements for the tool were fulfilled using a spatial logic framework for building designs. To elaborate on the spatial logic framework let us look at some of the fundamentals for spatial logic.

Definition of a halfplane: The concept of the halfplane helps us answer our basic question about how to represent spaces symbolically without any reference to a particular coordinate system or any other numerical reference. In geometry two halfplanes are divided by a line, and the points on that line do not belong to any halfplane. In our concept there is no line, only a conceptual border that divides two sets of points. Each set of points define one halfplane. More formally:

U is a region defined by a set of points $\boldsymbol{p}(\boldsymbol{x}, \boldsymbol{y})$
$U=\{p(x, y)\}$
$\boldsymbol{U}$ always can be divided into exactly two subsets A and $\boldsymbol{B}$, defined by:
$\mathrm{A}=\{p(x, y): f(x, y)>0\}$, and
$B=\{p(x, y): f(x, y)<=0\}$
$f(x, y)$ is a continuous function in $U$.
A and $\boldsymbol{B}$ are non-empty, closed sets.
Therefore A and $\boldsymbol{B}$ have the following characteristics:
$\mathrm{A} \cap \boldsymbol{B}$ is $\mathscr{\emptyset}$, and
$\mathrm{A} U \boldsymbol{B}$ is U .

The set $\boldsymbol{B}$ is the complement of set A, denoted by A', where we only consider elements in $\boldsymbol{U}$.

The predicate $\mathrm{hp}(\mathrm{x})$ is a general representation of a halfplane, according to its truth value. For instance, in Figure the halfplane a is shown by the shaded area. Its complement, $\bar{a}$ is the unshaded area bounded by U and can be represented by $\neg \mathrm{hp}(\mathrm{a})$, regardless of the specific truth value assigned to it. By convention, we assign with truth value True to the halfplane where its name lies, as $\boldsymbol{h p}(\boldsymbol{a})$ shows in Figure below


Figure 3: Half plane a in $U$

Given the sets $\mathrm{A}=\{\mathrm{p}(\mathrm{x}, \mathrm{y})\}$ and $\mathrm{J}=\{\mathrm{p}(\mathrm{x}, \mathrm{y})\}$, if $\mathrm{A} \subset \mathrm{J}$, the logical representation equivalent to this condition is
$\mathrm{hp}(\mathrm{a}) \rightarrow \mathrm{hp}(\mathrm{j})$
where $\boldsymbol{a}$ and j are halfplanes for A and J .

## Constraint

A constraint $\boldsymbol{C}$ is the set of logical formulas equivalent to a given topology.

## Region

Given n halfplanes, a region $\boldsymbol{R}$ is defined by a conjunctive formula of $\mathrm{n} \mathrm{hp}(\mathrm{x})$, as $R$ is $h p\left(a_{1}\right) \wedge h p\left(a_{2}\right) \wedge . . \wedge h p\left(a_{n}\right)$.
Since each halfplane can have truth value True or False, each region is an interpretation of the Formula above. This means, for a given $n$ halfplanes we have $2^{n}$ different regions.

## Minimal description

Given a region $\boldsymbol{F}$ defined by $\boldsymbol{h} \boldsymbol{p}\left(\boldsymbol{a}_{1}\right) \wedge \boldsymbol{h} \boldsymbol{p}\left(\boldsymbol{a}_{2}\right) \wedge \ldots \wedge \boldsymbol{h} \boldsymbol{p}\left(\boldsymbol{a}_{n}\right), \boldsymbol{F}$ has a reduced form $\boldsymbol{F}^{\wedge}$ if an $\boldsymbol{h} \boldsymbol{p}\left(\boldsymbol{a}_{\boldsymbol{i}}\right)$ were removed from $\boldsymbol{F}$, and $\boldsymbol{F}$ and $\boldsymbol{F}$ ' define the same region unambiguously. If $\boldsymbol{F}$ cannot be reduced further, $\boldsymbol{F}$ is said to be the minimal description and is represented by $\boldsymbol{F}_{\boldsymbol{m i n}}$. There always exists an $\boldsymbol{F}_{\boldsymbol{m i n}}$ for a given $\boldsymbol{F}$. graphically; $\boldsymbol{F}_{\boldsymbol{m i n}}$ means a formula with only $\boldsymbol{h} \boldsymbol{p}\left(\boldsymbol{a}_{\boldsymbol{i}}\right)$ that bounds the region $\boldsymbol{F}$.

## Application to the Floor Plan

For the floor plan that we are trying to validate let us consider a region U defined by points $\mathrm{p}(\mathrm{x}, \mathrm{y})$ and bounded by imaginary half planes $\alpha, \beta, \gamma, \delta$

$$
\therefore U \rightarrow h p(\alpha) \wedge h p(\beta) \wedge h p(\gamma) \wedge h p(\delta)
$$

Further let halfplanes a, b, c and d be used to define the rooms in the region U. Fig 4 depicts the positions of the hyperplanes.


Figure 4: Depiction of regions using halfplanes
Summarizing the above we have,

$$
\begin{aligned}
& h p(\gamma) \rightarrow h p(b) \rightarrow h p(a) \\
& h p(\gamma) \rightarrow h p(b) \\
& h p(\alpha) \rightarrow h p(c) \\
& h p(\delta) \rightarrow h p(d)
\end{aligned}
$$

For the floor plan we are primarily concerned with the regions that form a room, door, window etc. First let us start with the rooms as a region to begin with.

R1 - rest room
R2 - bed room
R3 - living room
R4 - kitchen
R5 - passage way
Halfplanes can have a truth value that is either true or false. This makes each region an interpretation of a formula which can be a combination of two pr more halfplanes. For the region to exist the formula should be true. A region R is said to be visible if and only if R has truth value true under C i.e. $\mathrm{R} \rightarrow \mathrm{C}$. for the floor plan to be validated this is an important condition.

For our floor plan the formulation or logical representation for of the various rooms (regions) gives us the following formulas:

We consider $h p(\alpha) \wedge h p(\beta) \wedge h p(\gamma) \wedge h p(\delta)$ forming U and being true always.
$\mathbf{R 1} \rightarrow$ restroom can be described as
$h p(a) \wedge h p(b) \wedge h p(d) \wedge \neg h p(c)$
Applying the minimal description rule $\mathrm{R} 1 \rightarrow$ restroom can be described as


Figure 5 Rest room
$\mathbf{R 2} \rightarrow$ bedroom can be described as
$h p(a) \wedge h p(b) \wedge \neg h p(d) \wedge \neg h p(c)$
Applying the minimal description rule $\mathrm{R} 2 \rightarrow$ bedroom can be described as


Figure 6 Bed room
$\mathbf{R 3} \rightarrow$ living room can be described as
$\neg h p(a) \wedge \neg h p(b) \wedge \neg h p(d)$
Applying the minimal description rule $\mathrm{R} 3 \rightarrow$ living room can be described as


## Figure 7 Living Room

$\mathbf{R 4} \rightarrow$ kitchen can be described as
$\neg h p(a) \wedge \neg h p(b) \wedge h p(d) \wedge h p(c)$
Applying the minimal description rule $\mathrm{R} 4 \rightarrow$ kitchen can be described as
$h p(c) \wedge \neg h p(d)$


Figure 8 Kitchen

## $\mathrm{R} 5 \rightarrow$ passageway can be described as <br> $$
\neg h p(a) \wedge h p(b) \wedge h p(d) \wedge \neg h p(c)
$$

Applying the minimal description rule $\mathrm{R} 5 \rightarrow$ passageway can be described as


Figure 9 Passageway

## Constraints:

$\mathrm{hp}(\mathrm{b}) \rightarrow \neg \mathrm{hp}(\mathrm{c})$
$\mathrm{hp}(\mathrm{c}) \rightarrow \neg \mathrm{hp}(\mathrm{b})$
$\mathrm{hp}(\mathrm{b}) \rightarrow \mathrm{hp}(\mathrm{a})$
$\neg \mathrm{hp}(\mathrm{b}) \rightarrow \neg \mathrm{hp}$ (a)
Possible topologies and regions:

| S.No | hp $(\mathrm{a})$ | hp $(\mathrm{b})$ | hp $(\mathrm{c})$ | hp $(\mathrm{d})$ | R |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | False | False | False | False | True |
| 2 | False | False | False | True | True |
| 3 | False | False | True | False | True |
| 4 | False | False | True | True | True |
| 5 | False | True | False | False | True |
| 6 | False | True | False | True | True |
| 7 | False | True | True | False | False |
| 8 | False | True | True | True | False |
| 9 | True | False | False | False | False |
| 10 | True | False | False | True | False |
| 11 | True | False | True | False | False |
| 12 | True | False | True | True | False |
| 13 | True | True | False | False | True |
| 14 | True | True | False | True | True |
| 15 | True | True | True | False | False |
| 16 | True | True | True | True | False |

The table above gives us a possible combination of the regions that can exist when the corresponding topologies are true or false. for eg. There cannot be a region that is a combination of $\mathrm{hp}(\mathrm{b})$ and $\mathrm{hp}(\mathrm{c})$ true at the same time because the visible regions for these hyperplanes do not coincide as shown in the fig 10. So whenever hp(b) and hp(c) become true the resulting region is false.


Figure 10 Feasible regions for $\mathbf{h p}(\mathbf{b}) \& \mathbf{h p}(\mathbf{c})$

The table above can help us to verify the existence of all the rooms in the plan. The entire region corresponding to the minimal description of the rooms should be true for the rooms to exist. So our requirements of the type: the Apartment should have a bed room should be satisfied. In our case

| Room | Region | S.No | R |
| :--- | :--- | :--- | :--- |
| Restroom | R1 | 13 | True |
| Bedroom | R2 | 14 | True |
| Living room | R3 | 3 | True |
| Kitchen | R4 | 4 | True |
| Passageway | R5 | 6 | True |

## Adjacency:

## Region adjacency

The region adjacent to any visible region $R$ is defined as any region $R$ ' which graphically bounds R. There are two types of adjacency: border adjacency and corner adjacency. These are important concepts which support topological determinations.

## Border adjacency

Given a minimal description of a region $\boldsymbol{R}$ expressed by $\mathbf{h p}\left(\mathbf{x}_{1}\right) \wedge \mathrm{hp}\left(\mathrm{x}_{2}\right) \wedge \ldots \wedge \mathrm{hp}\left(\mathrm{x}_{\mathrm{n}}\right)$, a region $\boldsymbol{R}_{\text {adj }}$ is border adjacent to $\boldsymbol{R}$ iff it differs in only one $\mathrm{hp}\left(\mathrm{x}_{\mathrm{i}}\right)$, such as $\mathrm{hp}\left(\mathrm{x}_{\mathrm{i}}\right)$ in $\boldsymbol{R}$ is $\neg \mathrm{hp}\left(\mathrm{x}_{\mathrm{i}}\right)$ in $\boldsymbol{R}_{\text {adjj }}$. For example, the floor plan shown below restroom has the minimal description $h p(a) \wedge h p(d)$ therefore it has two border adjacent regions:
$h p(a) \wedge \neg h p(d)$
$\neg h p(a) \wedge h p(d)$

## Corner adjacency

Given a minimal description of a region $\boldsymbol{R}$ expressed by $\mathbf{h p}\left(\mathbf{x}_{1}\right) \wedge \mathrm{hp}\left(\mathrm{x}_{2}\right) \wedge \ldots \wedge \mathrm{hp}\left(\mathrm{x}_{\mathrm{n}}\right)$, a region $\boldsymbol{R}_{a d j}$ is corner adjacent to $\boldsymbol{R}$ iff it differs in exactly two literals $\boldsymbol{h} \boldsymbol{p}\left(\boldsymbol{x}_{i}\right)$ and $\boldsymbol{h} \boldsymbol{p}\left(\boldsymbol{x}_{j}\right)$, where $\boldsymbol{h} \boldsymbol{p}\left(\boldsymbol{x}_{\boldsymbol{i}}\right)$ and $\boldsymbol{h} \boldsymbol{p}\left(\boldsymbol{x}_{\boldsymbol{j}}\right)$ in $\boldsymbol{R}$ are $\neg \boldsymbol{h} \boldsymbol{p}\left(\boldsymbol{x}_{\boldsymbol{i}}\right)$ and $\neg \mathrm{hp}\left(\mathrm{x}_{\mathrm{j}}\right)$ in $\mathrm{R}_{\text {adj. }}$. For example, the floor plan shown below restroom has the minimal description $h p(a) \wedge h p(d)$ therefore it has the corner adjacent regions:

$$
\neg h p(a) \wedge \neg h p(d)
$$

For the floor plan under study we can use the adjacency property to verify if a room is adjacent to another. For example if a requirement says that bedroom should be adjacent to the rest room then it can be verified by using the logical formulas for the respective rooms and the adjacency property. The formulas for the rooms are:
Restroom $h p(a) \wedge h p(d)$
Bedroom $h p(b) \wedge \neg h p(d)$
Also, from the constraints we have $\mathrm{hp}(\mathrm{b}) \rightarrow \mathrm{hp}(\mathrm{a})$

Thus, restroom is region adjacent to the bedroom as it differs in only one $\mathrm{hp}(\mathrm{xi})$ [ $\mathrm{hp}(\mathrm{d})$ ] as shown in fig 11.


Figure 11 Adjacency between restroom and bedroom

This verifies our requirement.

## Relative position

As the mapping from a numeric to a logic representation does not carry out information on spatial relation among halfplanes, it is necessary to give a semantic denotation for each halfplane. This denotation can be given by the following declaration:

- The universe of discourse $U$ is represented by a rectangle, whose boundaries are numbered from 1 to 4 in a clockwise direction, as shown in Figure12


3
Figure 12

- Each halfplane has its symbol and denotation according to the endpoints of its border. Table below shows the possible combinations among sides for the floor plan, where the border of each halfplane is represented by a dotted line. Halfplane labels are consistently placed above or to the left of the boundary of the halfplane. Arbitrarily name above as the upwards direction and left as westwards direction.
- The denotation for each halfplane is shown in Table, assuming Left( L ) as opposite direction of $\operatorname{Right}(\mathrm{R})$ and $\operatorname{Above}(\mathrm{A})$ as opposite direction of $\operatorname{Below}(\mathrm{B})$.
- The name of each halfplane is given to the side its denotation is, as shown in the Table

| Half plane | Begin end | Denotation | Abbreviation |
| :--- | :--- | :--- | :--- |
| A | $1-3$ | Left | L |
| B | $1-3$ | Centre | C |
| C | $1-3$ | Right | R |
| D | $4-2$ | Above | A |

With this systematic way of labeling halfplanes and denotations, we can infer the relative position of a region to another, in relation to a given halfplane. In order to compare the relative position between two regions it is necessary to eliminate the halfplanes in common and analyze the differences between the resulting formulas. Where the predicate components are the same the two regions lie in the same halfplane and no topological information is available from the formula; only differences contain topological information.
For the floor plan under consideration comparing rest room and bedroom
Restroom $h p(a) \wedge h p(d)$
Bedroom $h p(b) \wedge \neg h p(d)$

The difference is restroom $\mathrm{hp}(\mathrm{d})$ and bedroom $\neg \mathrm{hp}(\mathrm{d})$ therefore topographically restroom is above bedroom. If we consider above being north then requirements such as bed room should be at the south of restroom is satisfied by considering the relative position of the two rooms.

Adding portals to the floor plan: let us consider our original floor plan and lets add some portals in the plan. For simplicity sake lets just add all the doors to the floor plan. To represent them using the hyperplane let us consider the following new hyperplanes added to our original structure.
hp(e)
$h p(f)$
$\mathrm{hp}(\mathrm{g})$
$h p(h)$
hp(i)
the resulting fig is as shown below.


Figure 13 : depiction of doors using hyperplanes
Also lets consider a region such as :
$h p(d) \wedge \neg h p(d)$
This would represent the hyperplane d which was imaginary. It now represents a wall. So the portal on this wall can be represents by limiting the boundary condition for the portal on the wall. For example:

Representing the bedroom door: The bedroom door can be represented by the formula $h p(d) \wedge \neg h p(d) \wedge h p(f) \wedge h p(e)$ and pictorially we can depict it as :


Figure 14 representing the bedroom door

Properties such as visibility, audibility, transparency can be validated by assigning a value to the portal formula or the wall formula and checking if it's true for that value. For example if we had a requirement of the type that the window door should be transparent and sound proof and if the formula of the door is of the type $\mathrm{hp}(\mathrm{x})[\mathrm{v}, \mathrm{a}]$ where ' v ' and ' a ' represent the truth values of visibility and audibility then the door can be said to verify all its requirements if its formula is $h p(d) \wedge \neg h p(d) \wedge h p(f) \wedge h p(e)[1,0]$. The visible region through the door can also be found out using further analysis as shown in the figure.

Portal $h p(x i)(1,0)$


Figure 15: Visibility through portals.

## Conclusion:

Thus the requirements that we can verify using the spatial logic approach are of the type:

Architectural requirements

1. Apartment Level:

1 Area of the apartment should be at least 10000 sq units.
2 The apartment should have 1 bedroom, 1 living room, 1 kitchen, 1 restroom and one passageway.
3 The apartment should have easy access to exit in case of fire.
2. Room Level:

1 Occupancy of the bedroom should be two.
2 Bedroom should be adjacent to the restroom.
3 Proximity strength between restroom and bedroom is 1 .
4 Bedroom should have air tight and sound proof doors.
5 Orientation of the bedroom should be towards the west.

And to finally conclude the validation matrix for the above requirements is as shown:

| Requirement no | Property of Spatial logic | Verified |
| :---: | :---: | :---: |
| 1.1 | - | No |
| 1.2 | Region | Yes |
| 1.3 | Adjacency, Relative Position | Yes |
| 2.1 | - | No |
| 2.2 | Adjacency | Yes |
| 2.3 | Adjacency | Yes |
| 2.4 | Portal and Imaginary hyperplane values | Yes |
| 2.5 | Relative position | Yes |

Future Work: The project started with the idea of installing and validating electronic sensors in a building. Using the spatial logic framework one can try to validate their locations in a room to start with and then carry on to a floor and then to a building. Also UML diagrams for other views like the electrical viewpoint, plumbing viewpoint etc can be made. Further work can be done on an approach to validate and verify quantitative requirements of the architectural viewpoint that could not be validated using the spatial logic approach.

- Class notes
- Papers
- A logic-based framework for shape representation by Jose C Damski and John S Gero
- A Referential Scheme for Modeling and Identifying Spatial Attributes of Entities in Constructed Facilities by M. Kiumarse Zamanian \& Steven J. Fenves
- Implementing Topological Predicates for Complex Regions by Marcus Schneider
- A Spatial Logic based on Regions and Connection by David A. Randell, Zhan Cui \& Anthony G. Cohn

