Exploratory Study of Demand Management Using Auction-Based Arrival Slot Allocation

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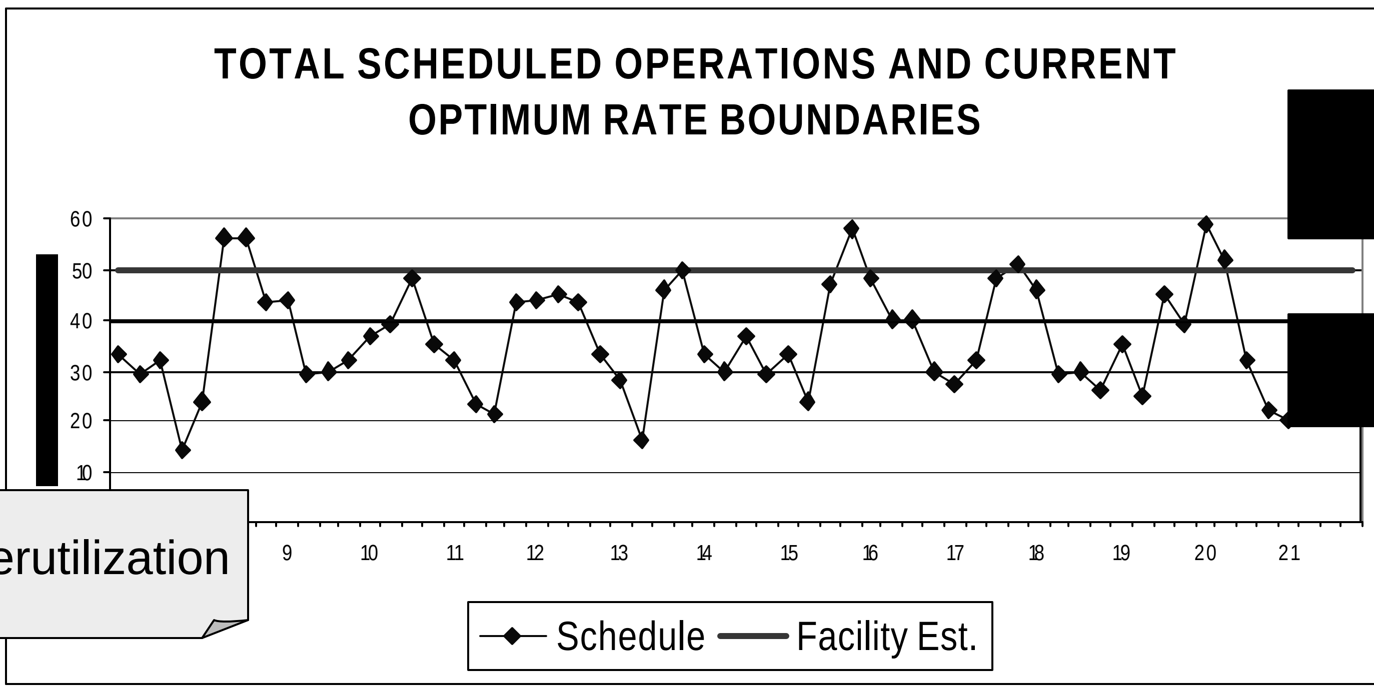
Outlines

- Slot auction model
- Auctioneer optimization model
- Airline optimization model
- Illustration : a case study
- Summary and future work

Asynchronous non-uniform scheduling

ATL – Atlanta Airport

(FAA Airport Capacity Benchmark Report 2001)



Underutilization

[redacted]

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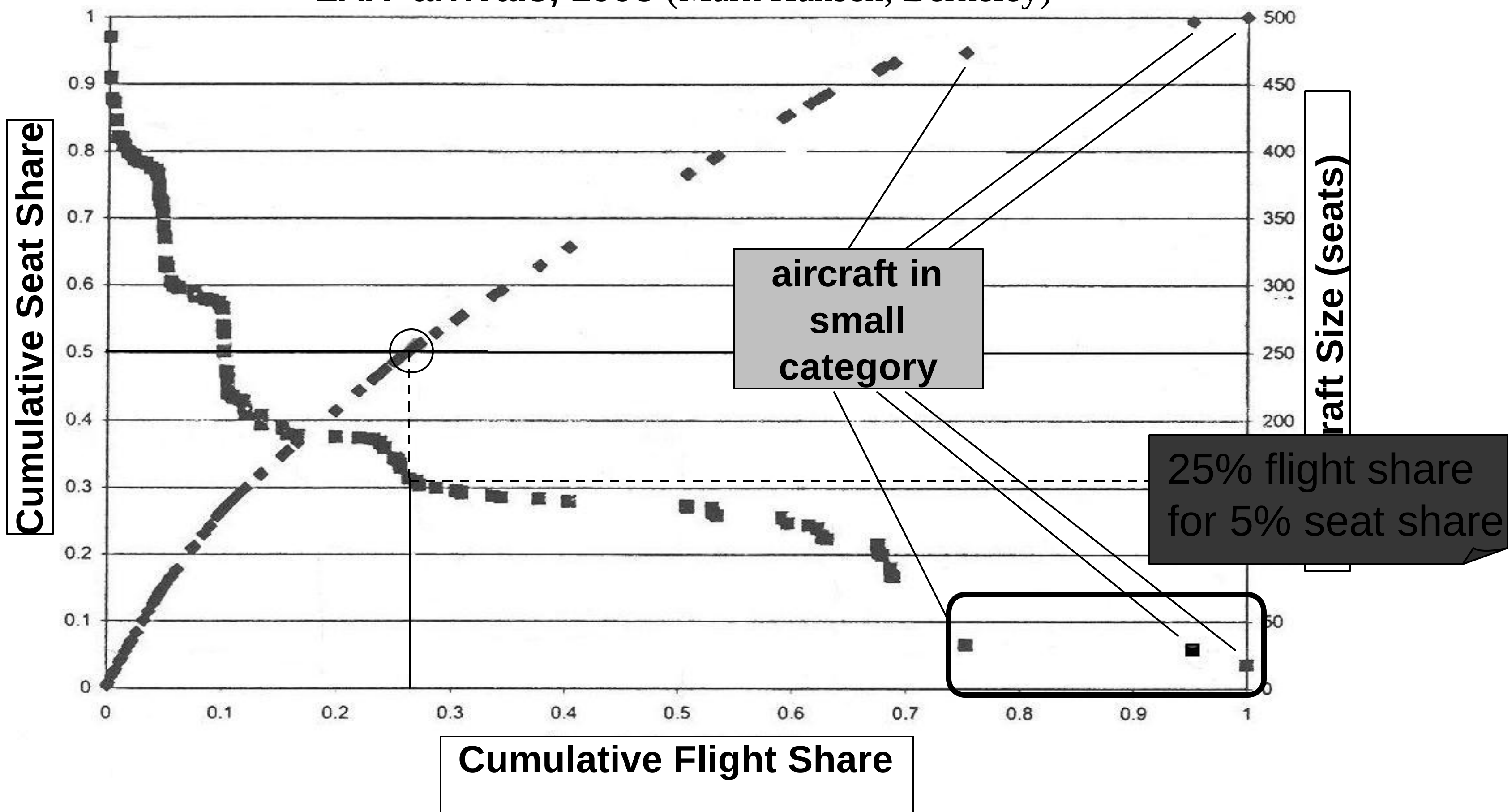
Problem Identification

Small aircraft makes inefficient use of slots

LAX arrivals, 1998 (Mark Hansen, Berkeley)



Auction Model Design issues

- Feasibility
  - package slot allocation for departure and arrival slots
  - incremental (airports and slots to auction off)
- Optimality
  - efficiency : throughput (enplanement opportunity) and delay
  - regulatory standards: safety, flight priorities
  - equity:
    - stability in schedule
    - airlines' need to leverage investments
    - airlines' competitiveness : new-entrants vs. incumbents
- Flexibility
  - primary market at strategic level
  - secondary market at tactical level

Design approach

- Objective:
  - provide an optimum fleet mix at optimum safe arrival capacity
  - ensure fair market access opportunity
  - reduce queuing delay
- Assumptions
  - airlines could make use of slots they bid
- Auction process:
  - an interactive and iterative process to enable flexibility and optimization
  - a mixture of simultaneous auction model and package model
- Auction rules: Bidders are ranked using a linear combination of:
  - flight OD pair
  - #seats
  - airline’s prior investment
  - historical slot occupancy rate
  - bid

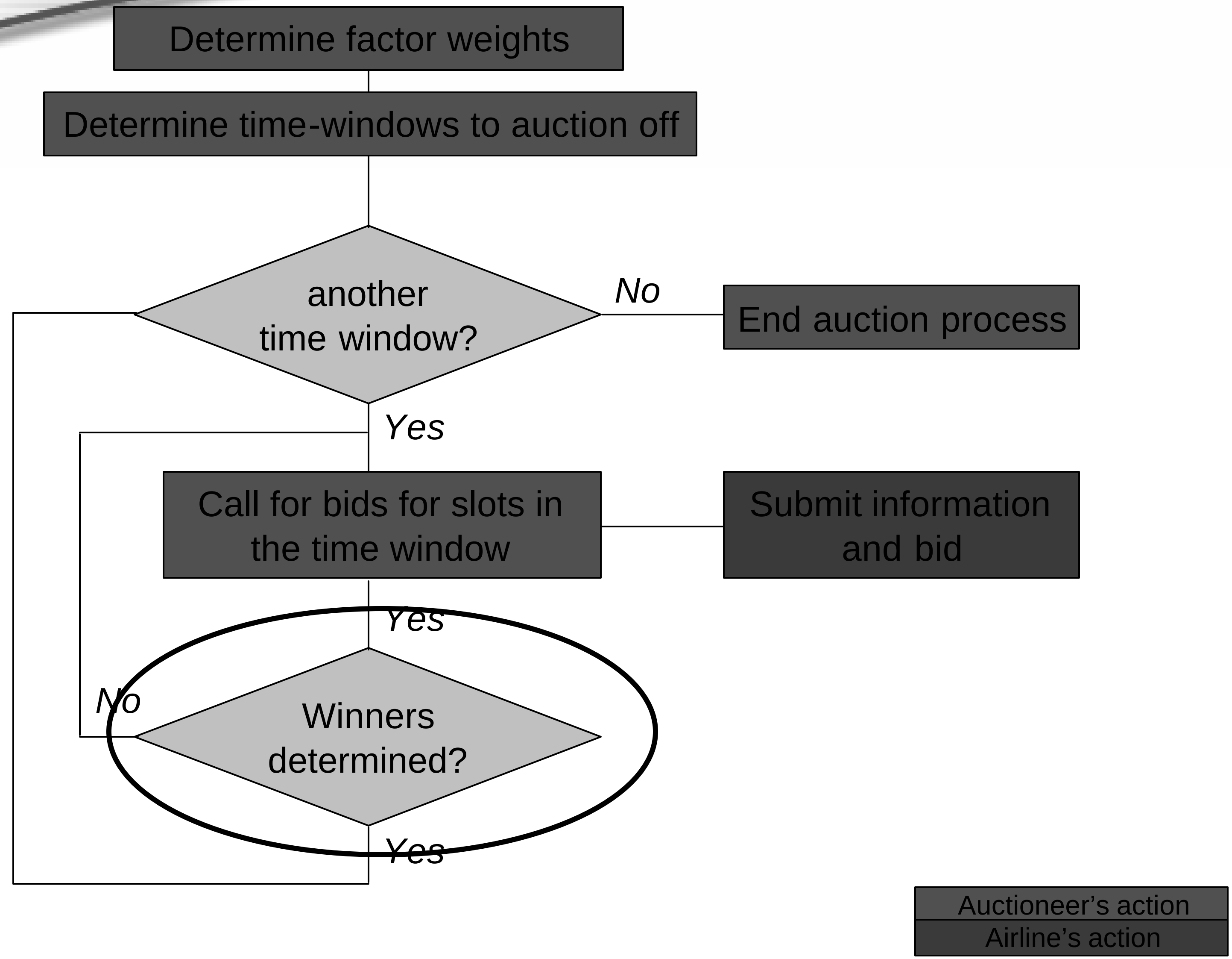
Background on auction models

- Simultaneous multiple-round auction
  - have discrete, successive rounds, with length of each round announced in advance. After each round closes, round results are processed and made public
  - → Account for departure/arrival slots interdependence but subject to aggregation risks

- Package bidding
  - bidders submit bids for multiple combinations of lots rather than just individual lots. Package biddings are either accepted or rejected in their entirety
  - → Eliminate aggregation risks

Auction Model Process

Incremental auction prioritizing most congested periods



Auctioneer Optimization Model

Airlines are ranked by a linear combination of :

- –#seats
- –flight OD pair
- –airline's prior investment
- –historical slot occupancy rate
- –monetary offer

BID

Ranking function:

$$t\left(B_{a,s}\right) = W^T \cdot B_{a,s}$$

S^T	slot vector, $\lvert S^T \rvert$=AAR
A	airline vector
W^T	vector of factor weights
$B_{a,s}$	bid of airline *a* for slot *s*

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A	airline vector
W^T	vector of factor weights
$B_{a,s}$	bid of airline *a* for slot *s*

Ranking function:

$$t(B_{a,s}) = W^T \cdot B_{a,s}$$

Bidding matrix X=A.S^T

$$(X)_{a,s} = \begin{cases} 1 & \text{if airline a is ranked highest for slot s after a round} \\ 0 & \text{otherwise} \end{cases}$$

Slots	8:00:00	8:02:30	8:05:00	8:07:30	8:10:00	8:12:30	8:15:00	8:17:30
Minimum Bid	**5000**	**7500**	**8600**	**10000**	**12000**	**8300**	**6600**	**5200**
AC1								
AC2								
AC3								
AC4								
AC5								
AC6								
AC7								

Auctioneer Optimization Model

→ **Objective function: Allocate slots to the highest ranked airlines**

$$\textbf{Max} \quad \sum_{a}\sum_{s} t\,(B_{a,s})\cdot(X)_{a,s}$$

Subject to:

$$\begin{cases} \sum_{a}(X)_{a,s} = 1 & \forall s \\ (M^{T})_{s}\cdot(X)_{a,s} <= (B_{a,s})_{5} & \forall a,s \\ \sum_{s}(X)_{a,s} <= 1 & \end{cases}$$

Airline constraint

Variables:

S^T	slot vector, $\lvert S^T\rvert$=AAR
A	airline vector
W^T	vector of factor weights
$B_{a,s}$	bid of airline *a* for slot *s*
$X = A*S^T$	bidding matrix

$$(X)_{a,s} = \begin{cases} 1 & \text{if airline a is ranked highest for slot s after a round} \\ 0 & \text{otherwise} \end{cases}$$

Airline Optimization Model

Objective function: Maximize revenue and ultimately maximize profit

$$\textbf{Maximize} \sum_{s} (P_s - B_s)$$

Subject to:

$$B_s \le M \cdot y_s$$

$$(B_0^T)_s \le B_s + M \cdot (1 - y_s)$$

Lower bound for bids

$$B_s \le a \cdot P_s$$

$$\left(B_s^{'} + \frac{\max_a (t(B_{a,s})) - t(B_{A,s})}{(W)_5} \cdot (B_0^T)_s \right) \cdot y_s \le a \cdot P_s \cdot y_s$$

$$\left(B_s^{'} + \frac{\max_a (t(B_{a,s})) - t(B_{A,s})}{(W)_5} \cdot (B_0^T)_s \right) \le B_s + M \cdot (1 - y_s)$$

Airlines' package bidding constraints

Variables:

$\{B_s\}$	set of monetary bids
$\{P_s\}$	airline expected profit by using a slot
M	big positive value
y_s	binary value
$y_s = \begin{cases} 1 \\ 0 \end{cases}$	if airline bids for slot s otherwise
B_0^T	airport threshold vector
α	airline threshold fraction
B_s'	old bid for slot s in previous round

Illustration : an example

Winner-determining factors

Factors	*Weight*
Number of seats	0,32
Previous Airline infrastructure investment	0,25
Historic slot occupancy frequency	0,19
OD-Pair	0,13
Bid	0,11

Ranking function :

$$t(B_{a,s}) = W^T \cdot B_{a,s}$$

Bidding matrix, initial round

Slots	8:00:00	8:02:30	8:05:00	8:07:30	8:10:00	8:12:30	8:15:00	8:17:30
Min Monetery Bid	**5000**	**7500**	**8600**	**10000**	**12000**	**8300**	**6600**	**5200**
NW (0.25)	S (30) 0 (0.4)			H (205) 1 (0.25)			L (128) 0 (0.35)	
AA (0.19)		L (147) 0 (0.5)		H (283) 1 (0.4)				L (291) 1 (0.35)
UA (0.15)			S (18) 0 (0.25)		H (392) 1 (0.3)		S (18) 0 (0.2)	
CA (0.13)		S (30) 0 (0.35)		S (30) 0 (0.15)		L (150) 0 (0.35)		
SW (0)		S (18) 0 (0.15)	S (30) 0 (0.06)		S (18) 0 (0.1)			S (30) 0 (0.4)
US (0.21)	S (30) 0 (0.35)		L (120) 0 (0.5)	L (85) 0 (0.2)		S (30) 0 (0.45)		
DAL (0.11)	S (18) 0 (0.2)				H (202) 1 (0.3)		S (30) 0 (0.4)	

%investment

Aircraft type (#seats) OD pair (Slot occupancy rate)

Illustration: an example

Bid score matrix, initial round

Slots	8:00:00	8:02:30	8:05:00	8:07:30	8:10:00	8:12:30	8:15:00	8:17:30
Min Monetery Bid	**5000**	**7500**	**8600**	**10000**	**12000**	**8300**	**6600**	**5200**
NW (0.25)	1.48			3.23			3.18	
AA (0.19)		3.44		3.94				3.98
UA (0.15)			1.04		7.75		0.96	
CA (0.13)		1.18		0.96		2.2		
SW (0)		0.62	0.83		0.62			0.62
US (0.21)	1.34		2.96	1.68		1.11		
DAL (0.11)	0.87				4.3		1.1	

Air carrier monetary bids

Round 1

Slots	8:00:00	8:02:30	8:05:00	8:07:30	8:10:00	[illegible]	[illegible]	8:17:30
Min Monetery Bid	**5000**	**7500**	**8600**	**10000**	**12000**	[illegible]	[illegible]	**5200**
NW (0.25)	5000			64109		[illegible]	[illegible]	
AA (0.19)		7500		8600		[illegible]	[illegible]	5200
UA (0.15)			149482		9200	[illegible]	[illegible]	
CA (0.13)		161590		0		8300		
SW (0)		199773	164945		0			164036
US (0.21)	11364		8100	185291		90545		
DAL (0.11)	32727				297745		131400	

Withdraw from auction for these slots

Illustration: an example

Bid score matrix, initial round

Slots	8:00:00	8:02:30	8:05:00	8:07:30	8:10:00	8:12:30	8:15:00	8:17:30
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Min Monetery Bid	**5000**	**7500**	**8600**	**10000**	**12000**	[illegible]	[illegible]	**5200**
NW (0.25)	S (30) 0 (0.4)			H (205) 1 (0.25)		[illegible]	[illegible]	
AA (0.19)		L (147) 0 (0.5)		H (283) 1 (0.4)		[illegible]	[illegible]	L (291) 1 (0.35)
UA (0.15)			S (18) 0 (0.25)		H (392) 1 (0.3)	[illegible]	[illegible]	
CA (0.13)		S (30) 0 (0.35)		S (30) 0 (0.15)		L (150) 0 (0.35)		
SW (0)		S (18) 0 (0.15)	S (30) 0 (0.06)		S (18) 0 (0.1)			S (30) 0 (0.4)
US (0.21)	S (30) 0 (0.35)		L (120) 0 (0.5)	L (85) 0 (0.2)		S (30) 0 (0.45)		
DAL (0.11)	S (18) 0 (0.2)				H (202) 1 (0.3)		S (30) 0 (0.4)	

Illustration: an example

Bid score matrix, initial round

Slots	8:00:00	8:02:30	8:05:00	8:07:30	8:10:00	8:12:30	8:15:00	8:17:30
Min Monetery Bid	**5000**	**7500**	**8600**	**10000**	**12000**	**8300**	**6600**	**5200**
NW (0.25)	1.48			3.23			3.18	
AA (0.19)		3.44		3.94				3.98
UA (0.15)			1.04		7.75		0.96	
CA (0.13)		1.18		0.96		2.2		
SW (0)		0.62	0.83		0.62			0.62
US (0.21)	1.34		2.96	1.68		1.11		
DAL (0.11)	0.87				4.3		1.1	

Air carrier monetary bids

Round 1

Slots	8:00:00	8:02:30	8:05:00	8:07:30	8:10:00			8:17:30
Min Monetery Bid	**5000**	**7500**	**8600**	**10000**	**12000**			**5200**
NW (0.25)	5000			64109				
AA (0.19)		7500		8600				5200
UA (0.15)			149482		9200			
CA (0.13)		161590		0		8300		
SW (0)		199773	164945		0			164036
US (0.21)	11364		8100	185291		90545		
DAL (0.11)	32727				297745		131400	

Withdraw from auction for these slots

Bid score matrix

Round 1

Slots	8:00:00	8:02:30	8:05:00	8:07:30	8:10:00	8:12:30	8:15:00	8:17:30
	5000	**7500**	**8600**	**10000**	**12000**	**8300**	**6600**	**5200**
	1.48			3.94			3.18	
		3.44		3.94				3.98
			2.96		7.75		3.18	
		3.44				2.2		
		3.44	2.96					3.98
US (0.21)	1.48		2.96	3.94		2.2		
DAL (0.11)	1.48				7.75		3.18	

Break ties by adding 10% of airline profit to bids

Illustration: an example

Air carrier monetary bids

Round 12 (final)

Slots	8:00:00	8:02:30	8:05:00	8:07:30	8:10:00	8:12:30	8:15:00	8:17:30
Min Monetery Bid	**5000**	**7500**	**8600**	**10000**	**12000**	**8300**	**6600**	**5200**
NW (0.25)	187500						48000	
AA (0.19)		58620		609800				15440
UA (0.15)					201040			
CA (0.13)						103340		
SW (0)								
US (0.21)			48900					
DAL (0.11)								

Bid score matrix

Round 12 (final)

Slots	8:00:00	8:02:30	8:05:00	8:07:30	8:10:00	8:12:30	8:15:00	8:17:30
Min Monetery Bid	**5000**	**7500**	**8600**	**10000**	**12000**	**8300**	**6600**	**5200**
NW (0.25)	5.5						3.87	
AA (0.19)		4.19		11.63				4.19
UA (0.15)					10.05			
CA (0.13)						3.45		
SW (0)								
US (0.21)			3.51					
DAL (0.11)								

Auction winners

Slots	8:00:00	8:02:30	8:05:00	8:07:30	8:10:00	8:12:30	8:15:00	8:17:30
Minimum Bid	**5000**	**7500**	**8600**	**10000**	**12000**	**8300**	**6600**	**5200**
NW (0.25)	S (30) 0 (0.4)			H (205) 1 (0.25)			L (128) 0 (0.35)	
AA (0.19)		L (147) 0 (0.5)		H (283) 1 (0.4)				L (291) 1 (0.35)
UA (0.15)			S (18) 0 (0.25)		H (392) 1 (0.3)		S (18) 0 (0.2)	
CA (0.13)		S (30) 0 (0.35)		S (30) 0 (0.15)		L (150) 0 (0.35)		
SW (0)		S (18) 0 (0.15)	S (30) 0 (0.06)		S (18) 0 (0.1)			S (30) 0 (0.4)
US (0.21)	S (30) 0 (0.35)		L (120) 0 (0.5)	L (85) 0 (0.2)		S (30) 0 (0.45)		
DAL (0.11)	S (18) 0 (0.2)				H (202) 1 (0.3)		S (30) 0 (0.4)	

Summary and future work

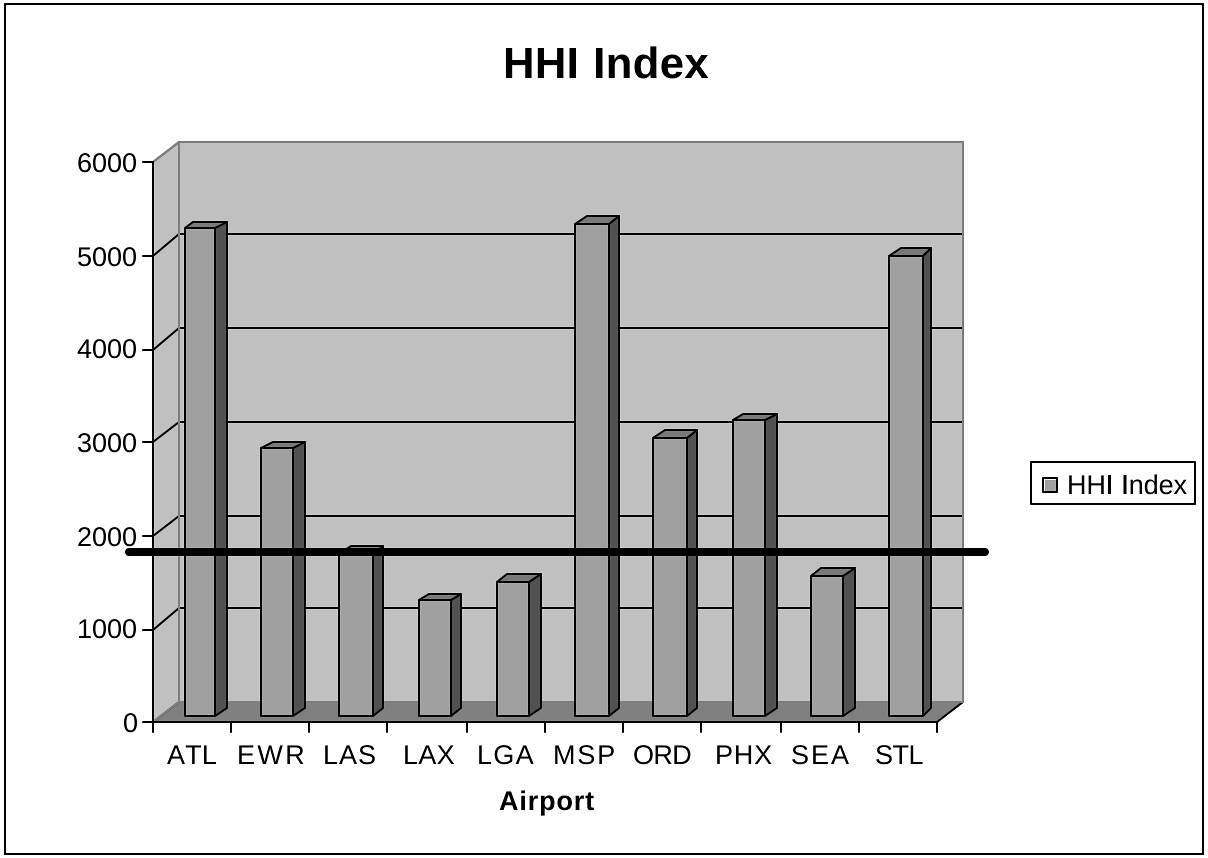
- Summary
  - Generic market-based approach to airport demand management allows to evaluate many alternatives by varying the weighting factors,
  - Legal and procedural challenges would require rigorous cost-benefit evaluation.
- Future work
  - Modeling airlines' behavior,
  - Cross-airport package bidding,
  - Conduct sensitivity analysis for different auction formats (with different values of factor weights)

back-up slides

Problem Identification

Lack of competition

- Hirschman-Herfindahl Index (HHI) is standard measure of market concentration
- Department of Justice uses to measure the competition within a market place
- HHI=♦$(100*s_i)^2$ w/ s_i is market share of airline i
- Ranging between 100 (perfect competitiveness) and 10000 (perfect monopoly)
- In a market place with an index over 1800, the market begins to demonstrate a lack of competition



Potential solutions

- Congestion pricing
  - monitoring and updating constraints
- Promote use of larger aircraft
  - airline economic constraints
- Improve local infrastructure
  - environmental constraints
- Rerouting flights
  - market constraints

Design scope

- Scope:
  - Strategic auction for arrival slots at individual airports
- Objective:
  - Provide an optimum fleet mix at optimum safe capacity
  - Ensure fair market access opportunity
  - Reduce queuing delay
- Definitions:
  - Slot: The concession or the entitlement to use runway capacity of a certain airport by an air carrier on a specific date and at a specific time **[CITE: EEC COM(2001)335]**
  - Grandfather right: Air carriers having historical use rate of a slot of at least 80% will have precedence.

Auctioneer Optimization Model

Variables:

S^T	slot vector, $\lvert S^T \rvert$=AAR
A	airline vector
W^T	vector of factor weights
$B_{a,s}$	bid of airline a for slot s
$X= A*S^T$	bidding matrix

$$(X)_{a,s} = \begin{cases} 1 & \text{if airline a is ranked highest for slot s after a round} \\ 0 & \text{otherwise} \end{cases}$$

M^T	initial monetary offer vector

Ranking function: $t(B_{a,s}) = W^T * B_{a,s}$

Objective function: **Max** $\sum_a \sum_s t(B_{a,s}) * (X)_{a,s}$

Subject to:

$$\sum_a (X)_{a,s} = 1 \qquad \forall s$$

$$(M^T)_s * (X)_{a,s} <= (B_{a,s})_5 \qquad \forall a, s$$

Safe separation b/w leading and trailing aircraft:

Mean of the calculated inter-arrival times by aircraft weight categories

Leading	Trailing			
	Small	Large	Heavy	B757
Small	1.33	1.13	1.07	1.1
Large	2.74	1.21	1.07	1.1
Heavy	3.91	2.467	1.73	2.26
B757	3.35	1.92	1.69	1.7

(Mark Hansen)

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Objective function: $$\textbf{Max} \sum_a \sum_s t(B_{a,s}) * (X)_{a,s}$$

Subject to:
$$\sum_a (X)_{a,s} = 1 \qquad \forall s$$
$$(M^T)_s * (X)_{a,s} <= (B_{a,s})_5 \qquad \forall a, s$$

Safe separation b/w leading and trailing aircraft:
$$\sum_s (X)_{a,s} * f(AT(a,s), AT(a',s+1)) <= 60$$

Mean of the calculated inter-arrival times by aircraft weight categories

Leading	Trailing			
	Small	Large	Heavy	B757
Small	1.33	1.13	1.07	1.1
Large	2.74	1.21	1.07	1.1
Heavy	3.91	2.467	1.73	2.26
B757	3.35	1.92	1.69	1.7

(Mark Hansen)

Auctioneer Optimization Model

Objective function:

$$\textbf{Max} \sum_a \sum_s t\,(B_{a,s}) * (X)_{a,s}$$

Subject to:

Capacity constraint: One slot allocated to one flight

$$\sum_a (X)_{a,s} = 1 \qquad \forall s$$

Ranking function:

$$t\,(B_{a,s}) = W^T * B_{a,s}$$

Variables:

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$X= A*S^T$	bidding matrix

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Auctioneer Optimization Model

Objective function:

$$\text{Max} \sum_a \sum_s t(B_{a,s}) * (X)_{a,s}$$

Subject to: $\sum_a (X)_{a,s} = 1 \quad \forall s$

Minimum bid: At any round, monetary offers should be equal or greater than initial thresholds

$$(M^T)_s * (X)_{a,s} <= (B_{a,s})_5 \qquad \forall a, s$$

Ranking function:

$$t(B_{a,s}) = W^T * B_{a,s}$$

Variables:

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A	airline vector
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Auctioneer Optimization Model

Objective function:

$$\textbf{Max} \sum_a \sum_s t(B_{a,s}) * (X)_{a,s}$$

Subject to:

$$\sum_a (X)_{a,s} = 1 \qquad \forall s$$

$$(M^T)_s * (X)_{a,s} <= (B_{a,s})_5 \qquad \forall a,s$$

Airline constraints: Only one slot is needed in a set of adjacent slots

$$\sum_s (X)_{a,s} <= 1$$

Ranking function:

$$t(B_{a,s}) = W^T * B_{a,s}$$

Variables:

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W^T	vector of factor weights
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Objective function:

$$\textbf{Max} \quad \sum_{a}\sum_{s} t\,(B_{a,s}) * (X)_{a,s}$$

Subject to:

$$\sum_{a} (X)_{a,s} = 1 \qquad \forall s$$

$$(M^{T})_{s} * (X)_{a,s} <= (B_{a,s})_{5} \qquad \forall a, s$$

Airlines' package bidding constraints

Ranking function:

$$t\,(B_{a,s}) = W^{T} * B_{a,s}$$

Variables:

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$B_{a,s}$	bid of airline *a* for slot *s*
$X = A * S^T$	bidding matrix
$(X)_{a,s} = \begin{cases} 1 & \\ 0 & \end{cases}$	1: if airline a is ranked highest for slot s after a round; 0: otherwise
M^T	initial monetary offer vector

Airline Optimization Model

Objective function: Maximize profit

$$\textbf{Maximize} \sum_{s} (P_s - B_s)$$

Subject to:

Airline either bids for slot s (monetary offer greater than minimum threshold) or not.

$$B_s \leq M * y_s$$

$$(B_0^T)_s \leq B_s + M * (1 - y_s)$$

Variables:

$\{B_s\}$	set of monetary bids
$\{P_s\}$	airline expected profit by using a slot
M	big positive value
y_s	binary value
$y_s = \begin{cases} 1 \\ 0 \end{cases}$	if airline bids for slot s otherwise
B_0^T	airport threshold vector

Airline Optimization Model

Objective function: Maximize profit

$$\textbf{Maximize} \sum_{s} (P_s - B_s)$$

Subject to:

$$B_s \leq M * y_s$$

$$(B_0^T)_s \leq B_s + M * (1 - y_s)$$

When offering bid for slot s, monetary amount should not pass a threshold

$$B_s \leq a * P_s$$

Variables:

$\{B_s\}$	set of monetary bids
$\{P_s\}$	airline expected profit by using a slot
M	big positive value
y_s	binary value
$y_s = \begin{cases} 1 \\ 0 \end{cases}$	if airline bids for slot s otherwise
B_0^T	airport threshold vector
♋	airline threshold fraction

Airline Optimization Model

Objective function: Maximize profit

$$\textbf{Maximize} \sum_s (P_s - B_s)$$

Subject to:

$$B_s \le M * y_s$$

$$(B_0^T)_s \le B_s + M * (1 - y_s)$$

$$B_s \le a * P_s$$

Given B_s' the bid airline offered in the previous round, in order to be ranked highest in this round, airline has to increase at least :

$$(B_0^T)_s \text{ —— } (W)_5$$

$$\frac{\max_a (t(B_{a,s})) - t(B_{A,s})}{(W)_5} * (B_0^T)_s \text{ —— } \max_a (t(B_{a,s})) - t(B_{A,s})$$

Variables:

$\{B_s\}$	set of monetary bids
$\{P_s\}$	airline expected profit by using a slot
M	big positive value
y_s	binary value
$y_s = \begin{cases} 1 \\ 0 \end{cases}$	if airline bids for slot s otherwise
B_0^T	airport threshold vector
♋	airline threshold fraction
B_s'	old bid for slot s in previous round

Airline Optimization Model

Objective function: Maximize profit

$$\textbf{Maximize} \sum_s (P_s - B_s)$$

Subject to:

$$B_s \le M * y_s$$

$$(B_0^T)_s \le B_s + M * (1 - y_s)$$

$$B_s \le a * P_s$$

Given B_s' the bid airline offered in the previous round, in order to be ranked highest in this round, airline has to increase at least :

$$\left(B_i^{'} + \frac{\max_a (t(B_{a,i})) - t(B_{A,i})}{(W)_5} * (B_0^T)_i \right) * y_i \le a * P_i * y_i$$

Old bid ($B_i^{'}$) — Minimum increase ($\frac{\max_a (t(B_{a,i})) - t(B_{A,i})}{(W)_5}$)

Variables:

$\{B_s\}$	set of monetary bids
$\{P_s\}$	airline expected profit by using a slot
M	big positive value
y_s	binary value
$y_s = \begin{cases} 1 \\ 0 \end{cases}$	if airline bids for slot s otherwise
B_0^T	airport threshold vector
♋	airline threshold fraction
B_s'	old bid for slot s in previous round

Airline Optimization Model

Objective function: Maximize profit

$$\textbf{Maximize} \sum_{s} (P_s - B_s)$$

Subject to:

$$B_s \le M * y_s$$

$$(B_0^T)_s \le B_s + M * (1 - y_s)$$

$$B_s \le a * P_s$$

$$\left(B_i^{'} + \frac{\max_a (t(B_{a,i})) - t(B_{A,i})}{(W)_5} * (B_0^T)_i \right) * y_i \le a * P_i * y_i$$

Old bid — Minimum increase

$$\underbrace{\left(B_i^{'} + \frac{\max_a (t(B_{a,i})) - t(B_{A,i})}{(W)_5} * (B_0^T)_i \right)}_{\text{Minimum new bid}} \le B_i + M * (1 - y_i)$$

Variables:

$\{B_s\}$	set of monetary bids
$\{P_s\}$	airline expected profit by using a slot
M	big positive value
y_s	binary value
$y_s = \begin{cases} 1 \\ 0 \end{cases}$	if airline bids for slot s otherwise
B_0^T	airport threshold vector
♋	airline threshold fraction
B_s'	old bid for slot s in previous round

Airline Optimization Model

Objective function: Maximize profit

$$\textbf{Maximize} \sum_{s} (P_s - B_s)$$

Subject to:

$$B_s \le M * y_s$$

$$(B_0^T)_s \le B_s + M * (1 - y_s)$$

$$B_s \le a * P_s$$

$$\left(B_i^{'} + \frac{\max\limits_{a} (t\,(B_{a,i})) - t\,(B_{A,i})}{(W)_5} * (B_0^T)_i \right) * y_i \le a * P_i * y_i$$

$$\left(B_i^{'} + \frac{\max\limits_{a} (t\,(B_{a,i})) - t\,(B_{A,i})}{(W)_5} * (B_0^T)_i \right) \le B_i + M * (1 - y_i)$$

Airlines' package bidding constraints

Variables:

$\{B_s\}$	set of monetary bids
$\{P_s\}$	airline expected profit by using a slot
M	big positive value
y_s	binary value
$y_s = \begin{cases} 1 & \text{if airline bids for slot s} \\ 0 & \text{otherwise} \end{cases}$	
B_0^T	airport threshold vector
♋	airline threshold fraction
B_s'	old bid for slot s in previous round