# Why Be Backward?

# Forward Evolution Equations for Barrier Options

Ali Hirsa
Morgan Stanley

with Peter Carr of the Courant Institute, NYU

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## Definitions

A European (vanilla) call option is an option that gives the holder the right to buy stock at a specified price (the **strike** price) at the expiration date (**maturity**).

A knock-out call option is the same as vanilla if it has not knocked out (the stock has not reached a specified level, **barrier**, throughout the life of the option.)

In the case of knocking from below, it is called **Up-and-Out**.

In the case of knocking from above, it is called **Down-and-Out**.

## Motivation

Propagating option prices in the maturity and/or strike directions

Enhances computational efficiency of calibration

Promotes computational efficiency in marking

# Outline

Review of the Dupire PDE

Markovian Stock Price Process

Backward PIDE for **Up-and-Out** and **Down-and-Out** Calls

Forward PIDE for **Down-and-Out** Calls

Forward PIDE for **Up-and-Out** Calls

## Outline (Cont'd)

Numerical Examples

Future Work

## Backward PDE for European Calls

Assuming that the stock price process follows the following

$$ds_t = [r(t) - q(t)]\, s_t dt + \sigma(s_t, t) s_t dW_t$$

Applying Ito's Lemma to show that $c(S,t)$ solves:

$$\frac{\partial c(S,t)}{\partial t} + \frac{\sigma^2(S,t)}{2} S^2 \frac{\partial^2 c(S,t)}{\partial S^2} + [r(t) - q(t)] S \frac{\partial c(S,t)}{\partial S} = r(t) c(S,t)$$

$$\begin{aligned}
c(S, T_0) &= (S - K_0)^+, \quad && S \in [0, \infty) \\
\lim_{S \downarrow 0} \frac{\partial^2}{\partial S^2} c(S,t) &= 0, && t \in [0, T_0] \\
\lim_{S \uparrow \infty} \frac{\partial^2}{\partial S^2} c(S,t) &= 0, && t \in [0, T_0].
\end{aligned}$$

## The Dupire PDE

As a starting point, we look at the Dupire PDE

$$\frac{\partial c}{\partial T} = \frac{\sigma^2(K,T)}{2} K^2 \frac{\partial^2 c}{\partial K^2} - [r(T) - q(T)]\, K \frac{\partial c}{\partial K} - q(T) c(K,T)$$

By having the market call prices, $c(K,T)$, the local volatility surface, $\sigma(K,T)$, can be calculated.

Or by having the local volatility surface, $\sigma(K,T)$, one can compute call prices, $c(K,T)$, for all strikes and maturities subject to:

$$\begin{aligned} c(K,0) &= (S_0 - K)^+, \quad K \in [0,\infty) \\ \lim_{K \downarrow 0} \frac{\partial^2}{\partial K^2} c(K,T) &= 0, \quad T \in [0,\bar{T}], \\ \lim_{K \uparrow \infty} \frac{\partial^2}{\partial K^2} c(K,T) &= 0, \quad T \in [0,\bar{T}]. \end{aligned}$$

## Markovian Stock Price Process

We assume that under a risk neutral measure $\mathbb{Q}$, the stock price $s_t$ satisfies the following stochastic differential equation:

$$ds_t = [r(t) - q(t)]\, s_t dt + \sigma(s_t, t) s_t dW_t + \int_{-\infty}^{\infty} s_t(e^x - 1)\, [\mu(dx, dt) - \nu(x, t) dx dt]\,,$$

for all $t \in [0, \bar{T}]$. Thus, the change in the stock price decomposes into three parts. The risk-neutral drift, the diffusion part, the jump part.

The random measure $\mu(dx, dt)$ counts the number of jumps of size $x$ in the log price at time $t$.

## Markovian Stock Price Process (cont'd)

The Lévy density $\{\nu(x,t), x \in \mathbb{R}, t \in [0,\bar{T}]\}$ is used to compensate the jump process

$$J_t \equiv \int_0^t \int_{-\infty}^{\infty} s_t(e^x - 1)\mu(dx, dt)$$

so that the last term is the increment of a $\mathbb{Q}$ jump martingale. Thus

$$\mathbb{E}_{\mathbb{Q}}[s_t|s_0] = s_0 e^{\int_0^t [r(u)-q(u)]du}.$$

## Backward PIDE for European Calls

$$\frac{a^2(S,t)}{2}\frac{\partial^2 c(S,t)}{\partial S^2} + \int_{-\infty}^{\infty}\left[c(Se^x,t) - c(S,t) - \frac{\partial}{\partial S}c(S,t)S(e^x-1)\right]\nu(x,t)dx$$
$$+[r(t)-q(t)]S\frac{\partial c(S,t)}{\partial S} - r(t)c(S,t) + \frac{\partial c(S,t)}{\partial t} = 0$$

*A fortiori*, the European call value function $c(S,t)$ solves a backward boundary value problem (BVP), consisting of the backward PIDE subject to the following boundary conditions:

$$\begin{aligned} c(S,T_0) &= (S-K_0)^+, && S \in [0,\infty) \\ \lim_{S\downarrow 0} c(S,t) &= 0, && t \in [0,T_0] \\ \lim_{S\uparrow\infty} \tfrac{\partial^2}{\partial S^2} c(S,t) &= 0, && t \in [0,T_0]. \end{aligned}$$

## Backward PIDE for Down-and-Out Calls

$$\frac{a^2(S,t)}{2}\frac{\partial^2 D(S,t)}{\partial S^2} + \int_{-\infty}^{\infty} \left[ D(Se^x,t) - D(S,t) - \frac{\partial}{\partial S} D(S,t) S(e^x - 1) \right] \nu(x,t)dx$$
$$+[r(t) - q(t)]S\frac{\partial D(S,t)}{\partial S} - r(t)D(S,t) + \frac{\partial D(S,t)}{\partial t} = 0$$

*A fortiori*, the down-and-out call value function $D(S,t)$ solves a backward boundary value problem (BVP), consisting of the backward PIDE subject to the following boundary conditions:

$$\begin{aligned} D(S,T_0) &= (S-K_0)^+, & S &\in [H,\infty) \\ \lim_{S\downarrow H} D(S,t) &= 0, & t &\in [0,T_0] \\ \lim_{S\uparrow\infty} \tfrac{\partial^2}{\partial S^2} D(S,t) &= 0, & t &\in [0,T_0]. \end{aligned}$$

## Backward PIDE for Up-and-Out Calls

$$\frac{a^2(S,t)}{2}\frac{\partial^2 U(S,t)}{\partial S^2} + \int_{-\infty}^{\infty} \left[ U(Se^x,t) - U(S,t) - \frac{\partial}{\partial S}U(S,t)S(e^x-1) \right] \nu(x,t)dx$$

$$+[r(t)-q(t)]S\frac{\partial U(S,t)}{\partial S} - r(t)U(S,t) + \frac{\partial U(S,t)}{\partial t} = 0$$

*A fortiori*, the up-and-out call value function $U(S,t)$ solves a backward boundary value problem (BVP), consisting of the backward PIDE subject to the following boundary conditions:

$$\begin{aligned} U(S,T_0) &= (S-K_0)^+, & S &\in [0,H] \\ \lim_{S\downarrow 0} U(S,t) &= 0, & t &\in [0,T_0] \\ \lim_{S\uparrow H} U(S,t) &= 0, & t &\in [0,T_0]. \end{aligned}$$

# Forward PIDE for European Calls

$$
\begin{aligned}
\frac{\partial}{\partial T}c(K,T) &= \frac{a^2(K,T)}{2}\frac{\partial^2}{\partial K^2}c(K,T) - [r(T)-q(T)]K\frac{\partial}{\partial K}c(K,T) - q(T)c(K,T) \\
&+ \int_{-\infty}^{\infty}\left[c(Ke^{-x},T) - c(K,T) - \frac{\partial}{\partial K}c(K,T)K(e^{-x}-1)\right]e^{x}\nu(x,T)dy.
\end{aligned}
$$

Boundary conditions are:

$$
\begin{aligned}
c(K,0) &= (S_0-K)^+, \quad K\in[0,\infty), \\
\lim_{K\downarrow 0}\frac{\partial^2}{\partial K^2}c(K,T) &= 0, \quad T\in[0,\bar{T}], \\
\lim_{K\uparrow \infty}\frac{\partial^2}{\partial K^2}c(K,T) &= 0, \quad T\in[0,\bar{T}].
\end{aligned}
$$

# Forward PIDE for Down-and-Out Calls

$$
\begin{aligned}
\frac{\partial}{\partial T} D_0^c(K,T) &= \frac{a^2(K,T)}{2}\frac{\partial^2}{\partial K^2} D_0^c(K,T) - [r(T)-q(T)]K\frac{\partial}{\partial K}U_0^c(K,T) - q(T)D_0^c(K,T) \\
&+ \int_{-\infty}^{\infty}\left[D_0^c(Ke^{-x},T) - D_0^c(K,T) - \frac{\partial}{\partial K}D_0^c(K,T)K(e^{-x}-1)\right] e^x \nu(x,T)dy.
\end{aligned}
$$

Boundary conditions are:

$$
\begin{aligned}
D_0^c(K,0) &= (S_0-K)^+, \quad K \in [H,\infty) \quad \text{and} \quad H < S_0, \\
\frac{\partial^2}{\partial K^2} D_0^c(H,T) &= 0, \quad T \in [0,\bar{T}], \\
\lim_{K\uparrow\infty} \frac{\partial^2}{\partial K^2} D_0^c(K,T) &= 0, \quad T \in [0,\bar{T}].
\end{aligned}
$$

# Forward PIDE for Up-and-Out Calls

$$
\begin{aligned}
\frac{\partial}{\partial T}U_0^c(K,T) &= \frac{a^2(K,T)}{2}\frac{\partial^2}{\partial K^2}U_0^c(K,T) - [r(T)-q(T)]K\frac{\partial}{\partial K}U_0^c(K,T) - q(T)U_0^c(K,T) \\
&+ \int_{-\infty}^{\infty}\left[U_0^c(Ke^{-x},T) - U_0^c(K,T) - \frac{\partial}{\partial K}U_0^c(K,T)K(e^{-x}-1)\right]e^x\nu(x,T)dy \\
&+ (H-K)\frac{a^2(H,T)}{2}\frac{\partial^3}{\partial K^3}U_0^c(H,T) \\
&+ (H-K)\int_{0+}^{\infty}\frac{\partial}{\partial K}U_0^c(He^{-x},T)\nu(x,T)dy - \int_{0+}^{\infty}U_0^c(He^{-x},T)e^x\nu(x,T)dy
\end{aligned}
$$

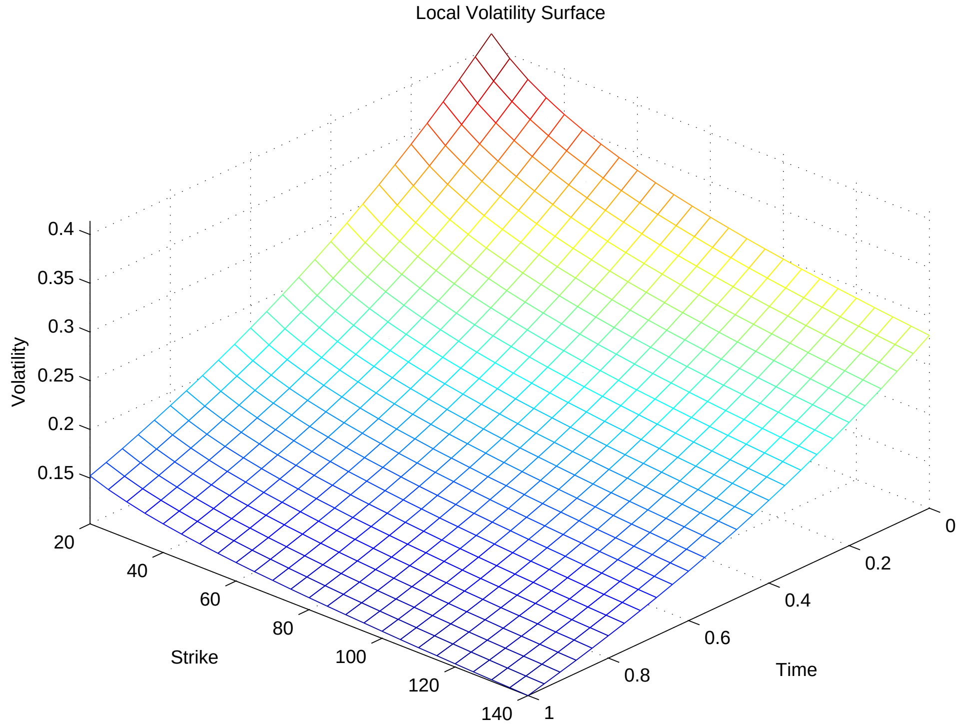
Boundary conditions are:

$$
\begin{aligned}
U_0^c(K,0) &= (S_0-K)^+, \quad K\in[0,H) \quad \text{and} \quad H>S_0, \\
\lim_{K\downarrow 0}\frac{\partial^2}{\partial K^2}U_0^c(K,T) &= 0, \quad T\in[0,\bar{T}], \\
\frac{\partial^2}{\partial K^2}U_0^c(H,T) &= 0, \quad T\in[0,\bar{T}].
\end{aligned}
$$

## Local Volatility Surface

In our numerical examples, we consider the following local volatility surface

$$\sigma(K, T) = 0.3e^{-T}(100/K)^{0.2}$$



## The Variance Gamma Process

In our numerical examples, $\nu(x)dx$ is the *Lévy density* for the VG process in the following form

$$\nu(x) = \frac{e^{-\lambda_p x}}{\nu x} \text{ for } x > 0 \quad \text{and} \quad \nu(x) = \frac{e^{-\lambda_n |x|}}{\nu |x|} \text{ for } x < 0$$

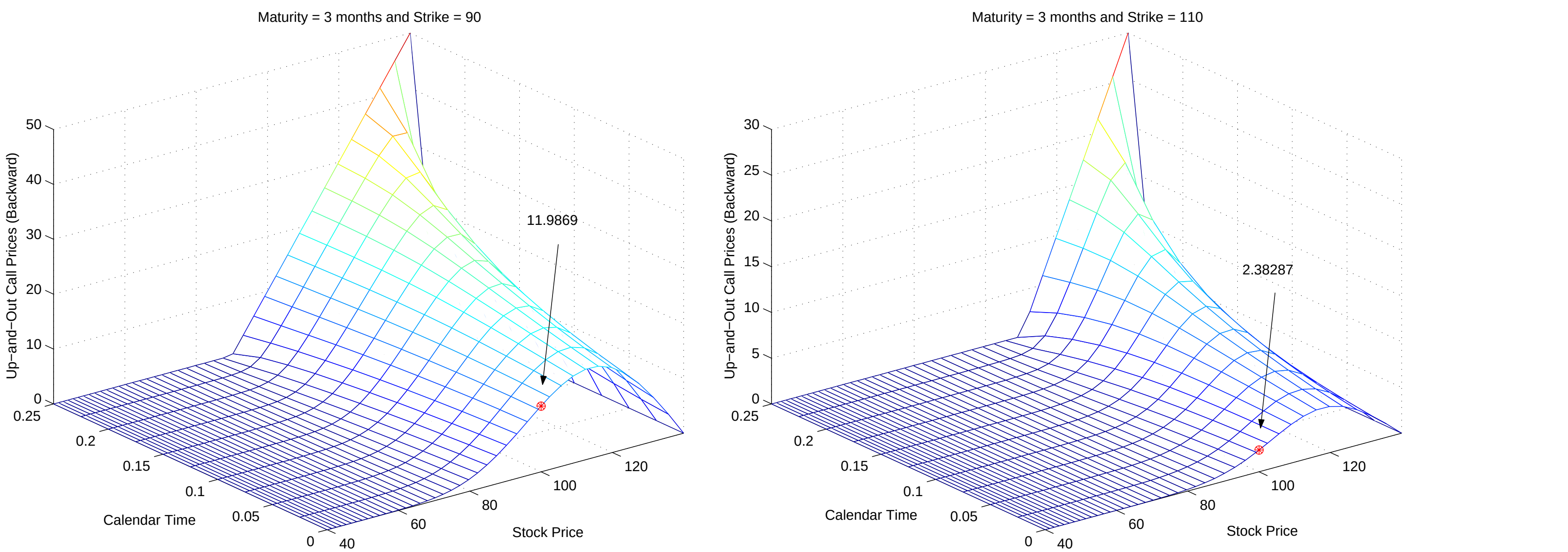
and

$$\lambda_p = \left(\frac{\theta^2}{\sigma^4} + \frac{2}{\sigma^2 \nu}\right)^{\frac{1}{2}} - \frac{\theta}{\sigma^2} \qquad \lambda_n = \left(\frac{\theta^2}{\sigma^4} + \frac{2}{\sigma^2 \nu}\right)^{\frac{1}{2}} + \frac{\theta}{\sigma^2}.$$

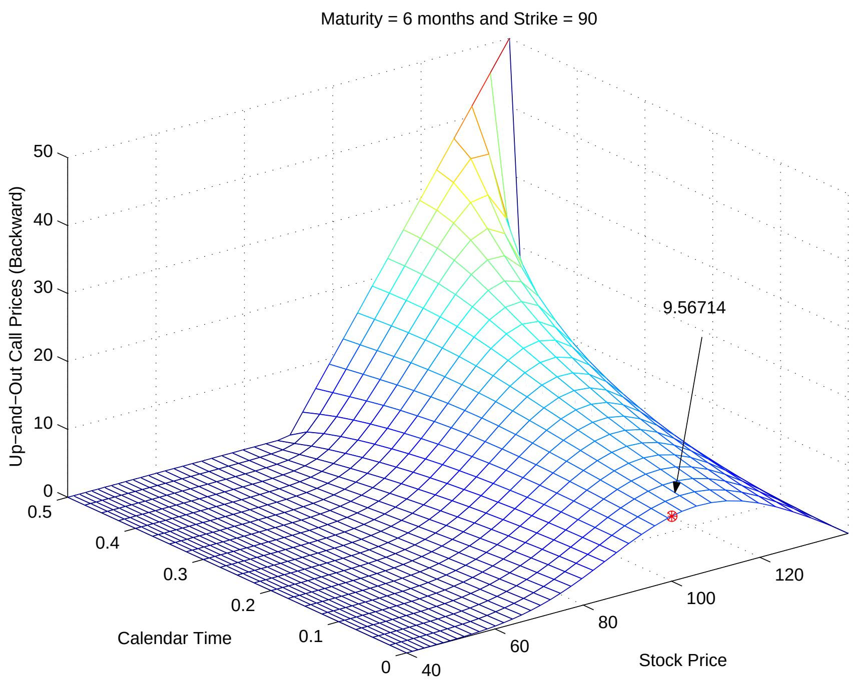
where $\sigma$, $\nu$, and $\theta$ are VG parameters.

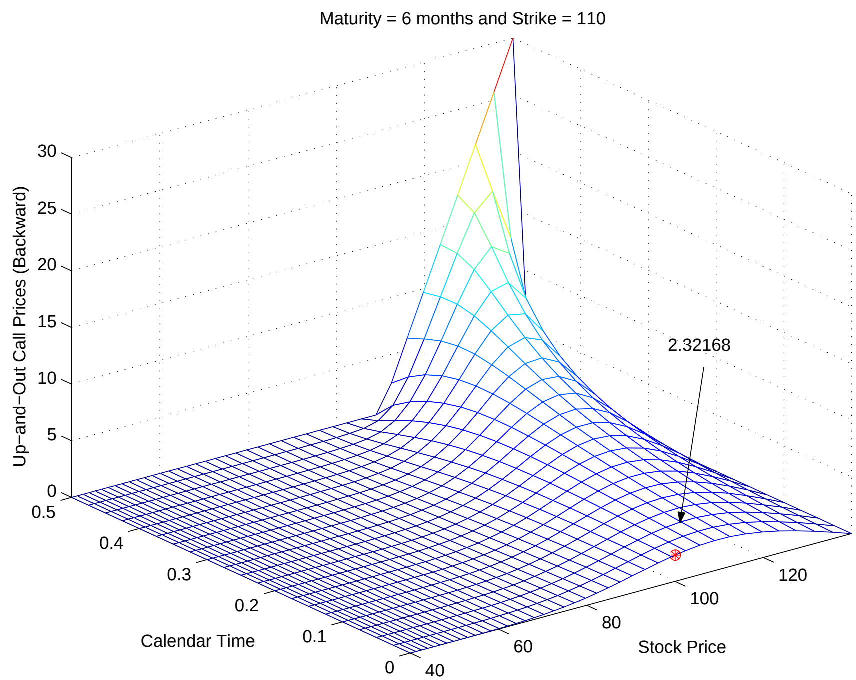
## Illustration 1(a): Up-and-Out Call Prices (Backward)

The variables are: spot $S_0$=100, Up-Barrier $H = 140$, risk-free rate $r = 0.06$, dividend rate $q = .02$, and VG parameters $\sigma = 0.3$, $\nu = 0.25$, $\theta = -0.3$.

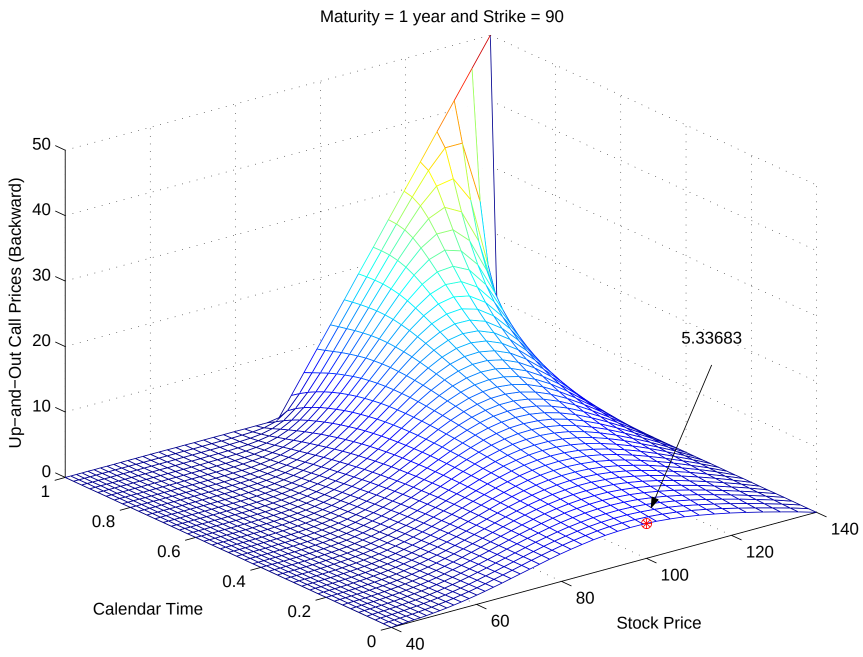


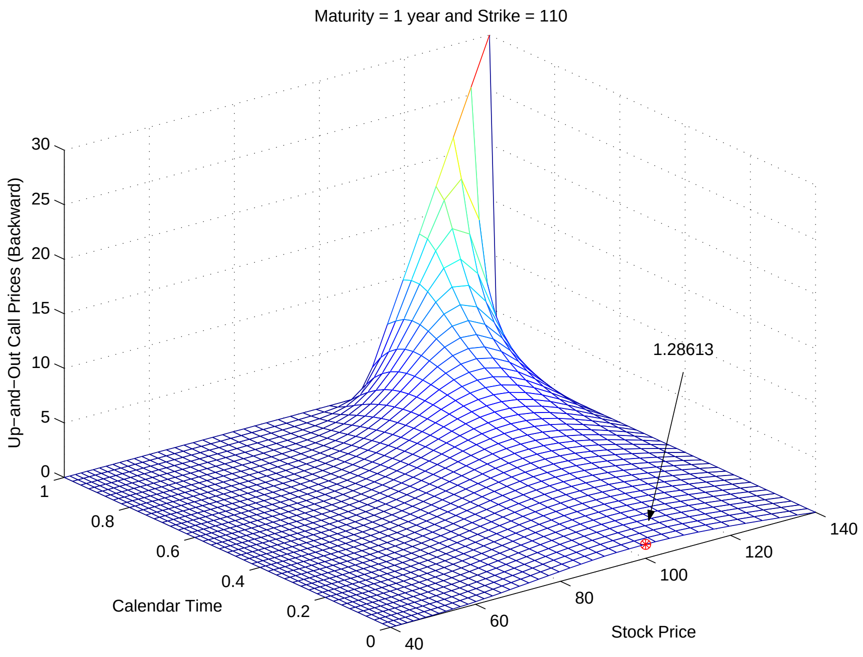
# Illustration 1(b): Up-and-Out Call Prices (Backward)



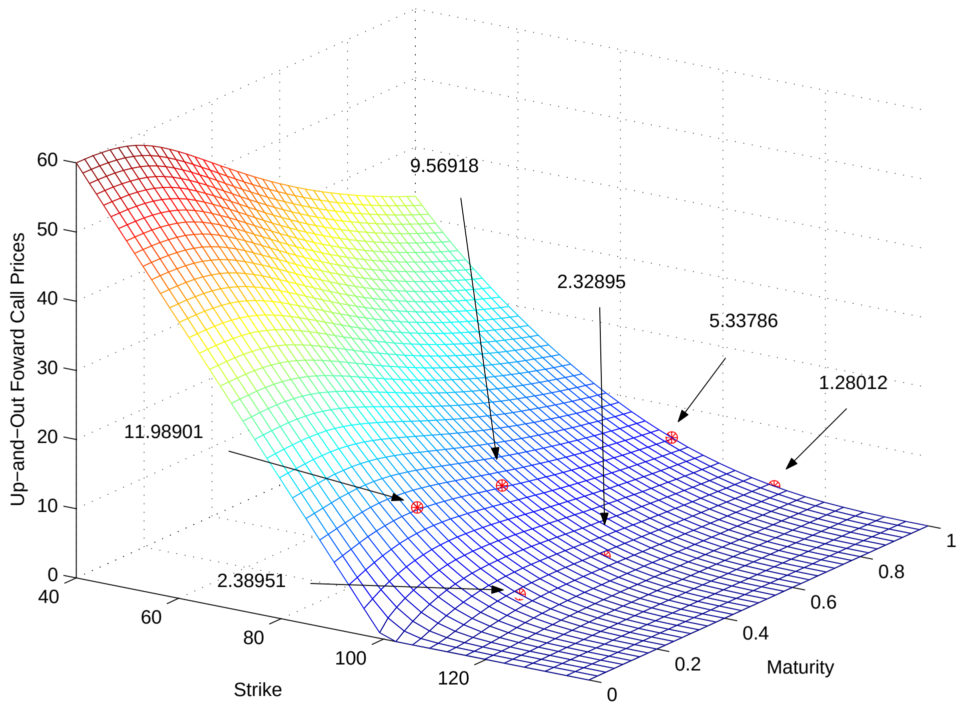
# Illustration 1(c): Up-and-Out Call Prices (Backward)



# Illustration 2: Up-and-Out Call Prices (Forward)

The variables are: spot $S_0$=100, Up-Barrier $H = 140$, risk-free rate $r = 0.06$, dividend rate $q = .02$, and VG parameters $\sigma = 0.3$, $\nu = 0.25$, $\theta = -0.3$.

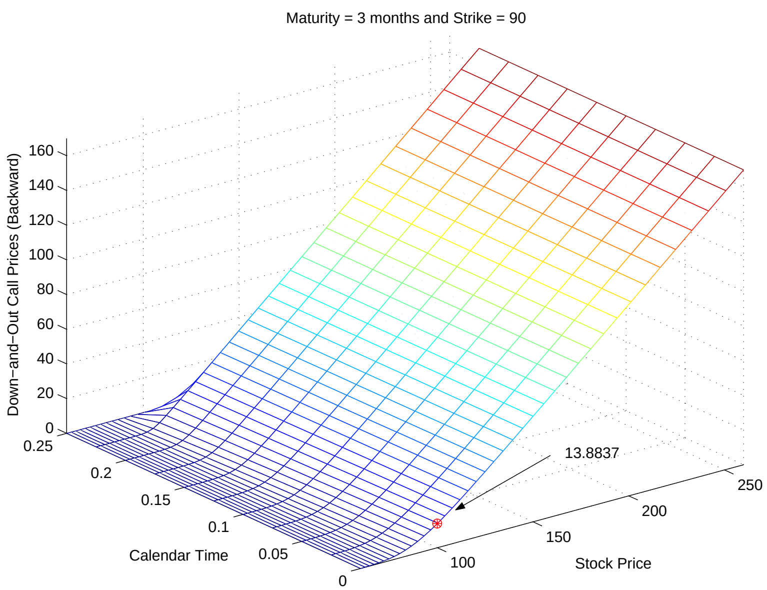


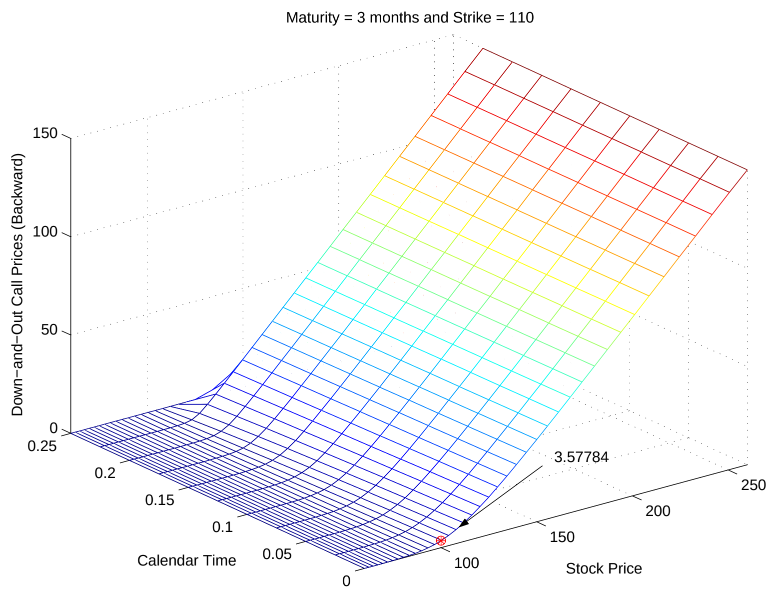
# Up-and-Out Call Prices (Forward vs. Backward)

| Maturity | | $T_1 = 0.25$ | | $T_2 = 0.5$ | | $T_3 = 1.0$ | |
|---|---|---|---|---|---|---|---|
| Barrier | Strike | **Bwd** | *Fwd* | **Bwd** | *Fwd* | **Bwd** | *Fwd* |
| 140 | 90 | **11.9869** | 11.98901 | **9.56714** | 9.56918 | **5.33683** | 5.33786 |
| | 110 | **2.38287** | 2.38951 | **2.32168** | 2.32895 | **1.28613** | 1.28012 |

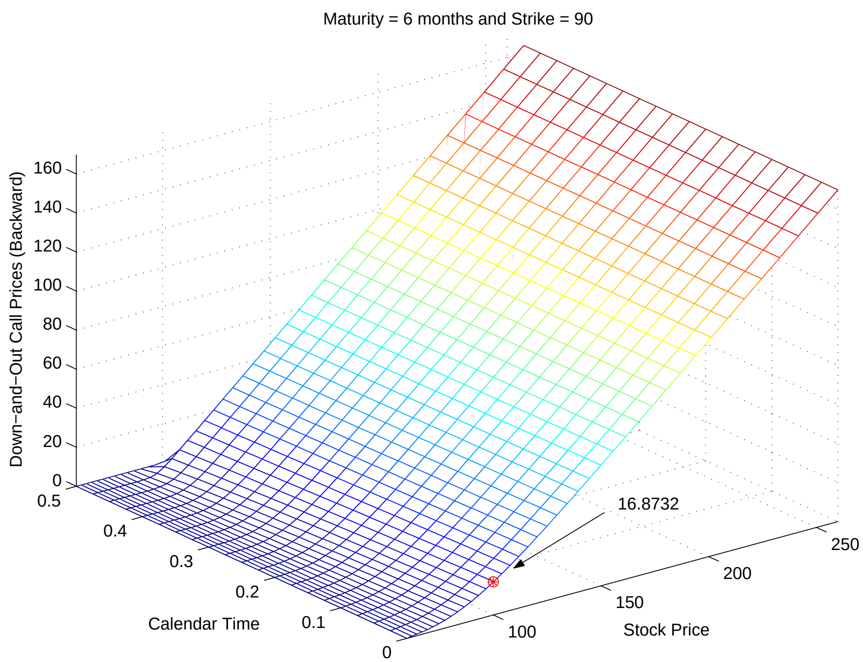
## Illustration 3(a): Down-and-Out Call Prices (Backward)

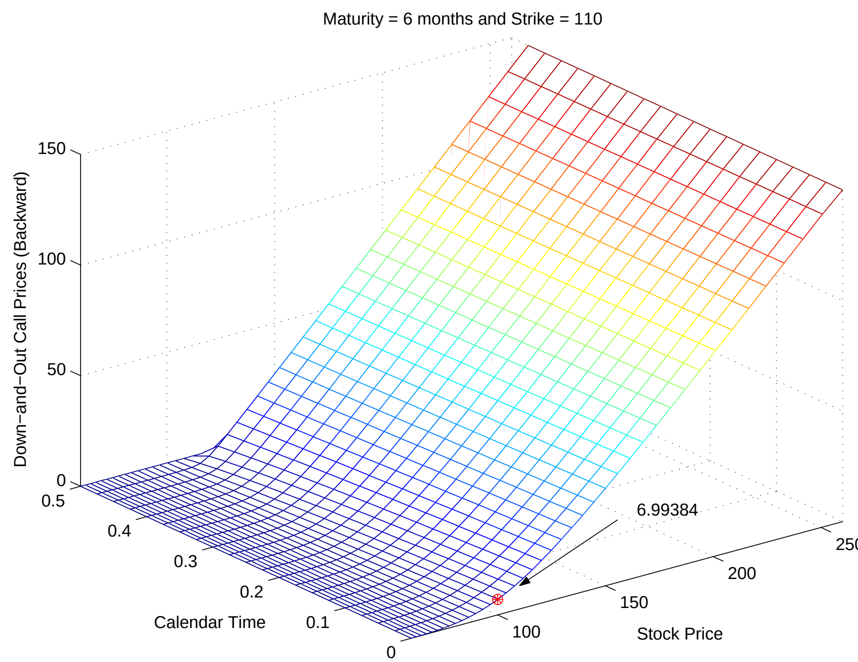
The variables are: spot $S_0$=100, Down-Barrier $H = 60$, risk-free rate $r = 0.06$, dividend rate $q = .02$, and VG parameters $\sigma = 0.3$, $\nu = 0.25$, $\theta = -0.3$.



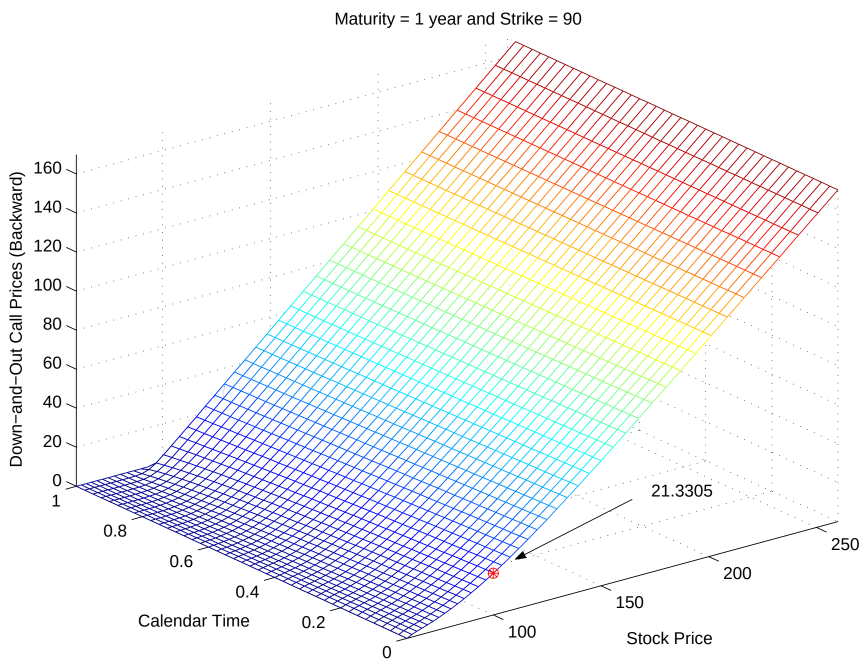



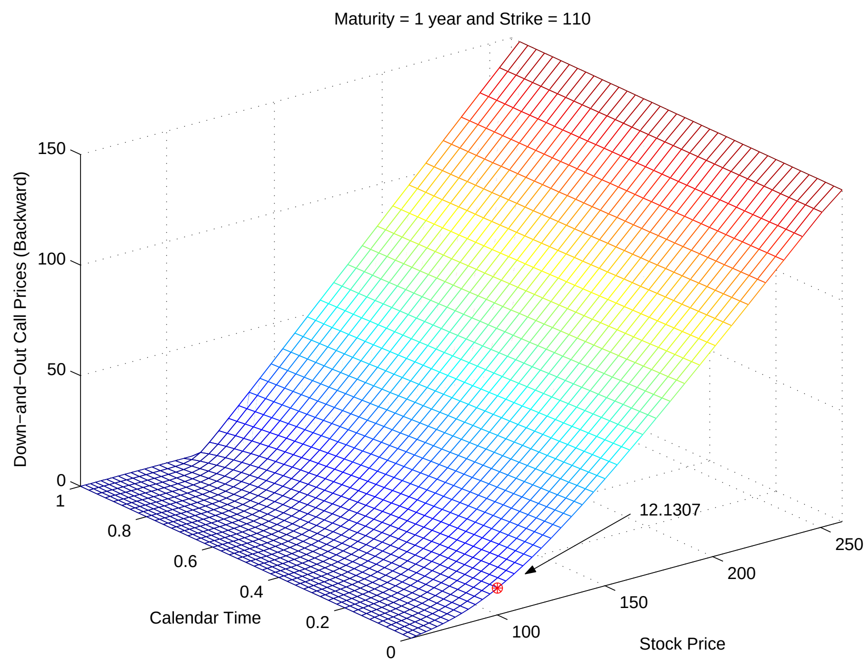
# Illustration 3(b): Down-and-Out Call Prices (Backward)



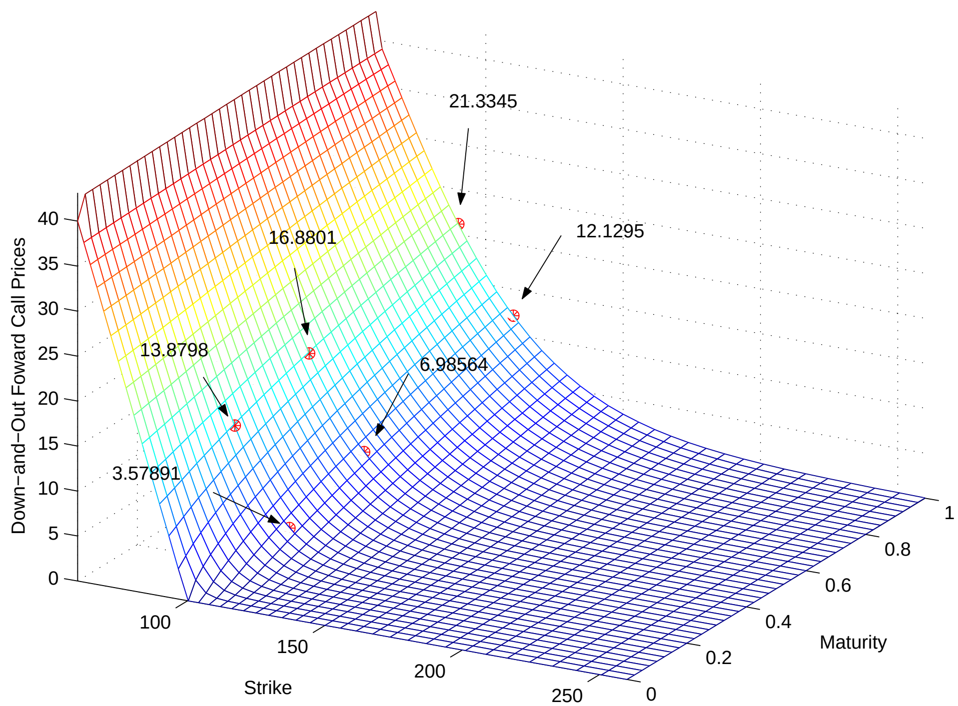
# Illustration 3(c): Down-and-Out Call Prices (Backward)



# Illustration 4: Down-and-Out Call Prices (Forward)

The variables are: Spot $S_0$=100, Down-Barrier $H = 60$, risk-free rate $r = 0.06$, dividend rate $q = .02$, and VG parameters $\sigma = 0.3$, $\nu = 0.25$, $\theta = -0.3$.



## Down-and-Out Call Prices (Forward vs. Backward)

| Maturity | | $T_1 = 0.25$ | | $T_2 = 0.5$ | | $T_3 = 1.0$ | |
|---|---|---|---|---|---|---|---|
| Barrier | Strike | **Bwd** | *Fwd* | **Bwd** | *Fwd* | **Bwd** | *Fwd* |
| 60 | 90 | **13.8837** | 13.8798 | **16.8732** | 16.8801 | **21.3305** | 21.3345 |
| | 110 | **3.57784** | 3.57891 | **6.99384** | 6.98564 | **12.1307** | 12.1295 |

## Future work

Bermudan, Compound, and American options in the Markov setting

Derivatives with stochastic volatility

## Tanaka-Meyer Formula

Assuming the Markovian stock price process, the Tanaka Meyer formula implies:

$$
\begin{aligned}
(S_T - K)^+ &= e^{\int_0^T r(u)du}(S_0 - K)^+ + \int_0^T e^{\int_t^T r(u)du} 1(S_t > K) dS_t \\
&+ \int_0^T e^{\int_t^T r(u)du} \left\{ \frac{a^2(S_t, t)}{2} \delta(S_t - K) - r(t)(S_t - K)^+ \right\} dt \\
&+ \int_0^T e^{\int_t^T r(u)du} \int_{-\infty}^{\infty} \left[ (S_t e^x - K)^+ - (S_t - K)^+ - 1(S_t > K) S_t (e^x - 1) \right] \mu(dx, dt),
\end{aligned}
$$

## Derivation of Forward PIDE for Up-and-Out Calls

Multiplying by $e^{-\int_0^T r(u)du} 1(\tau_H > T)$ and taking expectations on both sides under an equivalent martingale measure $\mathbb{Q}$, we have:

$$
\begin{aligned}
U_0^c(K,T) &= (S_0 - K)^+ E_0^Q 1(\tau_H > T) + \int_0^T e^{-\int_0^t r(u)du} E_0^Q \{1(\tau_H > T) 1(S_t > K)[r(t) - q(t)] S_{t-}\} dt \\
&+ \int_0^T e^{-\int_0^t r(u)du} \left\{ \frac{a^2(K,t)}{2} E_0^Q [1(\tau_H > T)\delta(S_t - K)] - r(t) E_0^Q [1(\tau_H > T)(S_t - K)^+] \right\} dt \\
&+ \int_0^T e^{-\int_0^t r(u)du} E_0^Q 1(\tau_H > T) \int_{-\infty}^{\infty} \left[ (S_t e^x - K)^+ - (S_t - K)^+ - 1(S_t > K) S_t (e^x - 1) \right] \nu(x,t) dx dt.
\end{aligned}
$$

## Derivation of Forward PIDE for Up-and-Out Calls

Differentiating w.r.t. $T$ implies:

$$
\begin{aligned}
&\frac{\partial}{\partial T} U_0^c(K,T) \\
= \quad & -e^{-\int_0^T r(u)du} E_0^Q\{\delta(\tau_H - T)(S_T - K)^+\} \\
+ \quad & e^{-\int_0^T r(u)du} E_0^Q\{1(\tau_H > T)1(S_{T-} > K)[r(T) - q(T)]S_{T-}\} \\
+ \quad & \frac{a^2(K,T)}{2} e^{-\int_0^T r(u)du} E_0^Q[1(\tau_H > T)\delta(S_{T-} - K)] \\
- \quad & r(T) e^{-\int_0^T r(u)du} E_0^Q[1(\tau_H > T)(S_{T-} - K)^+] \\
+ \quad & e^{-\int\limits_0^T r(u)du} E_0^Q \left\{ 1(\tau_H > T) \int\limits_{-\infty}^{\infty} \left[ (S_T e^x - K)^+ - (S_T - K)^+ - 1(S_T > K) S_T (e^x - 1) \right] \nu(x,T) dx \right\}.
\end{aligned}
$$

## Derivation of Forward PIDE for Up-and-Out Calls

Subtracting and adding $e^{-\int_0^T r(u)du} E_0^Q\{1(\tau_H > T)[r(T) - q(T)]K\ 1(S_T > K)\}$ to the second term on the RHS gives:

$$
\begin{aligned}
&\frac{\partial}{\partial T} U_0^c(K,T) \\
= \quad & -e^{-\int_0^T r(u)du} E_0^Q\{1(\tau_H < T)1(S_T \geq H)(S_T - K)\} \\
+ \quad & e^{-\int_0^T r(u)du} E_0^Q\{1(\tau_H > T)1(S_{T-} > K)[r(T) - q(T)](S_T - K)\} \\
+ \quad & e^{-\int_0^T r(u)du} E_0^Q\{1(\tau_H > T)[r(T) - q(T)]K1(S_T > K)\} + \frac{a^2(K,T)}{2}\frac{\partial^2}{\partial K^2} U_0^c(K,T) \\
- \quad & r(T)U_0^c(K,T) \\
+ \quad & e^{-\int\limits_0^T r(u)du} E_0^Q\left\{1(\tau_H > T)\int\limits_{-\infty}^{\infty}\left[e^x(S_T - Ke^{-x})^+ - 1(S_T > K)(S_T - K + S_T e^x - S_T)\right]\nu(x,T)dx\right\}
\end{aligned}
$$

# Derivation of Forward PIDE for Up-and-Out Calls (Cont'd)

$$
\begin{aligned}
&\frac{\partial}{\partial T} U_0^c(K,T) \\
&= \quad -e^{-\int_0^T r(u)du} E_0^Q \{1(\tau_H < T) 1(S_T \geq H)(H-K) + 1(\tau_H < T) 1(S_T \geq H)(S_T - H)\} \\
&+ \quad [r(T) - q(T)] U_0^c(K,T) - [r(T) - q(T)] K \frac{\partial}{\partial K} U_0^c(K,T) \\
&+ \quad \frac{a^2(K,T)}{2} \frac{\partial^2}{\partial K^2} U_0^c(K,T) - r(T) U_0^c(K,T) \\
&+ \quad e^{-\int_0^T r(u)du} E_0^Q \left\{ 1(\tau_H > T) \int_{-\infty}^{\infty} e^x \left[ (S_T - Ke^{-x})^+ - 1(S_T > K)(S_T - Ke^{-x} + K - K) \right] \nu(x,T) dx \right\}.
\end{aligned}
$$

The first term on the RHS is the sum of the payoffs from $H-K$ partial barrier up-and-out binary calls with barrier $H$ and one partial barrier up-and-out call, with barrier and strike $H$. For both options, the end of the barrier monitoring period is $T-$, while the options mature at $T$.

## Derivation of Forward PIDE for Up-and-Out Calls (Cont'd)

For the partial barrier up-and-out call with strike and barrier $H$, we have:

$$e^{-\int_0^T r(u)du} E_0^Q\{1(\tau_H < T)1(S_T \geq H)(S_T - H)\} = \frac{a^2(H,T)}{2}\frac{\partial^2}{\partial K^2}U_0^c(H,T).$$

However, $\frac{\partial^2}{\partial K^2}U_0^c(H,T) = 0$, since, loosely speaking, this is just the discounted probability of surviving beyond $T$ and that $S_T = H$. For the partial barrier up-and-out binary call with barrier $H$, we have:

$$e^{-\int_0^T r(u)du} E_0^Q\{1(\tau_H < T)1(S_T \geq H)\} = -\frac{a^2(H,T)}{2}\frac{\partial^3}{\partial K^3}U_0^c(H,T).$$

The third derivative does not vanish.

# Derivation of Forward PIDE for Up-and-Out Calls (Cont'd)

If we now account for jumps when valuing the first term, we obtain:

$$
\begin{aligned}
&\frac{\partial}{\partial T}U_0^c(K,T)\\
= \quad & (H-K)\left[\frac{a^2(H,T)}{2}\frac{\partial^3}{\partial K^3}U_0^c(H,T) - e^{-\int_0^T r(u)du}E_0^Q\{1(\tau_H<T)\int_{0+}^{\infty}1(S_Te^x \geq H)\nu(x,T)dx\}\right]\\
- \quad & e^{-\int_0^T r(u)du}E_0^Q\{1(\tau_H<T)\int_{0+}^{\infty}(S_{T-}e^x-H)^+\nu(x,T)dx\} - q(T)U_0^c(K,T)\\
+ \quad & [r(T)-q(T)]K\frac{\partial}{\partial K}U_0^c(K,T) + \frac{a^2(K,T)}{2}\frac{\partial^2}{\partial K^2}U_0^c(K,T)\\
+ \quad & e^{-\int_0^T r(u)du}E_0^Q\left\{1(\tau_H>T)\int_{-\infty}^{\infty}\left[(S_T-Ke^{-x})^+ - (S_T-K)^+ - \frac{\partial}{\partial K}(S_T-K)^+K(e^{-x}-1)\right]e^x\nu(x,T)dx\right\}
\end{aligned}
$$

## Derivation of Forward PIDE for Up-and-Out Calls (Cont'd)

$$
\begin{aligned}
&\frac{\partial}{\partial T}U_0^c(K,T)\\
= \quad & (H-K)\left[\frac{a^2(H,T)}{2}\frac{\partial^3}{\partial K^3}U_0^c(H,T) - e^{-\int_0^T r(u)du}E_0^Q\{1(\tau_H < T)\int\limits_{0+}^{\infty} 1(S_{T-} \geq He^{-x})\nu(x,T)dx\}\right]\\
- \quad & e^{-\int_0^T r(u)du}E_0^Q\{1(\tau_H < T)\int_{0+}^{\infty}(S_{T-} - He^{-x})^+ e^x \nu(x,T)dx\}\\
- \quad & q(T)U_0^c(K,T) - [r(T) - q(T)]K\frac{\partial}{\partial K}U_0^c(K,T) + \frac{a^2(K,T)}{2}\frac{\partial^2}{\partial K^2}U_0^c(K,T)\\
+ \quad & \int_{-\infty}^{\infty}\left[U_0^c(Ke^{-x},T) - U_0^c(K,T) - \frac{\partial}{\partial K}U_0^c(K,T)K(e^{-x}-1)\right]e^x\nu(x,T)dx
\end{aligned}
$$