Dynamic Routing of Aircraft under weather uncertainty

Arnab Nilim

Laurent El Ghaoui

Mark Hansen

University of California, Berkeley

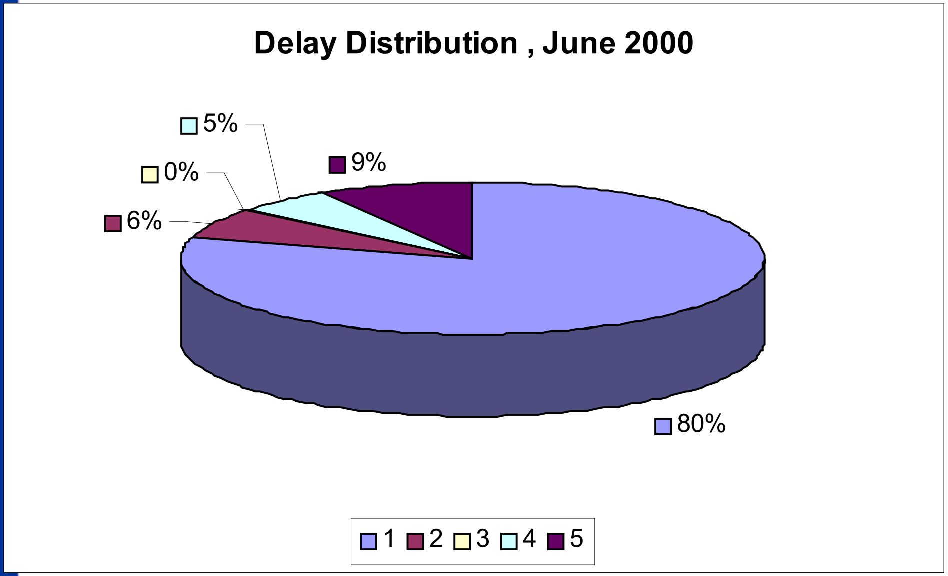
Vu Duong

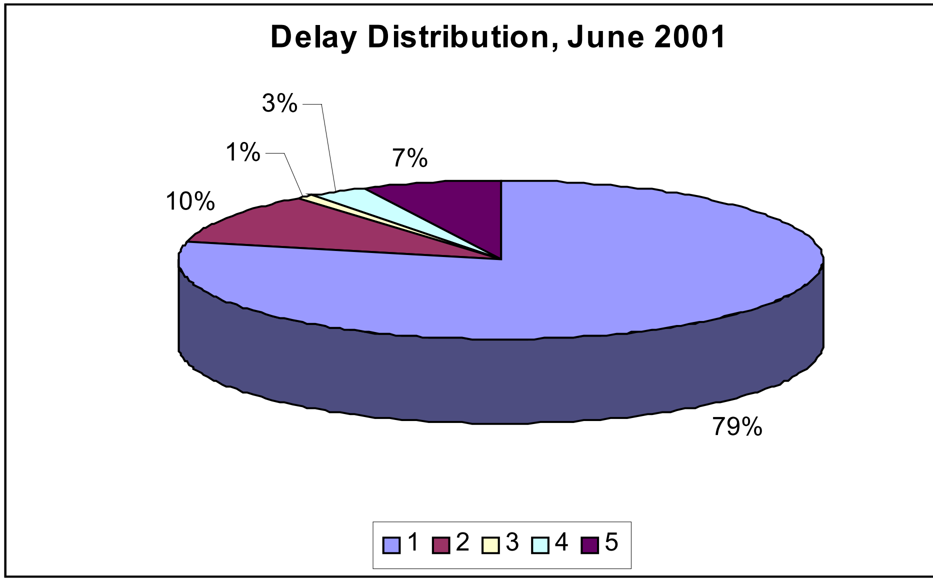
Euro Control Experimental Centre

Introduction

- Delays in commercial air travel
- Weather induced enroute delays
- Shortcomings of existing programs

Delay Distribution





1:Weather 3: Equipment 5: Other

2: Volume 4: Runway

Previous work

- Deterministic traffic flow management

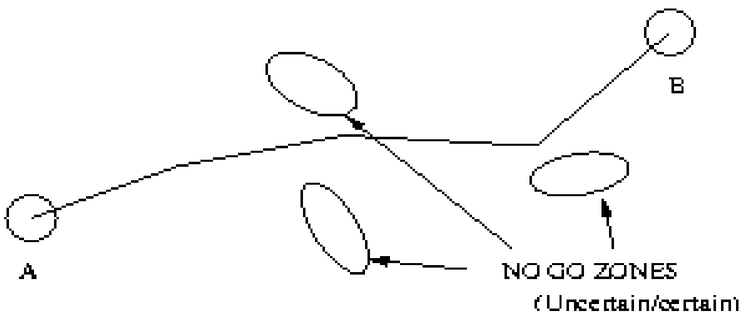
(Bertsimas (98), Goodhart (99), Burlingame(94) etc)

- Automation tool: explicitly dealing with the dynamics and the stochasticity of the weather

- Optimization under uncertainty

New architecture (???)

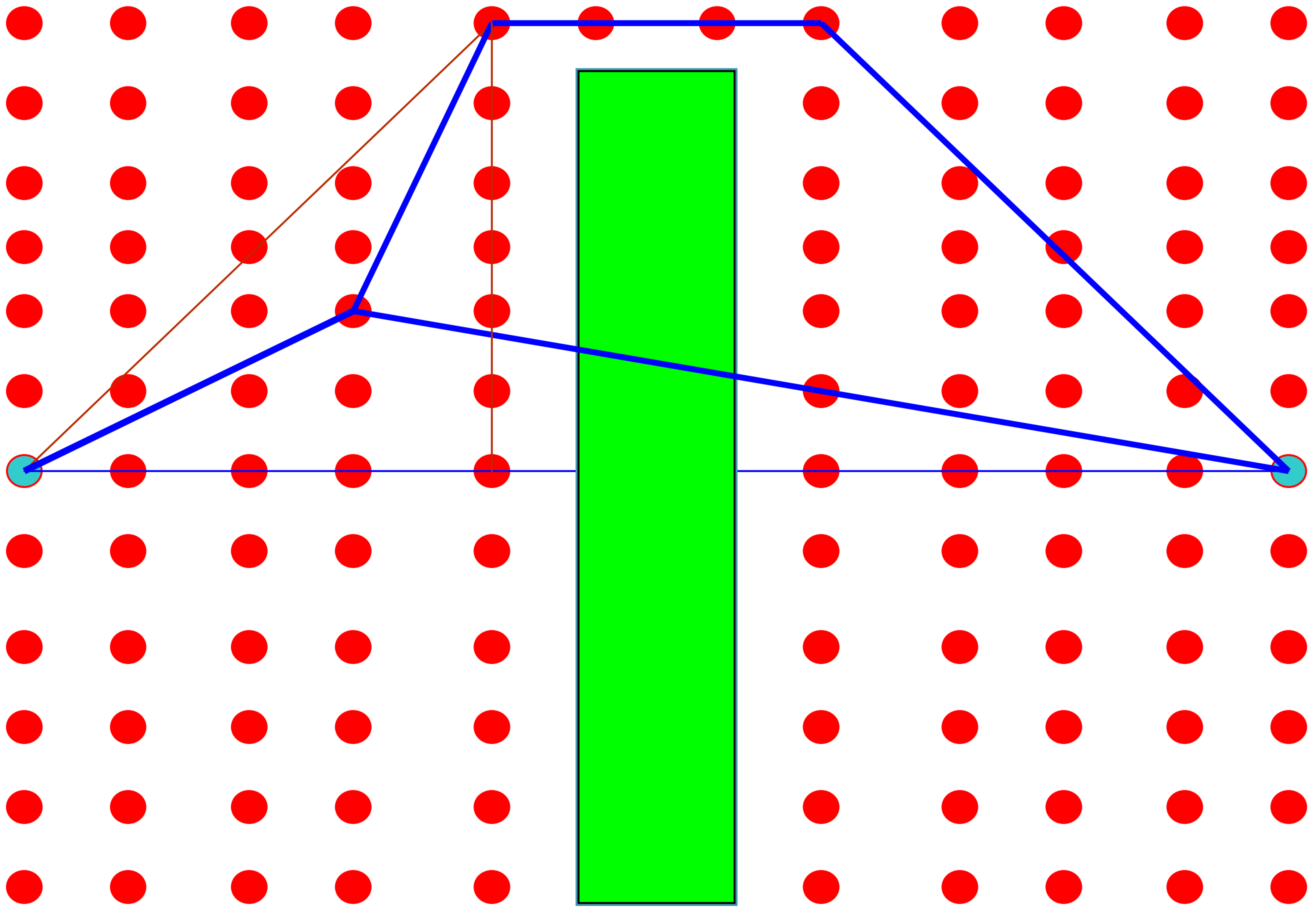
- Airspace vs Trajectory based architecture



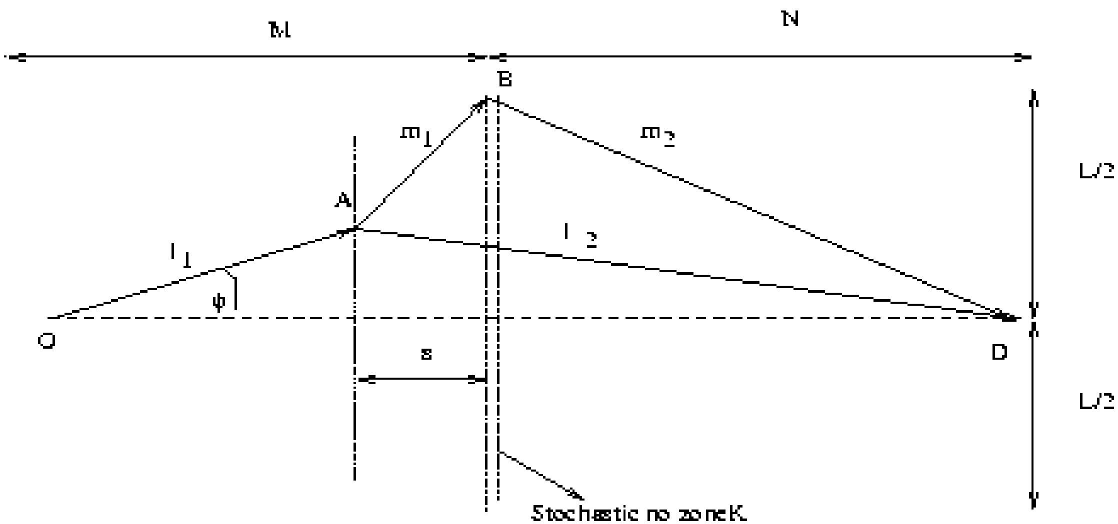
Research Agenda

- Dynamic Routing strategy of a single aircraft.
- Robust solution w.r.t. estimation of storm probabilities error.
- System level solution

Dynamic Routing of Aircraft under Uncertainty

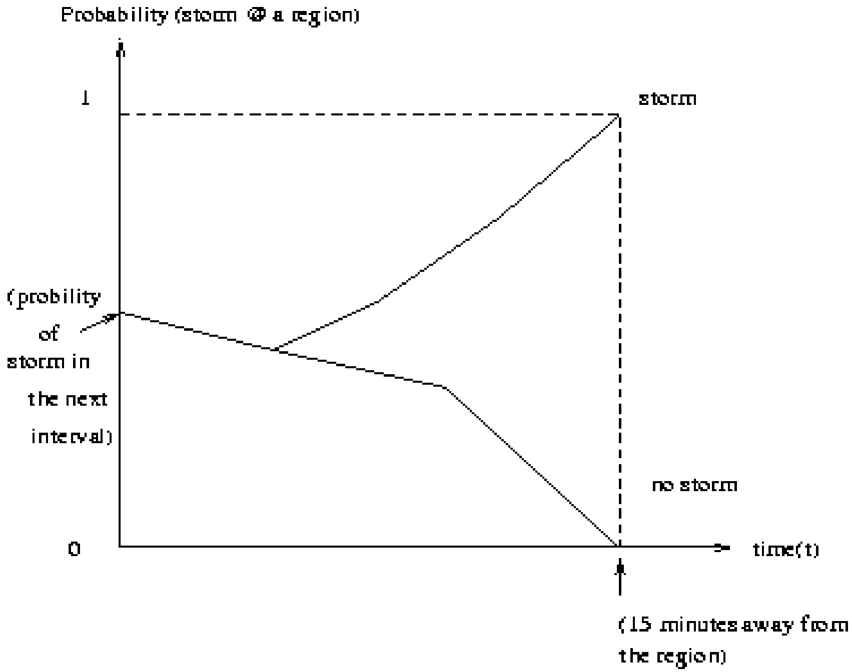


Simple optimization



$$\min_{\phi}\{l_1 + p(m_1 + m_2), l_1 + (1-p)l_2\}$$

Uncertainty



Stochastic dynamic Programming/Markov Decision Processes

State: $x_{k+1} = f_k(x_k, \mu_k, w_k), \forall k = 0,1,\ldots, n-1$

Control: $\vec{\mu} = [\mu_0, \mu_1, \ldots, \mu_{n-1}]$

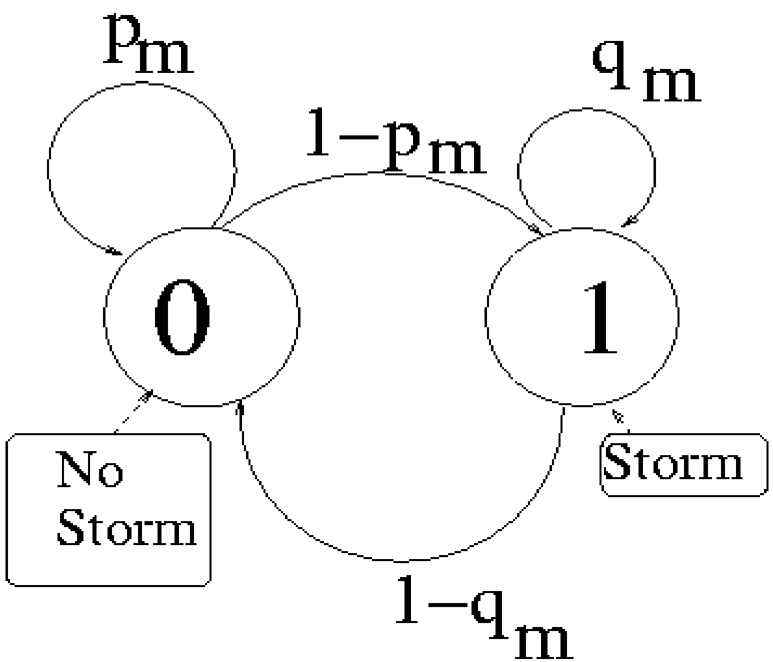
Expected cost function:

$$J_\mu(x_0) = E_{(w_k \forall k=0,1,\ldots n-1)}(g_n(x_n) + \sum_{k=0}^{n-1} g_k(x_k, \mu_k(x_k), w_k))$$

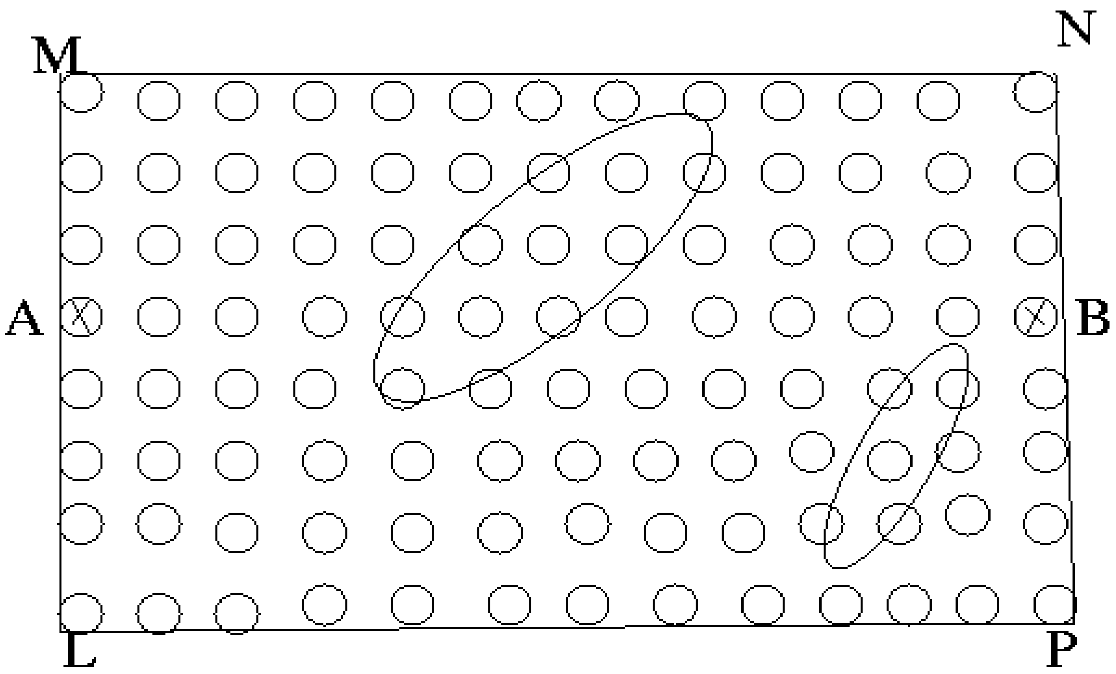
Markovian uncertainty:

$$v(i,n) = \min_{(1 \le k \le A_i)}[q_i^k + \sum_{j=1}^{N} p_{ij}^k v(j, n-1)]$$

Weather Model



Gridding



Algorithm

- Step 1:Calculate the total number of stages

$$n_{\max} = \frac{T - \mathrm{mod}[\frac{T}{15}]}{15} + 1$$

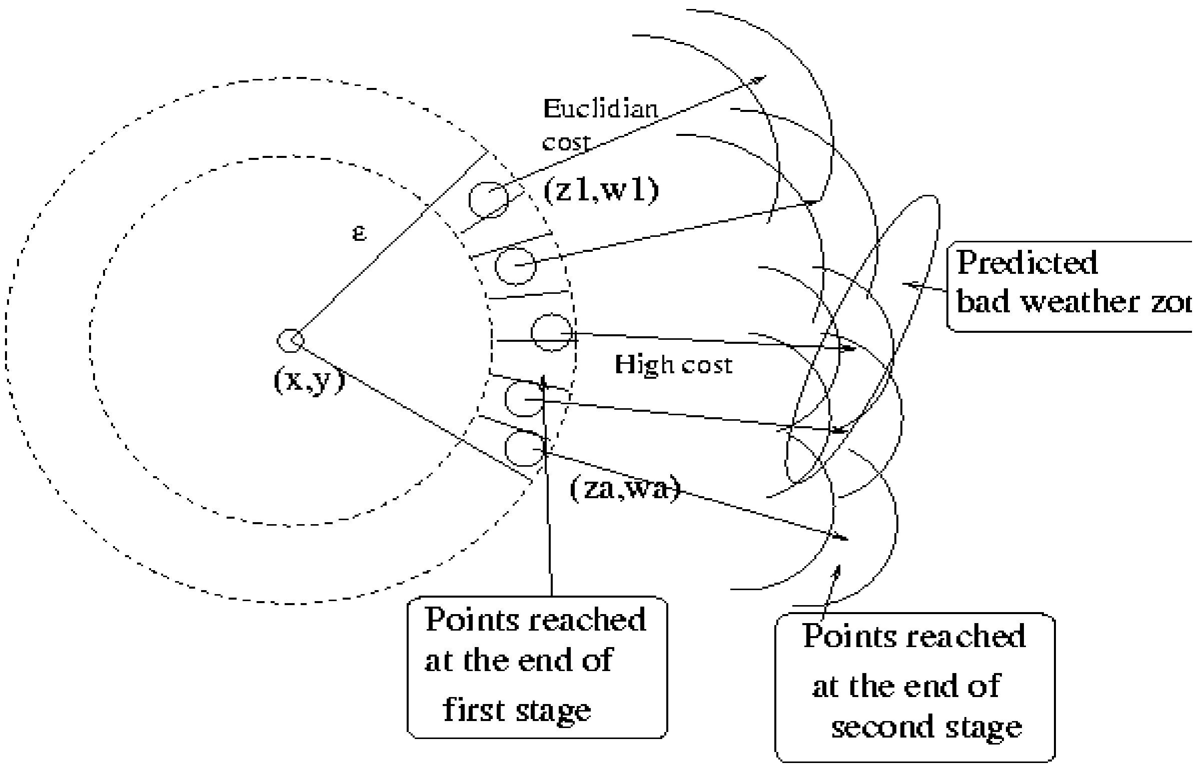
T: Worst case time

- Step 2: Discretize (airspace)

- Step 3: Pruning

Algorithm

Step 4: Next points



Algorithm

- Step 5: Assigning appropriate cost

$c(i, x, y, z_j, w_j)$: Cost to go from (x, y) to (z_j, w_j) in state i

- Step 6:Defining value function

$v(i, x, y, n)$: Expected minimum distance to go if the aircraft is at the point (x,y), with the state and it has n stages to go to reach the destination point

- Step 7: Assigning boundary conditions

Algorithm

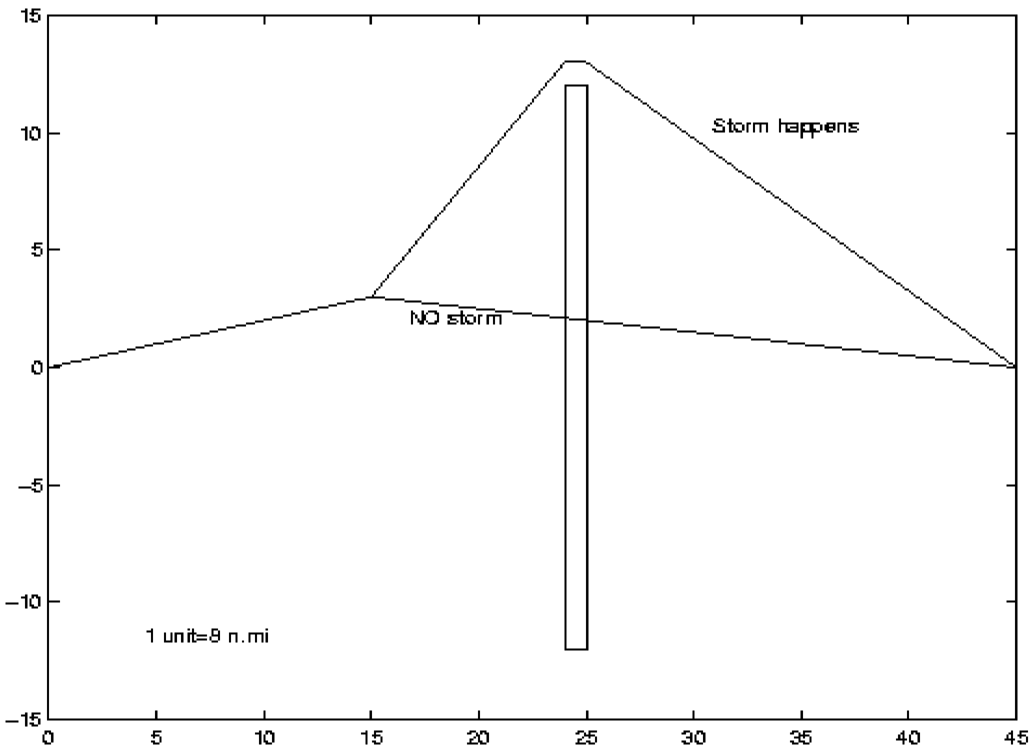
- Step 8:

$$v(i,x,y,n) = \min_{(z_1,w_1),\ldots,(z_a,w_a)} V$$

Where,

$$V = \begin{pmatrix} c(i,x,y,z_1,w_1) + \sum_{j=1}^{2^M} p_{ij} v(j,z_1,w_1,n-1) \\ c(i,x,y,z_2,w_2) + \sum_{j=1}^{2^M} p_{ij} v(j,z_2,w_2,n-1) \\ \ldots\ldots\ldots\ldots \\ c(i,x,y,z_a,w_a) + \sum_{j=1}^{2^M} p_{ij} v(j,z_a,w_a,n-1) \end{pmatrix}$$

Simulation



Improvements

	I.M. of our model over TS1	I.M. of our model over TS2
Scenario 1	66.42%	42.76%
Scenario 2	54.78%	49.81%

Conclusion

- Less circuitous route
- Less overloading in the neighboring sectors
- Complexity
- Robust solution w.r.t. errors in estimation of the storm probabilities
- Routing of multiple aircraft under uncertainty

Acknowledgements

Stuart Dreyfus, Christos Pappadimitrou, Pravin Varaiya, Jim Evans, Jim Wetherly