Optimization of Ski Resort Layouts

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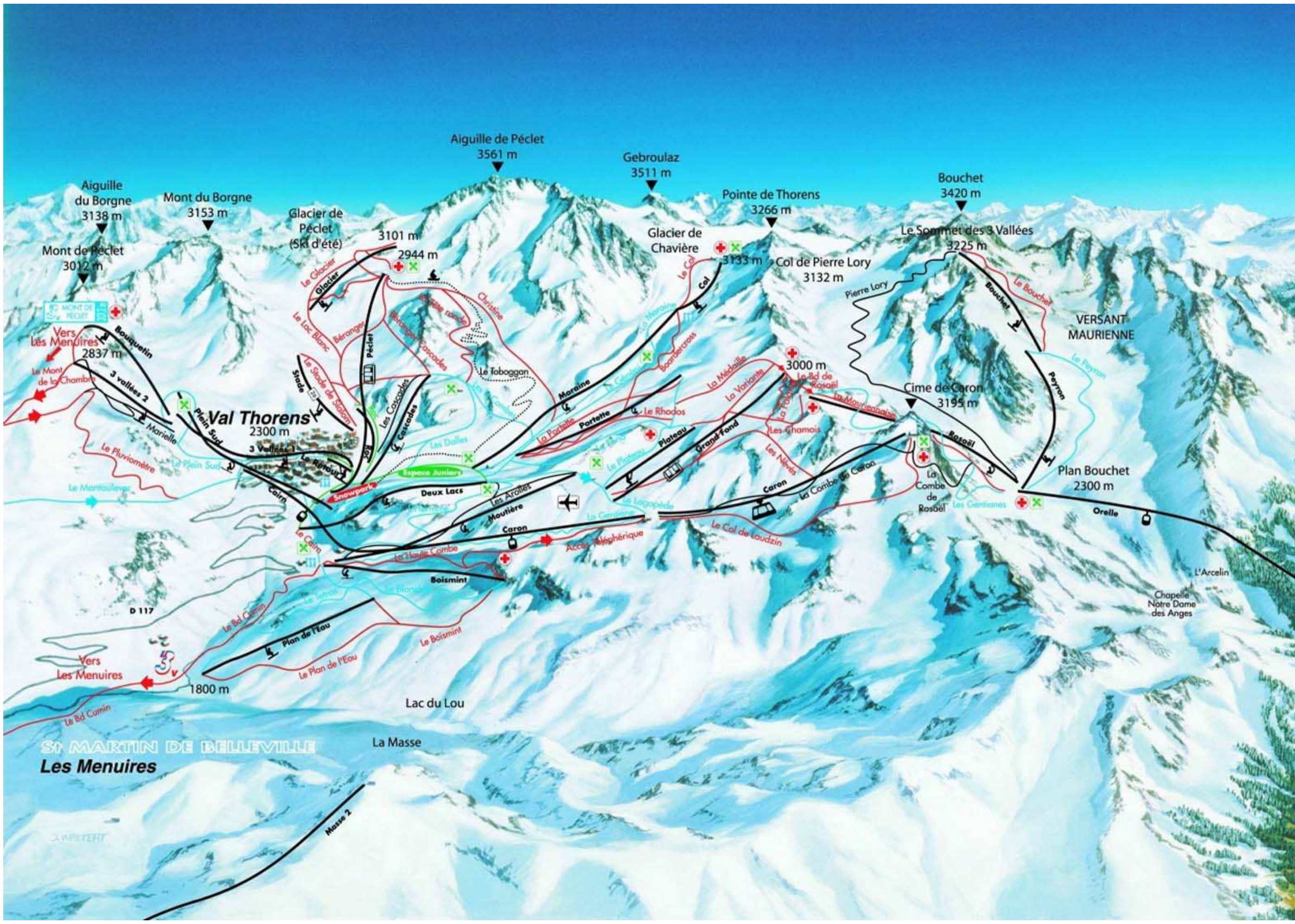


Project Description.

Our project in ENPM 641 focused on a ski resort chairlift as a system, with human actors as riders, operators, and ski patrol. The design optimized the lift for speed, efficiency, cost, etc while maintaining safety. The project, however, focused mainly on the functionality of a single lift.

In ENPM 642, the design expands to optimize a small ski resort containing multiple chairlifts. The objective is to equally disperse skiers and riders around the resort, on both the Runs and lifts to prevent crowding and lengthy lift lines. Each Run is categorized under a specified difficulty level and each lift operates at a constant speed and capacity. Riders are assumed to move at a certain rate down the mountain depending on the difficulty level of the Run. The design is determined through analysis of flow what type and how many lifts best optimize the mountain.

The optimization of the ski resort layout is expanded further in ENPM 643 through the use of full and fractional factorial design. The input parameters of each layout are observed to determine which parameter is most sensitive to the system response. For an existing mountain with a predetermined layout of lifts, a ski resort planner weighs the cost effectiveness of lifts with varying speed, number of seats per chair, and length to make each run. Rather than performing all possible experiments, this model provides a quick and accurate estimate to show which parameter has the greatest effect on increasing the lift and run capacity, which reduces the number of people waiting in the lift lines.

What is Factorial Design?

A full factorial design measures the system response of every possible combination of input variables and levels. These responses are analyzed to provide information about every main effect and interaction effect. Main effects of one independent variable are averaged across the levels of the other independent variables, revealing the sensitivity of the system response to each variable. The sensitivity of the interactions can also be calculated with a full factorial design. The interaction effects show how one independent variable depends upon the level of the other independent variables. A full factorial design is practical when few input variables are investigated. Testing all combinations of levels becomes too expensive and time-consuming with many variables.

When many variables are investigated, fractional factorial designs are useful to produce nearly the same result as full factorial designs, but with fewer experiments. The ASQC (1983) Glossary & Tables for Statistical Quality Control defines fractional factorial design in the following way: "*A factorial experiment in which only an adequately chosen fraction of the treatment combinations required for the complete factorial experiment is selected to be run*" (10). All fractional factorial designs should be balanced and orthogonal to obtain the most accurate results because it eliminates correlation between the estimates of the main effects and interactions. Any suitably chosen fractional factorial design has columns that are all pair-wise orthogonal and sum to zero.

Application of Factorial Design.

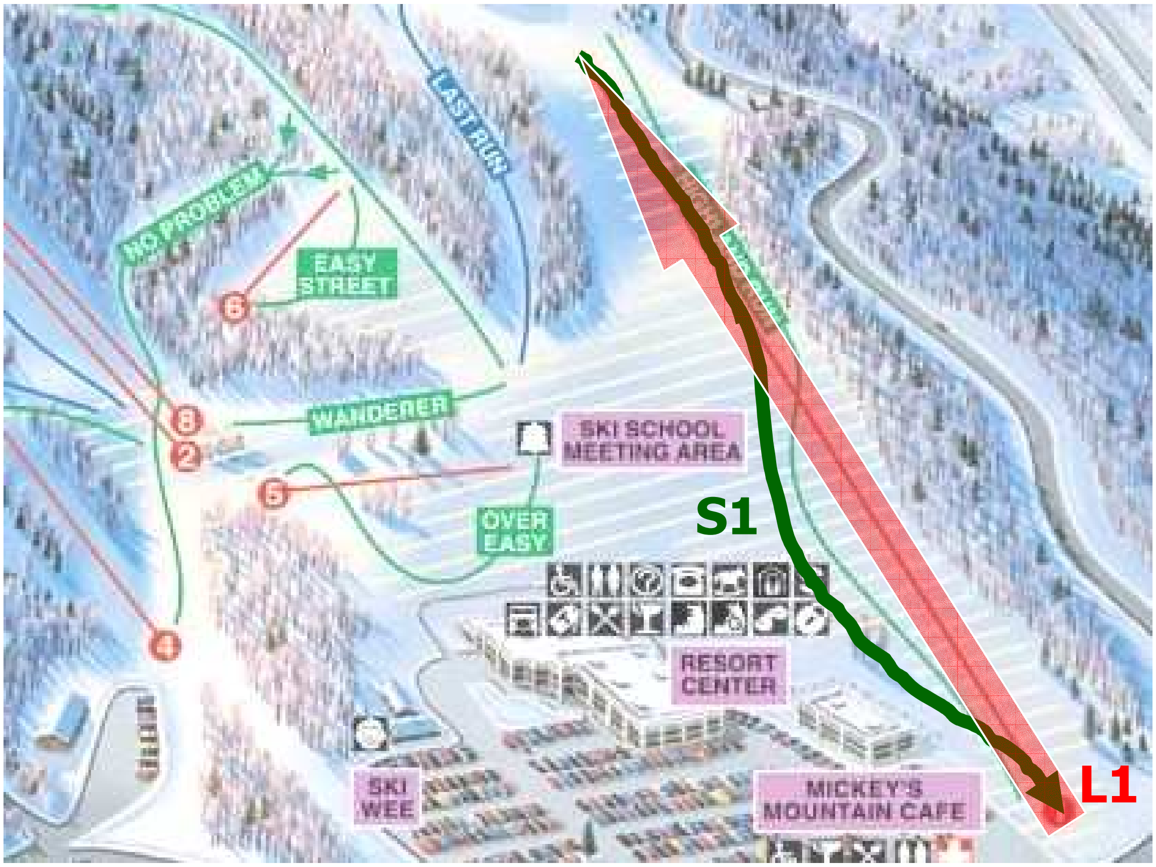
When planning the layout of a ski resort, the optimal design for maximizing lift and run capacity is not obvious without a method of weighing different input variables. It is easy to come up with a system that simply gets people to the top of a mountain and allows them to ski down, but it can be difficult to verify that it is done in the most efficient manner. By using factorial design, we are able to see how the parameters of the ski resort layout affect the overall capacity of the resort. The larger the capacity of a resort, the greater population it can accommodate. The information produced from this process is helpful in making decisions based on cost. When given multiple designs for a particular mountain, factorial design can help the resort planner budget money to maximize lift and run capacity while minimizing cost. For example, if high speed double lifts are less expensive than regular speed quad lifts, the planner may opt to implement a smaller chair size with a high-speed chairlift motor.

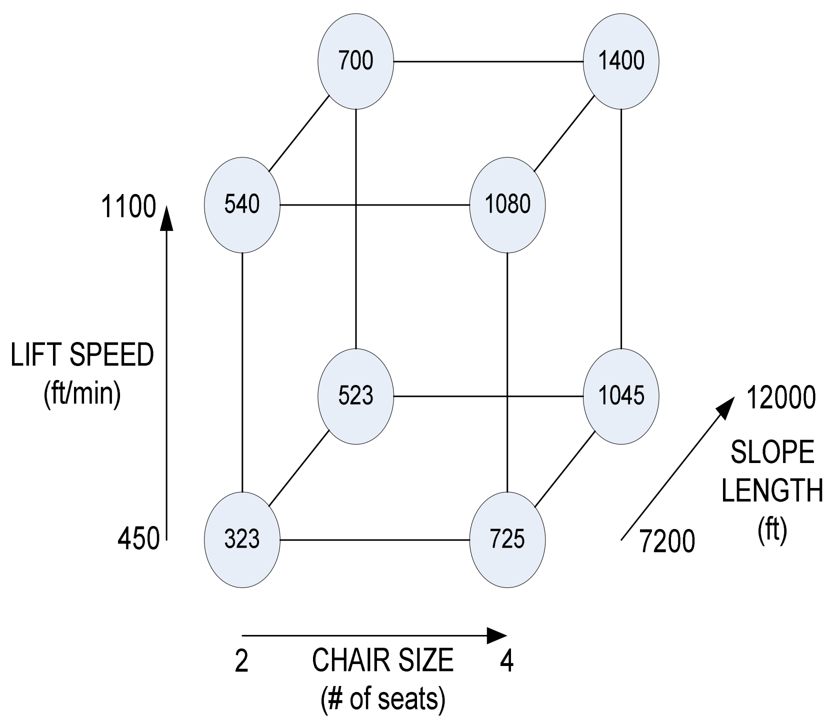
Requirements and Assumptions.

The main objective in optimizing a ski resort is to minimize the time waiting to board a lift. The model used to determine wait time has seven input variables: chair size, chair spacing, lift speed, lift length, difficulty of run, length of run, and resort population. In order to minimize wait time, lift and run capacity must be maximized. Of the seven variables, resort population has no effect on lift or run capacity, chair spacing is generally constant among all types of lifts, lifts generally stretch from the base to the summit, and difficulty of a run, which determines the average speed down a run, is a result of the pitch of its location on the mountain. The remaining three variables (chair size, lift speed, and length of run) were selected as controllable design parameters for a given mountain. Testing these three variables in a factorial design allows the resort planner to maximize total lift and run capacity, which will minimize the wait time for a given population, chair spacing, difficulty of run(s), and lift length.

System Models and Experiments.

- **1 Lift, 1 Run (full factorial design)**


NO PROBLEM
LAST RUN
EASY STREET
6
WANDERER
8
2
5
SKI SCHOOL MEETING AREA
OVER EASY
S1
4
RESORT CENTER
SKI WEE
MICKEY'S MOUNTAIN CAFE
L1


700
1400
540
1080
523
1045
323
725
1100
450
LIFT SPEED (ft/min)
12000
7200
SLOPE LENGTH (ft)
2
4
CHAIR SIZE (# of seats)

Graphical representation of three variable (2^3) full factorial design

Lift 1 Data				Slope 1 Data				
Specification	**Quantity**	**Units**		**Specification**	**Quantity**	**Units**		**Specific**
Lift Chair Size =	2	seats		Type of Slope =	Green			Resort
Lift Chair Spacing =	60	ft		Average Skier Speed =	10	mph		**Slope and Lif**
Lift Speed =	450	ft/min		Converted Skier Speed =	52800	ft/hr		
Converted Lift Speed =	27000	ft/hr		Slope Length =	7200	ft		
Lift Length =	6000	ft		Slope Capacity =	123	people		
Number of Chairs =	100	chairs						Slope Look
Lift Rate =	900	people/hr						Blac
Lift Capacity =	200	seats						Blu
Time Between Chairs =	8	seconds						Gree
Total People Waiting =	77	people						
Wait Time =	5.15	minutes						Chair Size Lo
Lift Cost Factor =	690							1
								2
								3
								4

Model used for calculation of yield for “Test 1” in the three variable (2^3) full factorial design

The model above shows the total slope and lift capacity for the minimum value of each of three input parameters, labeled below as chair size, lift speed, and run length. The slope and lift capacity shown in red above is copied into the main effects calculation below for test 1. The input parameters are varied according to the chart below, where “1” represents the maximum value and “-1” represents the minimum value of each variable. The slope and lift capacity is recorded as the yield for each test.

	min	max	
1 = chair size	2	4	seats
2 = lift speed	450	1100	ft/min
3 = run length	7200	12000	feet

Yield = Resort Capacity = Lift Capacity + Slope Capacity

Test	1	2	3	12	13	23	123	Yield
1	-1	-1	-1	1	1	1	-1	323
2	1	-1	-1	-1	-1	1	1	725
3	-1	1	-1	-1	1	-1	1	540
4	1	1	-1	1	-1	-1	-1	1080
5	-1	-1	1	1	-1	-1	1	523
6	1	-1	1	-1	1	-1	-1	1045
7	-1	1	1	-1	-1	1	-1	700
8	1	1	1	1	1	1	1	1400
Divisor	4	4	4	4	4	4	4	

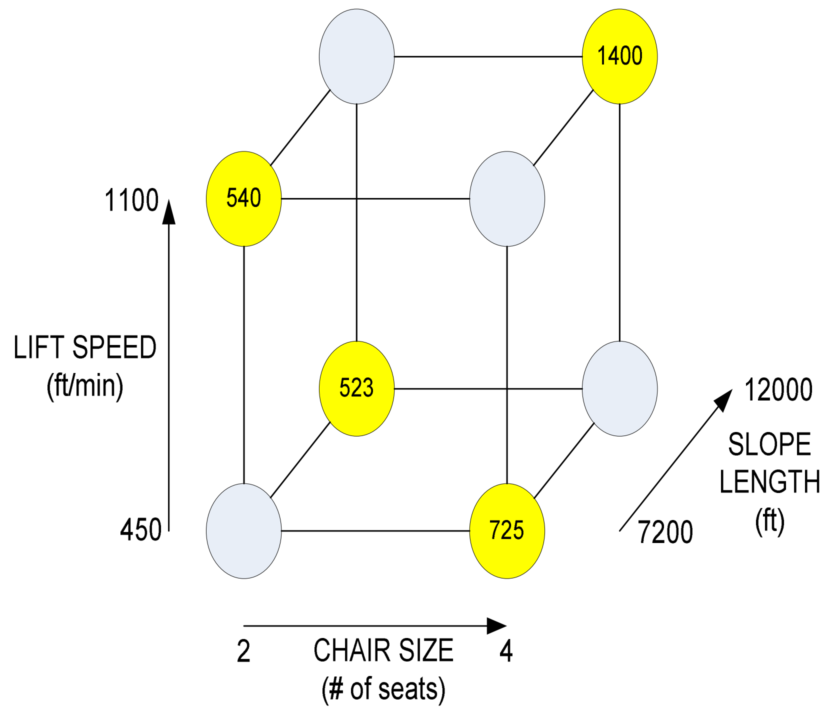
Test	1	2	3	12	13	23	123
1	-323	-323	-323	323	323	323	-323
2	725	-725	-725	-725	-725	725	725
3	-540	540	-540	-540	540	-540	540
4	1080	1080	-1080	1080	-1080	-1080	-1080
5	-523	-523	523	523	-523	-523	523
6	1045	-1045	1045	-1045	1045	-1045	-1045
7	-700	700	700	-700	-700	700	-700
8	1400	1400	1400	1400	1400	1400	1400
sum	2164	1104	1000	316	280	-40	40
E	**541**	**276**	**250**	79	70	-10	10

Most sensitive variable: chair size

Main effects of three variable (2^3) full factorial design

The next step is to determine the main effects and two and three factor interactions for each input variable. Each “1” and “-1” is multiplied by its corresponding yield and the resulting values are summed for each variable. The average of positive values in each column is subtracted from the average of negative values in each column to produce the main effects and interactions. The main effects are shown in bold. The farther away from zero of each main effect, the greater effect the variable has on system response. In the design above, variable 1 (chair size) is farther from zero than variables 2 and 3, so the total lift and run capacity is most sensitive to chair size. The full factorial design shows the effect based on the interaction of the variables. The interactions of variables in the design above do not have as great effect on the system as the individual variables.

1 Lift, 1 Run (1/2 fractional factorial design)



Graphical representation of three variable (2^{3-1}) fractional factorial design

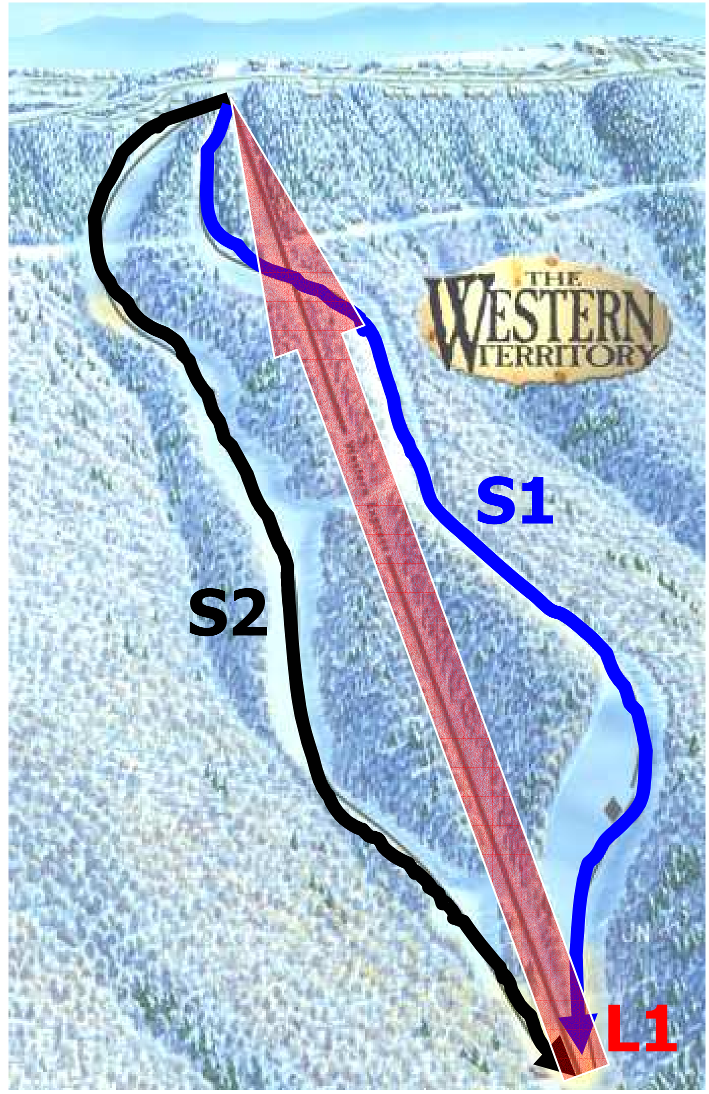
The graphical representation of the half fractional factorial design for the same ski resort layout is shown above. Only half the tests are required, but the tests are selected to make the design balanced and orthogonal, as shown graphically in yellow.

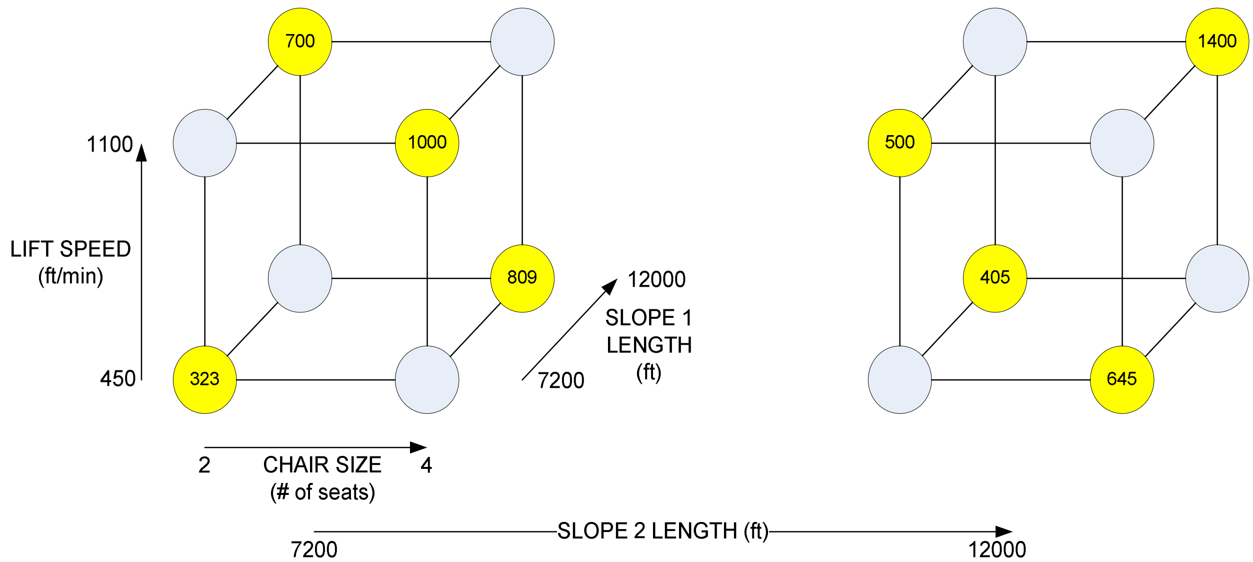
	min	max	
1 = chair size	2	4	seats
2 = lift speed	450	1100	ft/min
3 = run length	7200	12000	feet
Yield = Resort Capacity = Lift Capacity + Slope Capacity			
Test	1	2	3=12	Yield
1	-1	-1	1	523
2	1	-1	-1	725
3	-1	1	-1	540
4	1	1	1	1400
Divisor	2	2	2	
Test	1	2	3=12	
1	-523	-523	523	
2	725	-725	-725	
3	-540	540	-540	
4	1400	1400	1400	
sum	1062	692	658	
E	**531**	**346**	**329**	
Most sensitive variable: chair size				

Main effects of three variable (2^{3-1}) fractional factorial design

Although the fractional factorial design requires only half the tests, the calculations of main effects are performed the same way. The “1” and “-1” values for the third variable are simply the product of the first and second variables making the design orthogonal. Therefore the two and three factor interactions cannot be calculated using the fractional factorial design. The yield for each test is determined from the same model as the full factorial design.

1 Lift, 2 Runs


THE
WESTERN
TERRITORY
S1
S2
L1



Four variable (2^{4-1}) fractional factorial design

The addition of one more run to the model establishes a fourth variable. Instead of calculating all sixteen tests, only the eight shown in yellow were required for the one-half fractional factorial design. Again, the tests were specifically selected to make the design balanced and orthogonal.

	min	max	
1 = chair size	2	4	seats
2 = lift speed	450	1100	ft/min
3 = run1 length	7200	12000	feet
4 = run2 length	7200	12000	feet

Yield = Resort Capacity = Lift Capacity + Slope Capacity

Test	1	2	3	4=123	Yield
1	-1	-1	-1	-1	323
2	1	-1	-1	1	645
3	-1	1	-1	1	500
4	1	1	-1	-1	1000
5	-1	-1	1	1	405
6	1	-1	1	-1	809
7	-1	1	1	-1	700
8	1	1	1	1	1400
Divisor	4	4	4	4	

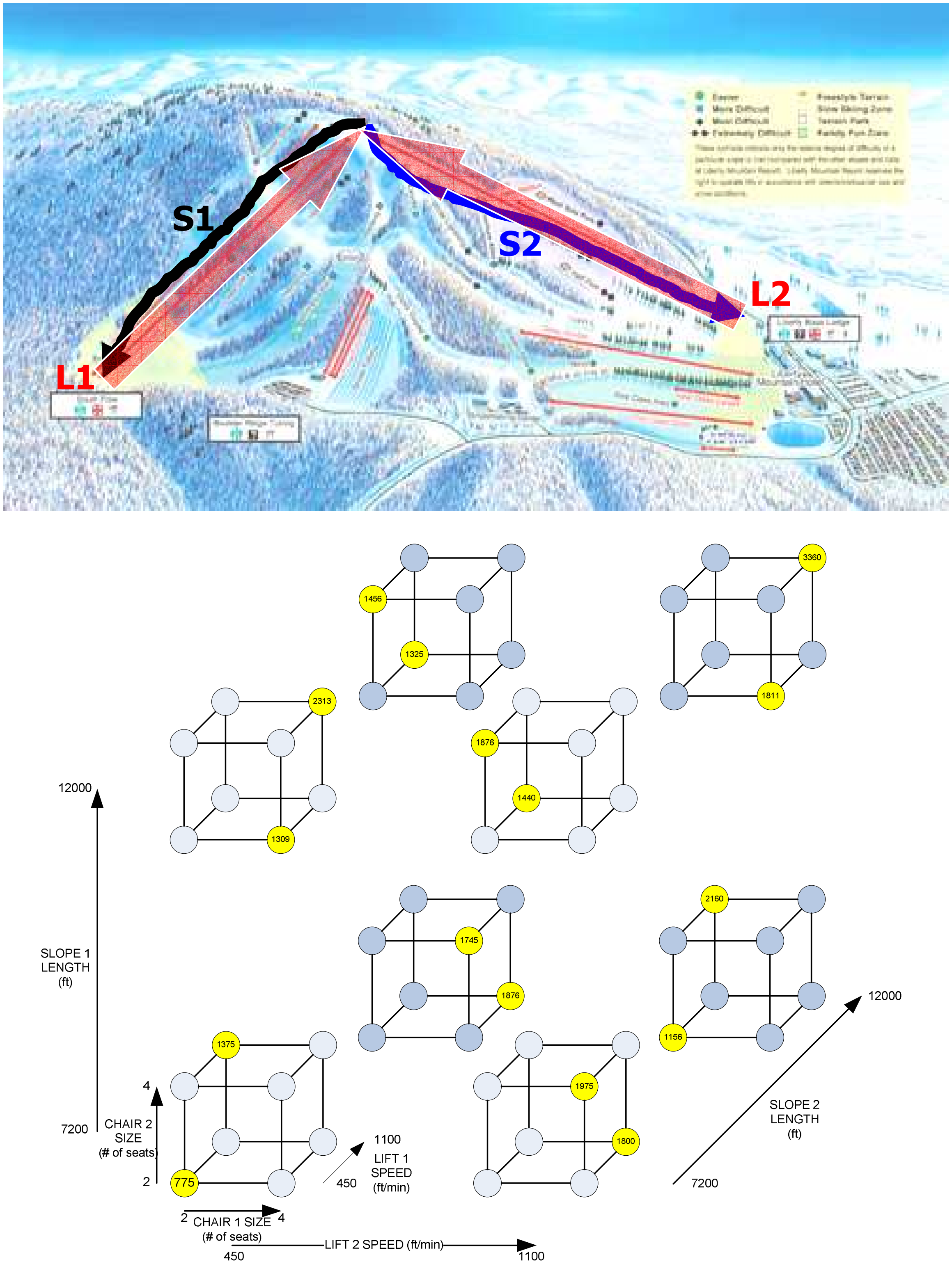
Test	1	2	3	4=123
1	-323	-323	-323	-323
2	645	-645	-645	645
3	-500	500	-500	500
4	1000	1000	-1000	-1000
5	-405	-405	405	405
6	809	-809	809	-809
7	-700	700	700	-700
8	1400	1400	1400	1400
sum	1926	1418	846	118
E	**481.5**	**354.5**	**212**	**29.5**

Most sensitive variable: chair size

Main effects of four variable (2^{4-1}) fractional factorial design

The main effect of the fourth variable is calculated the same as the other three, which were calculated the same as in the previous example.

2 Lifts Converging, 2 Runs



Six variable (2^{6-2}) fractional factorial design

As the number of variables increases, the number of fractional factorial tests selected can remain minimal as long as the design is balanced and orthogonal. The ¼ fractional factorial tests are shown in yellow. Each of the eight cubes shown is positioned at the vertices of one larger cube. Cubes in the back plane have darker shading.

	min	max	
1 = chair1 size	2	4	seats
2 = chair2 size	2	4	seats
3 = lift1 speed	450	1100	ft/min
4 = lift2 speed	450	1100	ft/min
5 = run1 length	7200	12000	feet
6 = run2 length	7200	12000	feet

Yield = Resort Capacity = Lift Capacity + Slope Capacity

Test	1	2	3	4	5=123	6=234	Yield
1	-1	-1	-1	-1	-1	-1	775
2	1	-1	-1	-1	1	-1	1309
3	-1	1	-1	-1	1	1	1456
4	1	1	-1	-1	-1	1	1745
5	-1	-1	1	-1	1	1	1325
6	1	-1	1	-1	-1	1	1876
7	-1	1	1	-1	-1	-1	1375
8	1	1	1	-1	1	-1	2313
9	-1	-1	-1	1	-1	1	1156
10	1	-1	-1	1	1	1	1811
11	-1	1	-1	1	1	-1	1876
12	1	1	-1	1	-1	-1	1975
13	-1	-1	1	1	1	-1	1440
14	1	-1	1	1	-1	-1	1800
15	-1	1	1	1	-1	1	2160
16	1	1	1	1	1	1	3360
Divisor	8	8	8	8	8	8	

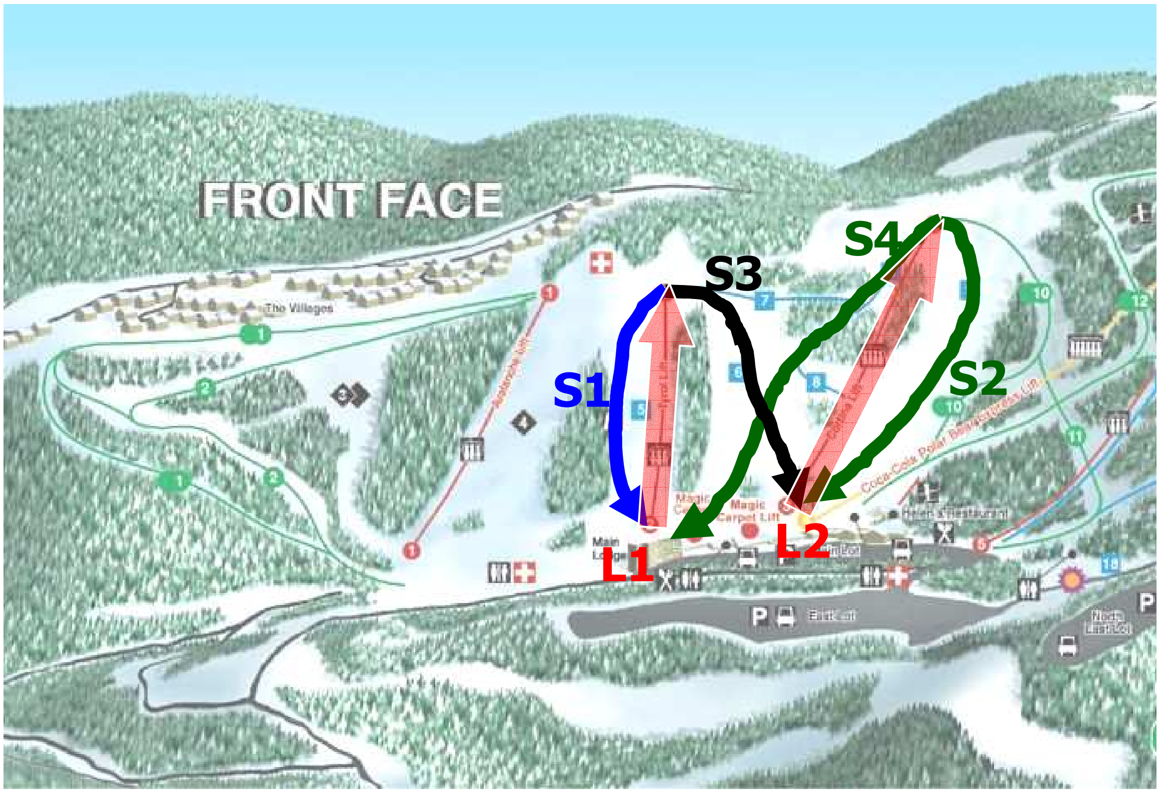
Test	1	2	3	4	5=123	6=234
1	-775	-775	-775	-775	-775	-775
2	1309	-1309	-1309	-1309	1309	-1309
3	-1456	1456	-1456	-1456	1456	1456
4	1745	1745	-1745	-1745	-1745	1745
5	-1325	-1325	1325	-1325	1325	1325
6	1876	-1876	1876	-1876	-1876	1876
7	-1375	1375	1375	-1375	-1375	-1375
8	2313	2313	2313	-2313	2313	-2313
9	-1156	-1156	-1156	1156	-1156	1156
10	1811	-1811	-1811	1811	1811	1811
11	-1876	1876	-1876	1876	1876	-1876
12	1975	1975	-1975	1975	-1975	-1975
13	-1440	-1440	1440	1440	1440	-1440
14	1800	-1800	1800	1800	-1800	-1800
15	-2160	2160	2160	2160	-2160	2160
16	3360	3360	3360	3360	3360	3360
sum	4626	4768	3546	3404	2028	2026
E	**578**	**596**	**443**	**425.5**	**253.5**	**253.3**

Most sensitive variable: chair size

Main effects of six variable (2^{6-2}) fractional factorial design

The column of “1” and “-1” values for variables 5 and 6 of the one-fourth fractional factorial design are carefully chosen products of other variables to make the design balanced and orthogonal.

2 Lifts in Parallel, 2 Runs



	min	max	
1 = chair1 size	2	4	seats
2 = chair2 size	2	4	seats
3 = lift1 speed	450	1100	ft/min
4 = lift2 speed	450	1100	ft/min
5 = run1 length	7200	12000	feet
6 = run2 length	7200	12000	feet
7 = run3 length	7200	12000	feet
8 = run4 length	7200	12000	feet

Yield = Resort Capacity = Lift Capacity + Slope Capacity

Test	1	2	3	4	5=234	6=134	7=123	8=124	Yield
1	-1	-1	-1	-1	-1	-1	-1	-1	627
2	1	-1	-1	-1	-1	1	1	1	1015
3	-1	1	-1	-1	1	-1	1	1	1039
4	1	1	-1	-1	1	1	-1	-1	1353
5	-1	-1	1	-1	1	1	1	-1	878
6	1	-1	1	-1	1	-1	-1	1	1298
7	-1	1	1	-1	-1	1	-1	1	1145
8	1	1	1	-1	-1	-1	1	-1	1587
9	-1	-1	-1	1	1	1	-1	1	878
10	1	-1	-1	1	1	-1	1	-1	1145
11	-1	1	-1	1	-1	1	1	-1	1298
12	1	1	-1	1	-1	-1	-1	1	1587
13	-1	-1	1	1	-1	-1	1	1	960
14	1	-1	1	1	-1	1	-1	-1	1320
15	-1	1	1	1	1	-1	-1	-1	1320
16	1	1	1	1	1	1	1	1	2160
Divisor	8	8	8	8	8	8	8	8	

Test	1	2	3	4	7=123	8=124	6=134	5=234
1	-627	-627	-627	-627	-627	-627	-627	-627
2	1015	-1015	-1015	-1015	-1015	1015	1015	1015
3	-1039	1039	-1039	-1039	1039	-1039	1039	1039
4	1353	1353	-1353	-1353	1353	1353	-1353	-1353
5	-878	-878	878	-878	878	878	878	-878
6	1298	-1298	1298	-1298	1298	-1298	-1298	1298
7	-1145	1145	1145	-1145	-1145	1145	-1145	1145
8	1587	1587	1587	-1587	-1587	-1587	1587	-1587
9	-878	-878	-878	878	878	878	-878	878
10	1145	-1145	-1145	1145	1145	-1145	1145	-1145
11	-1298	1298	-1298	1298	-1298	1298	1298	-1298
12	1587	1587	-1587	1587	-1587	-1587	-1587	1587
13	-960	-960	960	960	-960	-960	960	960
14	1320	-1320	1320	1320	-1320	1320	-1320	-1320
15	-1320	1320	1320	1320	1320	-1320	-1320	-1320
16	2160	2160	2160	2160	2160	2160	2160	2160
sum	3320	3368	1726	1726	532	484	554	554
E	**415**	**421**	**215.75**	**215.8**	**66.5**	**60.5**	**69.25**	**69.25**

Most sensitive variable: chair size

Main effects of eight variable (2^{8-4}) fractional factorial design

The graphical representation of an eight variable factorial design would have 32 cubes with 256 total vertices in the formation of a much larger set of cubes. The full factorial design of this magnitude would be cumbersome to compute. The one-eighth fractional factorial design requires only sixteen tests. This reduces experimentation time, yet still produces a reasonably accurate determination of the system sensitivity.

Summary of Results.

VARIABLES	1L, 1R (full)	1L, 1R (fract)	1L, 2R	2LC, 2R	2L, 4R
Chair 1 Size	**541**	**531**	**481.5**	**578.25**	**415**
Chair 2 Size	N/A	N/A	N/A	**596**	**421**
Lift 1 Speed	**276**	**346**	**354.5**	**443.25**	**215.75**
Lift 2 Speed	N/A	N/A	N/A	**425.5**	**215.75**
Slope 1 Length	**250**	**329**	**211.5**	**253.5**	**66.5**
Slope 2 Length	N/A	N/A	**29.5**	**253.25**	**60.5**
Slope 3 Length	N/A	N/A	N/A	N/A	**69.25**
Slope 4 Length	N/A	N/A	N/A	N/A	**69.25**
TOTAL VARIABLES	3	3	4	6	8

Comparison of main effects for each ski resort layout

Four ski resort layouts were chosen to study the system response of independent input variables. Each factorial design includes two input variables for each lift and one for each run, so the factorial design is more complex as the number of input variables increases and the resort grows. The numbers in circles on the geometric representation are lift and run capacities associated with specific test conditions. The main effects for each variable represent the average change of the system response due to a change in the variable.

For a resort with one lift and one run, the simplest model, the system response for the three input variables is calculated in two separate designs for comparison. The first uses a full factorial design, which is more accurate than a fractional factorial design, but shows nearly the same results. The conclusion is the same for both designs, indicating chair size is far more sensitive than lift speed, which is slightly more sensitive than run length.

When a second run is added to the layout, the model becomes slightly more complex by adding a fourth variable for the length of the second run. Since the results of the fractional factorial design for the previous model produce nearly the same results as the full factorial, a one-half fractional factorial design was chosen again. A full factorial design would produce sixteen rows, or require sixteen experiments, whereas the fractional design only requires eight.

The results are the same after adding a second run, in that chair size is more sensitive than speed of the lift, which is more sensitive than the length of each run.

Six variables are introduced in the model with two lifts converging and two runs. A 2^6 full factorial requires 64 experiments, so a ¼ fractional factorial is chosen requiring only sixteen experiments. Again the order of sensitivity of each variable is equal to the previous models, with the number of seats per chair having the greatest impact on system response.

When testing a layout with two lifts and four runs, eight variables affect the system response. Since 2^8 equals 256 experiments, a full factorial is extremely complex to calculate by hand. Therefore a 1/8 fractional factorial design is selected with only sixteen experiments. Chair size is again the most sensitive variable, followed by lift speed, and length of the run, which has very minimal effect in comparison to the other variables.

Conclusions.

When designing a ski resort on a given mountain, the length of each lift is usually predetermined to be as long as possible and the type of each slope is dependent on its desired location. In addition, the spacing between chairs is usually set at sixty feet and the resort population is desirably as large as possible. Therefore, chair sizes, lift speeds, and run lengths remain as independent input variables. These variables are evaluated with cost to maximize the lift and run capacity for the lowest possible cost. Our results show that for the four ski resort layouts tested, the number of seats on each chair is the most sensitive variable for increasing lift and run capacity. If there is a choice between a high-speed double lift and a regular quad lift, the regular quad lift would yield a greater lift and run capacity. Chair size is significantly more sensitive than the speed of the lifts, which is also significantly more sensitive than the length of each run. The cost of implementing each variable is not taken into account, even though it is often the most influential factor in business decisions. The sensitivity of variables determined in each design can be factored with the cost of implementation to determine the most efficient way to design a ski resort.

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