Factors Influencing Blind Collision Risk in En Route Sectors Under Free-Flight Conditions

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Abstract

We developed a simulation model of the flight of aircraft through an en route sector. We used the model in two experiments. The first studied the effect of directional clumping on risk. The second was a full factorial design with replication, which simulated a large number of scenarios to assess the impact of key input factors on free flight risk measures. We measured risk for a given flight by the mean distance of closest approach to other aircraft and the mean number of approaches closer than a specified distance. Using analysis of variance and regression analysis, we determined the most important influences on blind collision risk. The most important factors were time of entry into the sector and number of aircraft. Also influential were certain orientations of flight paths and the distribution of airspeeds. The least important factor was altitude. The relationship between number of aircraft and the two risk measures could be quantified in regression relationships.

Overview

The airspace over the US is congested, with a high and growing volume of traffic. In the interests of safety, the congestion is made worse by the constraint that aircraft be routed through a series of pre-established airways where they can be subject to the positive control of air traffic controllers. One proposal to mitigate the congestion problem is free flight . There are several variants of free flight, but all give pilots additional authority to fly directly from one point to another without restricting their movements to the established airways. In principle, free flight should save time and fuel. But extra freedom to navigate carries with it an increased risk that aircraft will pass dangerously close to each other, creating a conflict that might become a midair collision.

This analysis addressed the risks of free flight. In particular, the research aimed to better understand the relative importance of several risk factors. We did not consider the effects of actions by pilots and controllers to avoid or resolve conflicts between two aircraft wanting to use the same airspace at the same time. That is, we considered only blind conflicts . To progress from blind conflicts to actual conflicts will require analysis of all the clever technologies and control actions that keep the skies safe and is the ultimate goal of a complete safety analysis. Our more modest goal here was to understand in a quantitative way the background conditions that lead to risk.

We developed a simulation model of the flight of aircraft through an en route sector. Using methods of experimental design and multivariate statistical analysis, we designed and analyzed a large number of scenarios to assess the impact of several variables on measures of free flight risk. We measured risk for a given flight by the mean distance of closest approach to other aircraft and by the mean number of approaches closer than a specified distance. The most important risk factors were time of entry into the sector and number of aircraft. Also influential were certain orientations of flight paths and the distribution of airspeeds. The least important factor was altitude. The relationship between number of aircraft and the two risk measures could be quantified in regression relationships: log-linear in the case of distance of closest approach, linear in the case of number of conflicts.

Literature Review

The most relevant references in the literature are Barnett (2000) and Sherali et al. (2000). Barnett (2000) listed three types of studies of free flight: simple modeling exercises, full-scale simulations, and real-world experiments. Barnett s work was of the first type and Sherali et al. s was of the second. The study reported here is actually a fourth type that falls between these two: an abstract simulation lacking full detail but incorporating more complexity than is easily handled analytically. We were able to relax three of Barnett s assumptions: directional restrictions at each altitude, level flight only, and all aircraft traveling at the same speed. Besides treating more complicated flight trajectories, we were also able to generalize the distribution of times at which aircraft enter the sector. Perhaps most important, we were able to obtain numerical estimates of several measures of risk.

Barnett (2000) used geometrical probability arguments to compare collision risks of the current system of controlled flight with risks under free flight. As noted above, he made a number of simplifying assumptions to render the analysis tractable. He also executed a what if analysis of changes in one en route sector using actual data from a single day s flight operations. He described likely changes in the geometry of flight paths through a sector and the geometry of conflicts. His tentative conclusions were positive for the effect of free flight on sector throughput and safety.

Sherali et al. (2000) approached the problem differently. They made a finely detailed simulation model using actual sector boundaries and aircraft flight plans. Consistent with the recommendations of the FAA/Eurocontrol Collision Risk Modeling Group (1998), they developed detailed descriptions of conflict geometry, including minimum separation distance (which we determined also), duration of conflict, relative headings of aircraft, etc. They also developed a three-level severity scheme for conflicts and an aggregate measure that considered the number and duration of conflicts at each severity level across multiple sectors. They analyzed two pairs of adjacent sectors under three scenarios: current operations using observed flight plans, a restricted form of free flight with reduced vertical separation rules, and an unrestricted form. For the two versions of free flight, Sherali et al. used dynamic programming to compute wind-optimized routes assuming specific historical weather conditions. They computed the conflicts inherent in the actual and hypothetical flight plans. Overall, they estimated that unrestricted free flight would reduce the number of en route blind conflicts at all three levels of severity, and restricted free flight would reduce conflicts even more. They attributed this reduction to dispersal of flights away from current concentrations along narrow routes. They did not explain why the restricted form of free flight was superior to the unrestricted form. Finally, they translated the lower rate of conflicts into an estimated increase in traffic that could be handled without raising the conflict rate above current levels.

Besides these two papers, there are a number of related studies of collision risk:

- Ball et al. (1995) compared one day of actual flight operations with a simulated replay under free flight conditions. They found that free flight dispersed flight paths and thereby reduced en route blind conflicts by 15%. However, allowing free flight in the vicinity of airports (TRACON airspace) increased conflicts by 20%.
- The complicated problems of detecting and, especially, resolving multi-aircraft conflicts was treated in Kuchar and Yang (2000), Prandini et al. (2000), Bicci and Pallottino (2000), Tomlin et al. (1998), and Inselberg and Dimsdale (1994).
- Remington et al. (2000) conducted simulation experiments with live controllers operating under current and free flight conditions. They found that removal of fixed routes did little to increase the time to detect conflicts. Removal of altitude restrictions did slightly increase detection time, although color coding of altitude data helped compensate. The major influences on detection time were traffic load (increasing from 12 to 16 to 20 aircraft) and conflict geometry (duration and angle).

- Harrison and Moek (1992) studied vertical separation norms in European airspace.
- Barnett (1999) considered the risk of conflicts during landing on closely spaced parallel runways.
- Smith et al. (1998) developed an index of dynamic density to serve as a proxy for collision risk. This index tracks the moment-to-moment state of the airspace in terms of a composite, normalized measure of the typical distance between aircraft. It is not of a form that can give a good macroscopic picture of the factors affecting collision risk, though Smith et al. illustrated its value in retrospective interpretation of a series of avoidance maneuvers.
- Hockaday and Chatziioanou (1986) developed an analytical model of blind conflict risk due to random deviations from the filed flight plan in terminal airspace.
- Gifford and Sinha (1991) studied factors influencing the rate of near mid-air collisions in 1985. They found that the level of air traffic, airspace complexity (number of airports) and controller staffing levels all influenced risk.
- Datta and Oliver (1991) studied collision risk in terminal areas, as distinct from the en route phase of flight we studied. They developed a model of risk dependent on the number of aircraft of various types, flight path geometry, and type of air traffic control.

Modeling Assumptions

Since the free flight concept will primarily apply to the en route or cruise portion of a flight, we analyze a hypothetical en route sector. There are hundreds of such sectors in the continental US, all with various shapes and sizes. To simplify analysis, and to examine a geometry different from that in Barnett (2000), we assumed the sector to be circular in cross section with a radius of 100 miles. We also assume a very free form of free flight, in which aircraft can change altitude at will, so the sector can be pictured as a cylinder above the ground. This notional sector is quite large relative to current low and high altitude sectors. However, it is comparable to some current super-high altitude sectors (e.g., ZLC41) and to proposed sectors in the High Altitude Airspace Redesign experiments underway at the time of this writing.

We further assumed that every flight through the sector could be completely characterized by a vector of six components:

- Angles of entry into and exit from the sector. At one extreme, traffic would be random (isotropic), coming and going equally at all angles (and thereby producing some short paths that transit only a small portion of the sector. At the other extreme, traffic would be concentrated along a single narrow path. In between, as is often the case now, there might be a few preferred directions of flow, such as a main east-west path and a secondary north-south path. We assumed that flights move in a straight line while within the sector, never changing heading or circling in a holding pattern.
- Altitudes at entry into and exit from the sector. We assumed straight line flight in the vertical dimension too. At one extreme, flights would all enter the sector at the same altitude and never change altitude while passing through. At the other extreme, flights would all enter at different altitudes and every flight would change altitude within the sector. In between, as is often the case now, flights might enter at one of several altitudes and occasionally change from one level to another.
- Time of entry into the sector. At one extreme, entry times could be uniformly distributed over a fixed time interval, say one hour. At the other extreme, all aircraft might appear simultaneously somewhere along the sector boundary. In between, as is often the case now, there might be several surges in traffic, such as a morning rush from east to west and an afternoon rush from west to east.
- Indicated airspeed. The mix of speeds can be made as even or uneven as desired. Note that this is not speed over the ground but speed in the direction of flight, which can have a vertical component. We assumed that the speed of any individual aircraft is constant while in the sector, so controllers are not separating aircraft using instructions to speed up or slow down.

Simulation Methodology

We wrote a special purpose computer program to perform the simulations on a personal computer. The program allows for simulation of up to 10 groups of aircraft, each sharing a common distribution of values for the six parameters. For instance, one could define a group that is west-to-east, high altitude, late, and fast . The parameter values for an individual aircraft are drawn from a uniform distribution between the lower and upper limits specified for its group. The program can simulate up to 100 aircraft passing through the sector in each replication.

The inputs to each simulation are the number of groups and the associated lower and upper limits for the six parameters characterizing each flight in a group. These factors might be thought of in four categories: directions, altitudes, times, and speeds. A fifth input factor is the total number of aircraft in passing through the sector during a simulation run.

The program provides a number of outputs, which can be classified as pairwise or flightwise measures of risk. Pairwise measures use as their unit of analysis any pair of flights, whereas flightwise measures aggregate pairwise measures over all other flights with which a given flight interacts.

The pairwise measures are:

- % of pairs not sharing the sector. A pair of flights share the sector if they are ever in the sector at the same time. Clearly, a high value of this measure implies lower conflict risk, other things being equal. This measure is an average of binary indicators over all pairs. With N aircraft, there are N!/[(N-2)!2!] pairs.
- % of pairs within five miles, for pairs sharing the sector. This is a conditional probability. Given that a pair of flights does at some time jointly occupy the sector, a certain percentage of them will pass within five miles of each other. Since less than five miles of lateral separation currently defines an operational error , this choice is conventional. The one difference here is that our calculation of distance includes the vertical dimension, so we think

of each aircraft as flying at the center of a sphere with radius five miles. This measure is an average of binary indicators over the subset of pairs that do share the sector.

- Mean distance of closest approach, for pairs sharing the sector. This is a conditional mean. Given that a pair of flights does at some time share the sector, there will be a distance of closest approach between them. This measure is an average of that distance over the subset of pairs that do share the sector.

The flightwise measures are:

- Mean closest approach. This is a conditional mean, computed only for flights that at some point share the sector with one or more other flights. A given flight may share the sector with a number of other flights, each of which has a distance of closest approach. The minimum of all these distances is the overall distance of closest approach. This measure is the average over all flights that shared the sector with at least one other flight.
- Mean number of conflicts. Each flight has a count of other aircraft that came within five miles of it during the simulation. This measure is the average count over all flights.

This collection of performance measures is designed to provide a broad view of risk. For the most part, we consider the flightwise measures to be more readily interpretable. One advantage of reporting the mean closest approach is that it is a continuous measure that is not already binned according to a criterion that might not be useful for every purpose (e.g., 5 miles of separation). Another advantage is that it directly characterizes the most dangerous situation encountered while traversing the sector. On the other hand, the advantage of reporting the mean number of conflicts is that it is more directly indicates potential controller workload. In theory, free flight operations might move primary responsibility for safe separation from controllers to the flight deck, with controllers intervening only when they detect potential conflicts. One performance measure that we did not report is the conflict geometry, i.e., the number and relative positions and velocities of aircraft involved in a conflict. This is an important measure of the complexity of risk, since it can be very difficult to rapidly de-conflict multiple aircraft without creating new conflicts in the process.

We used the simulation program to compute the above risk measures for specified scenarios in two experiments. The first experiment focused on the effect of directional clumping, i.e., comparing traffic that could enter and leave the sector at any angle with traffic that is mostly confined to corridors through the sector. The second experiment was a full factorial design intended to find which of the five categories of input factor had significant main effects and interactions. In each experiment, we selected parameter values to give speeds, times and altitudes that were reasonable matches to current operations. It was not essential for our purposes, however, to make the kind of minutely realistic simulation, such as that of Sherali et al. (2000), which is most credible to air traffic controllers focussed hard on the status quo.

Experiment #1: Effect of Directional Clumping

Figure 1 illustrates with 50 aircraft the contrast we studied in the first experiment. On the left is a random (isotropic) traffic pattern, with flights entering and leaving at uniformly distributed angles. This is an idealized version of free flight. On the right is a situation more typical of current and anticipated operations with three groups of aircraft: a roughly left-to-right flow, a roughly bottom-to-top flow, and a small random flow. It is important to make this comparison because, in practice, free flight may look more clumped than random. This is because most flights will continue to be made to and from a restricted set of origin-destination pairs. Furthermore, winds aloft will tend to concentrate flights into narrow bands for reasons of fuel efficiency.

The specific parameter values for the simulated aircraft are listed in Table 1. Our goal in selecting these values was to depict a reasonable range of conditions. We performed simulations using 10, 50 and 100 aircraft per replication. For each scenario, the number of replications varied inversely with the time required for one replication. We used 100 replications when simulating 10 aircraft, 25 when simulating 50, and 10 when simulating 100. Given the greater internal averaging with larger numbers of aircraft per run, these sample sizes produced roughly equal levels of uncertainty for all three numbers of aircraft.

The pairwise results comparing random and clumped flights are shown in Figures 2, 3, and 4. These plots show estimated averages with error bars set at two standard errors. Figure 2 shows the percentage of aircraft pairs not sharing the sector at some time. Figure 3 shows the percentage of aircraft pairs sharing the sector that closed to within five miles of each other. Figure 4 shows the mean closest approach for aircraft pairs sharing the sector. For all three risk measures, clumping most of the flights into two corridors significantly increased the risk, as one would expect. More surprising was the finding that the number of flights had no statistically significant effect on the pairwise risk measures (but see below).

The flightwise results are shown in Figures 5 and 6, which report, respectively, the mean closest approach and the mean number of conflicts. Once again, directional clumping substantially increased the risk of blind conflict. Unlike the pairwise results, these flightwise results also showed a significant influence for the number of aircraft. This is reasonable, since flightwise results require comparison against all pairs together. The combination of directional clumping and 50 or more flights reduced the mean distance of closest approach to an uncomfortably small level. Under these conditions, the average flight was very likely to encounter at least one conflict while traversing the sector, with some flights exposed to multiple conflicts.

These relationships can be given quantitative form. Figure 7 plots mean closest approach against the number of flights on a log-log scale, superimposing ordinary least squares (OLS) fits for both clumped and random directions. These patterns can be summarized by the log-linear regression relationship

$$\text{Ln[Mean Closest Approach]} = \underset{(0.08)}{5.63} - \underset{(0.04)}{0.60}\ \text{Clumped} - \underset{(0.03)}{0.82}\ \text{Ln[Number of Flights]}$$

for which the adjusted R-square value is 79%. (The numbers in parentheses are estimated standard errors. In all the regression analyses of this experiment, one could improve somewhat on the regression estimates by using weighted least squares, but the OLS parameter estimates are unbiased.) Each 10% increase in the number of aircraft using the sector resulted in about an 8% decrease in the mean distance of closest approach. Clumping of the particular type we simulated reduced the distance by about a factor of exp[-0.60] = 0.55, i.e., a 45% reduction.

Figure 8 shows the relationship between mean number of conflicts and number of flights, again with separate OLS fits superimposed for both random and clumped directions. The fits through the origin were, for random directions

$$\text{Number of Conflicts} = \underset{(0.001)}{0.011}\ \text{Number of Flights} \quad (\text{Rsquare} = 89\%)$$

and for clumped directions

$$\text{Number of Conflicts} = \underset{(0.001)}{0.020}\ \text{Number of Flights} \quad (\text{Rsquare} = 93\%).$$

The effect of each additional flight was 77% worse when flight directions were clumped. Thus, density of flights clearly matters. (The regression residuals showed slight curvature, suggesting that nonlinear models could improve the fit slightly, but the simpler models were adequate for our purposes. We consider log-linear models below.)

Experiment #2: Full Factorial Experiment

The parameter values in Table 1 were selected to cover a reasonable range of values for the factors not of primary interest. However, the levels chosen for these background factors surely interacted to influence the estimated effects of directional clumping. To get a more comprehensive view of the factors influencing risk, we designed a second experiment to explicitly consider the separate and combined influences of all five factors: directions, times, altitudes, speeds, and number of aircraft.

To untangle the separate and combined effects of the five factors, we executed a full 4^5 factorial experimental design with three replications per cell. That is, we created 1,024 scenarios and ran the simulation program three times for each scenario, for a total of 3,072 simulation runs. Each scenario used one of four levels for each of the five factors. All scenarios involved two groups of aircraft, one heading left to right (defined as a course of 0 degrees), the other heading in a direction that varied by scenario.

We chose the four levels for each factor in order to assess the impacts of various patterns. For the directional factor, we created four parameter combinations graded by the degree to which the two traffic streams intersected, ranging from flow in the same direction to flow in opposite directions. For the time, altitude, and speed factors, the parameter combinations were formed from the four combinations created by equal or unequal means for the two groups of aircraft and zero or nonzero variances. For the number of aircraft factor, we used a total of 10, 25, 50 and 100 flights, with each aircraft equally likely to be assigned to either group. Table 2 lists the parameter values for each factor for each group of aircraft.

We analyzed the simulation outputs using analysis of variance (ANOVA) to test the significance of main effects and interactions for the five factors. Because the distribution of both risk measures was skewed and had higher variance for scenarios with higher means, the raw data did not satisfy the assumptions for ANOVA inference. However, the distributions of the logarithm of the mean closest approach and of the started logarithm of the number of conflicts were much closer to the ideal of a Normal distribution with constant variance.

Tables 3 and 4 show the ANOVA results, sorted to highlight the most important effects. Both tables show that the chief influences on the response variables were sector arrival times and number of flights. These two terms alone accounted for the majority of variation in the response. Other main effects and interactions were statistically significant but negligible in size (except for the Times x Flights interaction term for conflicts).

We next analyzed the mean levels of risk for the four levels of each factor. Knowing that the time factor is important does not explain which combinations of mean and variance accounted for the highest and lowest risk. Figures 9 — 13 present these results. Note that the two risk measures tend to move in opposite directions in the figures.

Figure 9 shows the effect on both flightwise performance measures of the distributions of the time of entering the sector for both groups of flights. As one might expect, the worst case is when the entry times are constant and equal for both groups (ConsSame). This scenario forces all aircraft in both groups to arrive simultaneously at the sector boundary. The best case is when the times are randomly distributed without any overlap (RandDiff). Both the best and worst cases are extreme situations and unlikely to arise in practice. However, including all four combinations of mean and variance allowed us to confirm some commonsense general conclusions: variance in the sector arrival times is good, since it spreads out the traffic, and differences in the means are good for the same reason.

Figure 10 shows the effect of the directions of flight. By both measures, the riskiest situation was when the two streams were flying in opposite directions (180_Deg). This configuration maximizes the number of other aircraft to which a given flight is exposed.

Figure 11 shows the effect of various distributions of air speeds. Speeds had no appreciable influence on the mean distance of closest approach. This was as it should have been, since speed does not change geometry. However, having identical speeds for all aircraft in a group generated the most conflicts. Having homogeneous speeds prolongs the time during which two flights are at risk to interact. In contrast, if two aircraft travel at different speeds, they can drift apart. As Datta and Oliver noted (1991), it is relative velocities that matter.

Figure 12 shows the influence of the distributions of altitudes. The altitude variable had little effect. Obviously, using different altitudes must reduce risk, but the effect was surprisingly small and not statistically significant for the factor levels used in our experiments. In contrast, the lateral dimension of separation varied much more than the vertical.

Figure 13 shows the influence of the number of flights using the sector. For both risk measures, there was a strong relationship between more flights and more risk. Consistent with the results shown in Figure 7, the distance relationship can be modeled well by a power law or log-linear model

$$\text{Ln[Mean Closest Approach]} = \alpha + \beta \ \text{Ln[Number of Flights]}$$

where the intercept α and slope β depend on the values of the other factors. The same type of model can also be fit to the conflict relationship

$$\text{Ln[Mean Number of Conflicts]} = \alpha + \beta \ \text{Ln[Number of Flights]}$$

although Figure 8 shows that the slope (i.e., the power to which the number of flights is raised) was close to unity for this measure. The upper portion of Figure 14 shows the substantial variation across scenarios in the estimate of slope β for the distance relationship. Most scenarios displayed decreasing returns to scale in the number of aircraft (i.e., slope between 0 and —1). The lower portion of Figure 14 shows that the slopes for the conflict relationship clustered close to unity across scenarios, justifying the linear model in Figure 8. Figure 15 documents that log-linear models fit well across most scenarios, with an average Rsquare value of 91% for distance of closest approach and 94% for number of conflicts.

Summary and Conclusions

We abstracted the essential features of free flight in a simulation model of blind conflicts, i.e., close approaches developed in the absence of maneuvers to avoid or resolve dangerous proximities. Our model assumed linear flight trajectories at constant airspeed within a cylindrical sector with a 100-mile radius.

We primarily assessed conflict risk using two flightwise measures: mean distance of closest approach to a given flight by other aircraft with which it shared the sector, and mean number of conflicts for a given flight, defining a conflict as an approach within five miles. We also developed three pairwise measures of risk (percentage of pairs of flights not sharing the sector, percentage of pairs that share the sector and approach within five miles of each other, and mean closest approach for pairs that share the sector). We consider the flightwise measures more useful because they are more readily interpreted.

We used the model to better understand the quantitative relationships between the two flightwise risk measures and five factors: flight directions, altitudes, airspeeds, sector entry times, and number of aircraft.

Our first experiment considered the difference in risks between completely random flight directions and a clumped pattern involving one main flight corridor, one secondary corridor, and a random background.

For the specific factor levels chosen, we found:

- Clumping decreased the mean distance of closest approach by about 45%. Since actual operations are more likely to be clumped than random, this result reminds us that the big sky notion implicit in the concept of free flight has to be regarded with some caution. (However, as Sridhar et al. (2000) demonstrated, it is possible to reduce conflicts when going from the current very constricted set of routes to a sky made somewhat bigger by allowing shortcuts.)
- Flying more aircraft through the sector decreased the distance of closest approach in a nonlinear way well approximated by a power law: every 10% increase in the number of aircraft resulted in about an 8% decrease in the mean distance of closest approach.
- Flying more aircraft also increased the number of conflicts experienced by an average flight. This relationship was approximately linear, with a higher rate of increase when flight directions were clumped. Even with random directions, 100 flights produced an average of about 1 conflict per aircraft. This increased to almost 2 conflicts per aircraft when directions were clumped. Note that this linear relationship is not inconsistent with the quadratic relationship noted by Datta and Oliver (1991) and Sherali et al. (2000). We saw that any one flight has an expected number of conflicts proportional to N, the number of aircraft using the

sector. However, the total number of conflicts in the sector from all aircraft is N times the value expected for each flight, giving an N^2 relationship for total conflict count.

The first experiment quantified the importance of flight clumping and number of aircraft on both measures of conflict risk. However, these relationships were surely mediated by the levels of the other three factors and by our particular choice of flight trajectories. To systematically explore the full set of relationships, we ran a second experiment.

The second experiment was a 4^5 factorial design with three replications per cell. This design allows assessment of all main effects and interactions. We examined the case of two groups of aircraft, each characterized by six attributes: directions of entering and leaving the sector, altitudes at entry and exit, airspeed, and time of sector entry. For each aircraft in either group, these six attributes were assigned values at random from a uniform distribution with specified upper and lower limits. Altogether, there were 24 parameters needed to fully specify the experiment (6 attributes x 2 limits per attribute x 2 groups of aircraft). We chose values for the 24 parameters to roughly match current practice but also to expose the separate effects of means and variances.

In the second experiment, we found:

- The two most influential factors were time of entry and number of aircraft. In other words, the airline schedule is the most significant influence on blind collision risk.
- There was an approximate power law relationship between mean closest approach and number of aircraft. However, the parameters of the relationship varied with the levels of the other factors, so we cannot speak of a single, universal relationship.
- There was also an approximate power law relationship between mean number of conflicts per aircraft and number of aircraft that held over a wide range of scenarios. Most of the time, this relationship was close to linear.

- The most dangerous configuration of flight paths was when the two streams were opposed, since this maximized the number of aircraft pairs that could interact. (It also reduces the time available for controllers to react to a developing conflict.)
- The mix of altitudes had only a minor influence on risk. This increases our confidence in Barnett s (2000) analysis, which ignored the vertical dimension.
- A wider range of aircraft speeds was safer than a narrower range. A wide range allowed aircraft pairs to drift apart over time. This finding contradicts Barnett s speculation (p. 844) that equal speeds might be more favorable than mixed speeds. (Note, though, that controllers find it easier to handle traffic with little variation in airspeed; here we see again the distinction between blind conflicts and operational errors.)

To summarize what we learned from this simulation model in general terms, we have documented that the risk of blind conflict under free flight is heavily dependent on the levels and combinations of several factors. We have identified situations that are inherently more or less risky, and established that the variation in risk across these situations is substantial. In turn, this knowledge could be used to guide the choice of scenarios for the next stage of conflict analysis, i.e., studies along the lines of Remington et al. (2000) but even more realistic, using both simulated controllers and pilots to assess the risk of actual conflict. We have also filled a gap in the set of methods for risk assessment, working with more complexity than Barnett (2000) but with less encumbering detail than Sherali et al. (2000).

One way to summarize many of our results is that density matters. Since aircraft density changes as aircraft move, it may be that the time-average density would be a powerful summary measure of risk. Future research might well try to operationalize the notion of aircraft density. In general, density is a count divided by a volume. To be useful, a computation of density would have to consider a relevant volume, such as the 3-D convex hull of the aircraft positions at a given moment. However, there are some problems with this approach. First, computing a convex hull is not cheap. Second, the convex hull does not serve well when there are several widely separated clusters of aircraft or even single aircraft, since the hull would have to include all the empty spaces among clusters. There may be some merit to defining density as the count of aircraft in a sector divided by the length of the minimal spanning tree (MST) connecting them. An MST is easier to compute than a convex hull and less sensitive to the presence of a few isolated aircraft. This approach may be less cumbersome and more interpretable than that of Smith et al. (1998).

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Figure 1: Comparing random and clumped directional flow through a circular sector

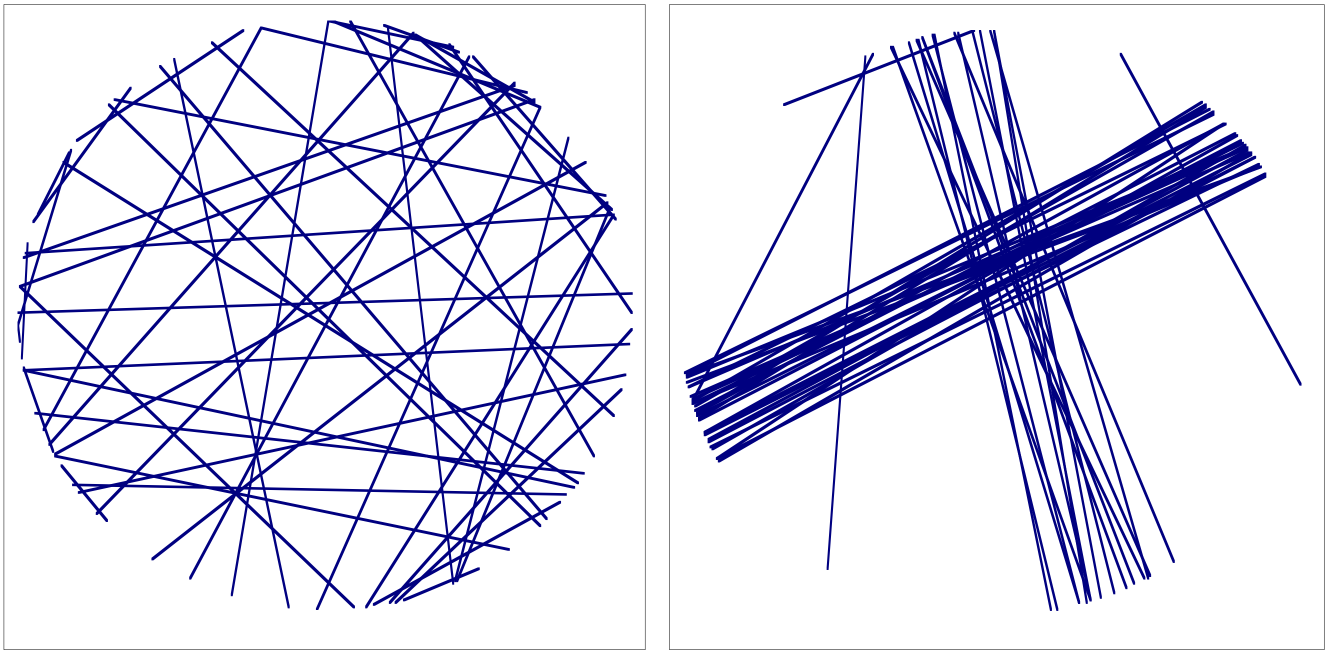


Figure 2: Percentage of aircraft pairs not sharing the sector

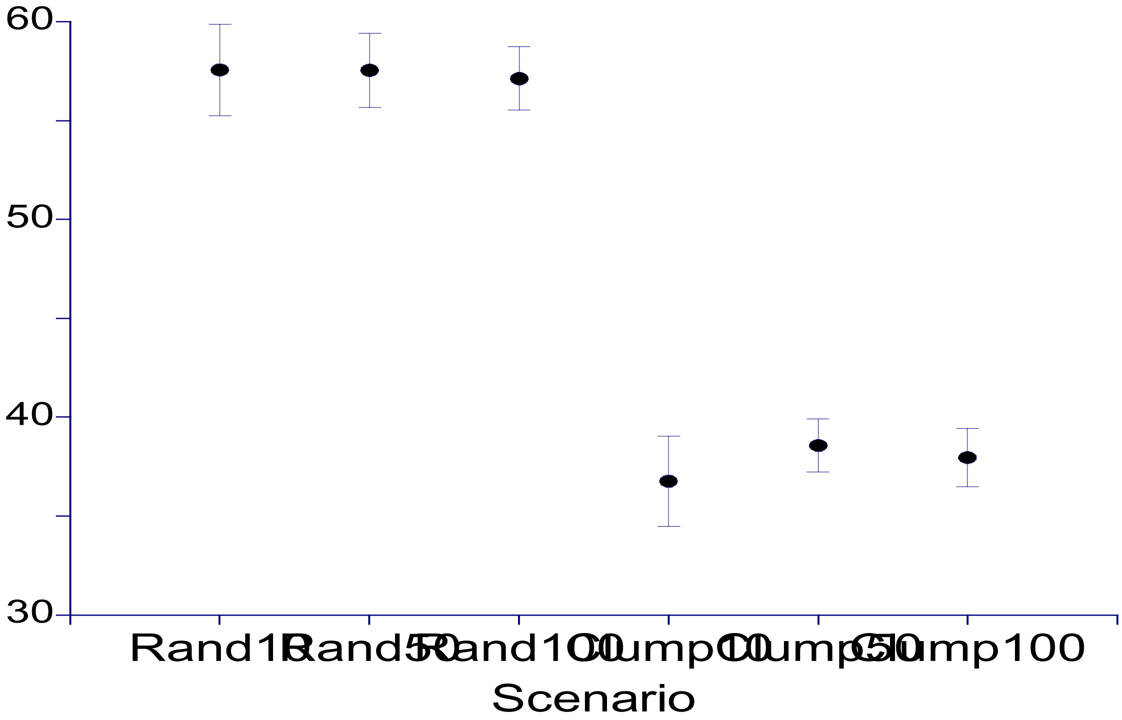


Figure 3: Percentage of aircraft pairs sharing the sector that approach within 5 miles

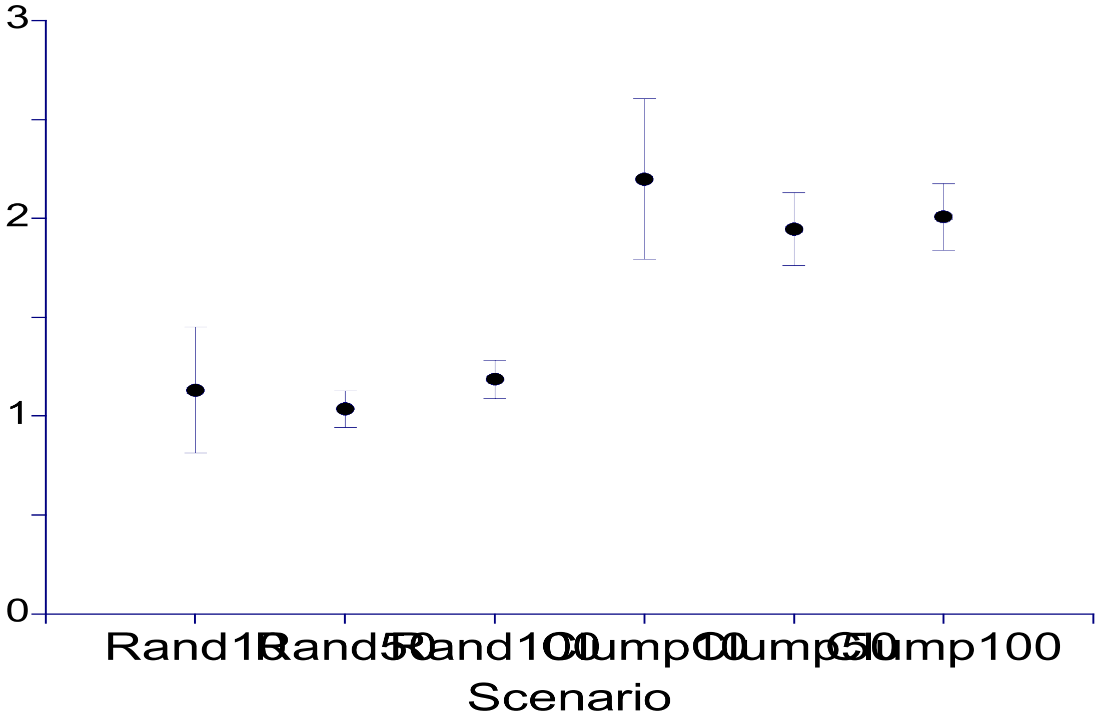


Figure 4: Mean closest approach for aircraft pairs sharing the sector

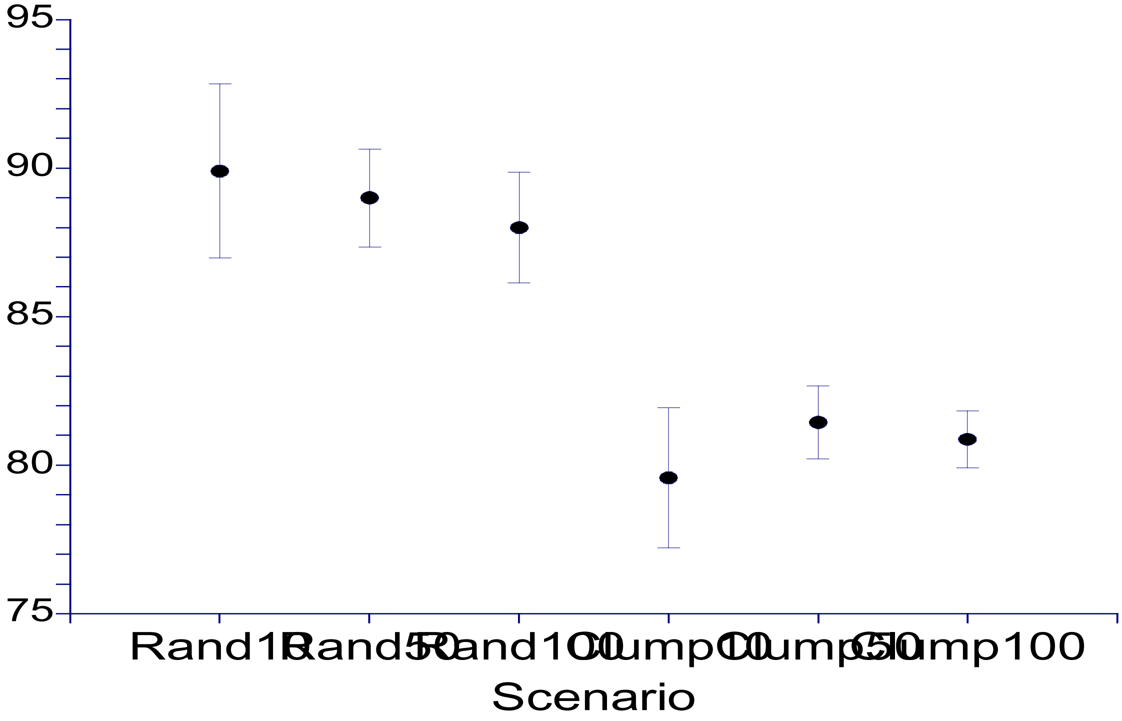


Figure 5: Mean closest approach for a given flight

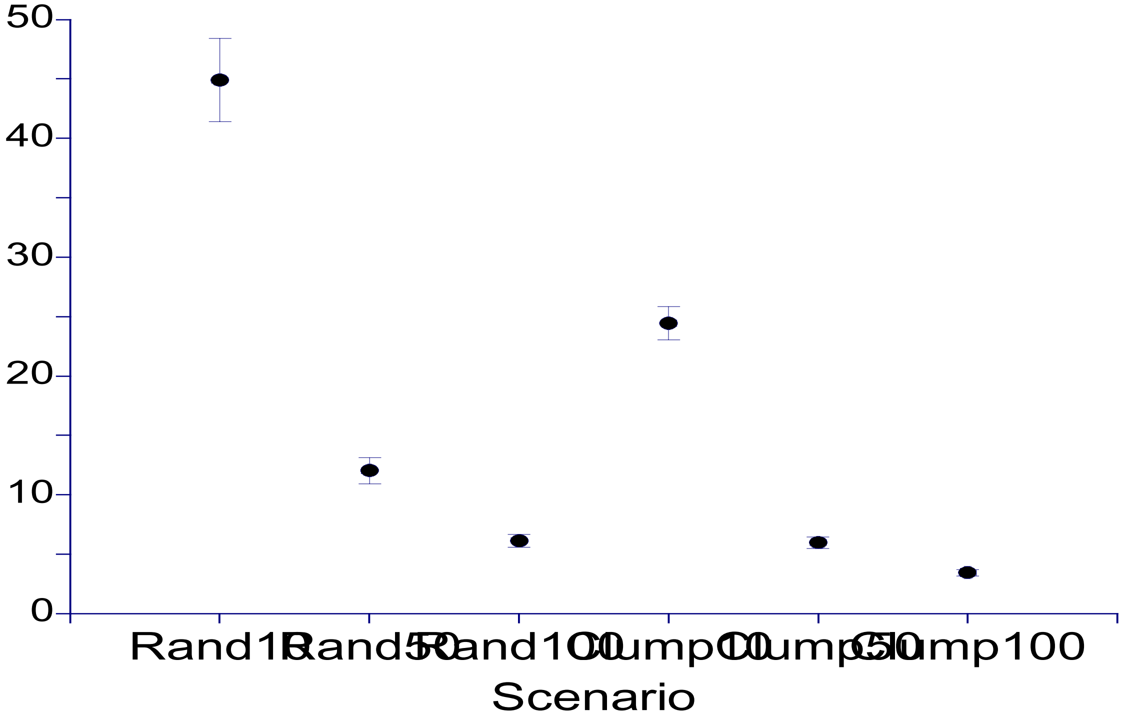


Figure 6: Mean number of conflicts for a given flight

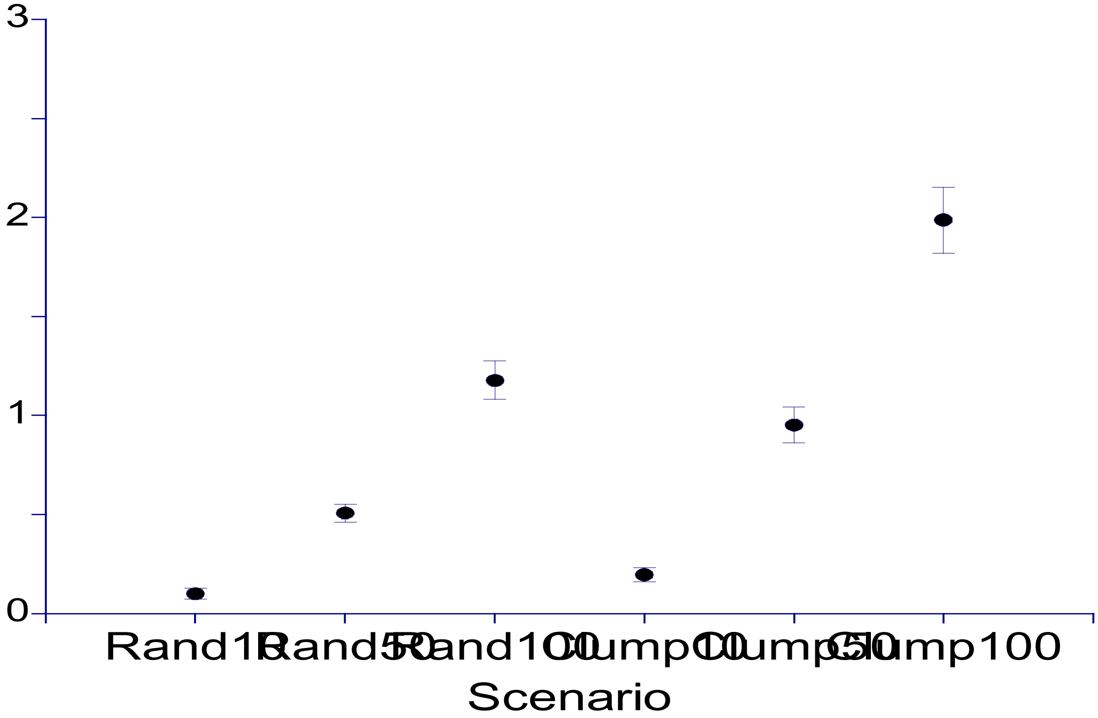


Figure 7: Mean closest approach as a function of number of flights

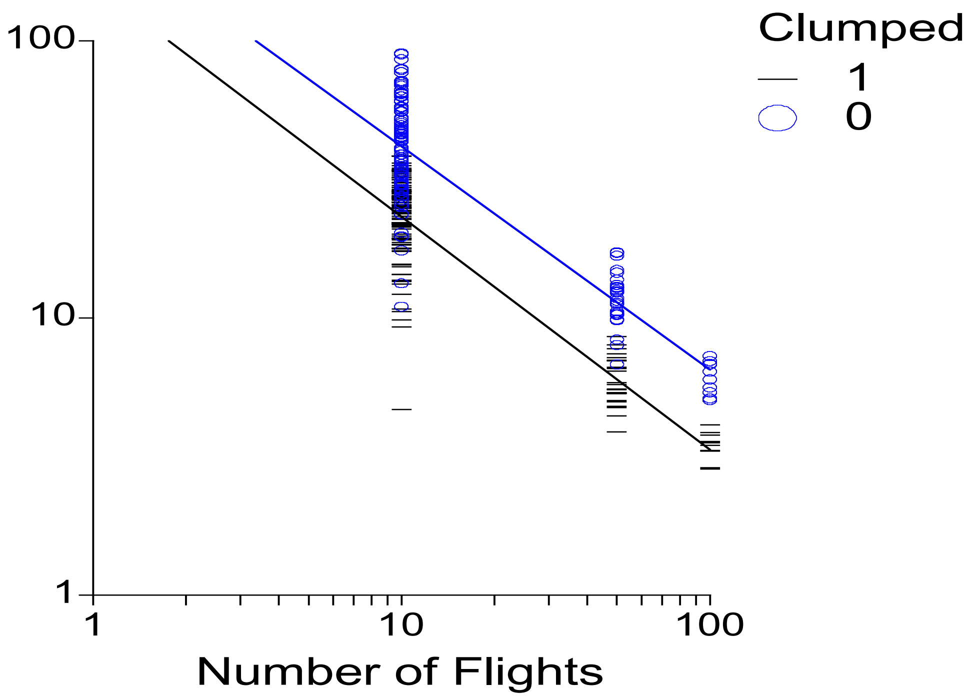


Figure 8: Mean number of conflicts as a function of number of flights

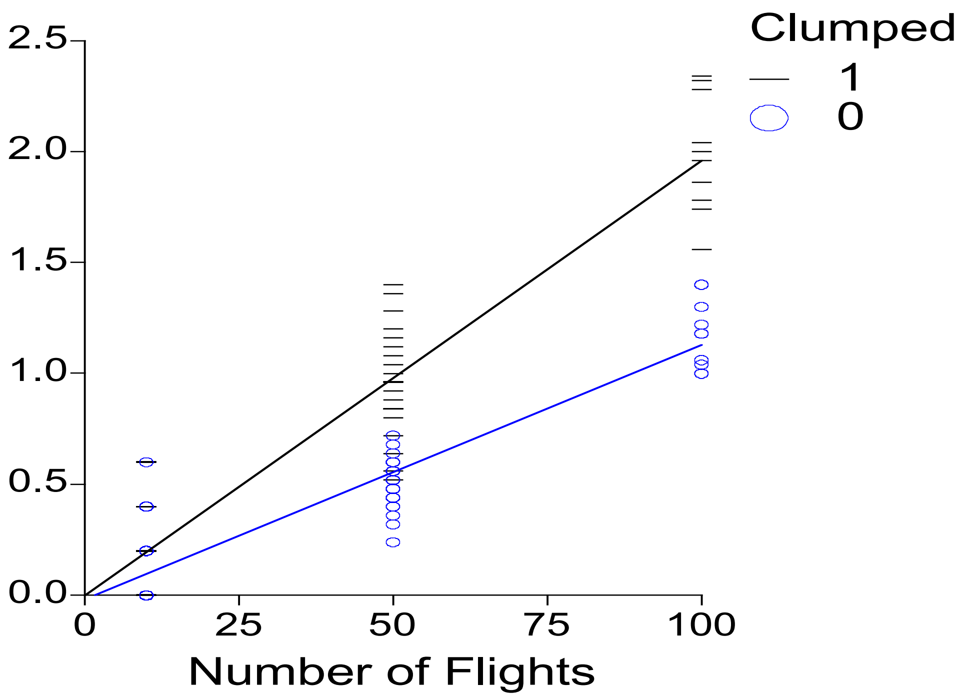


Figure 9: Effect of times of entering sector

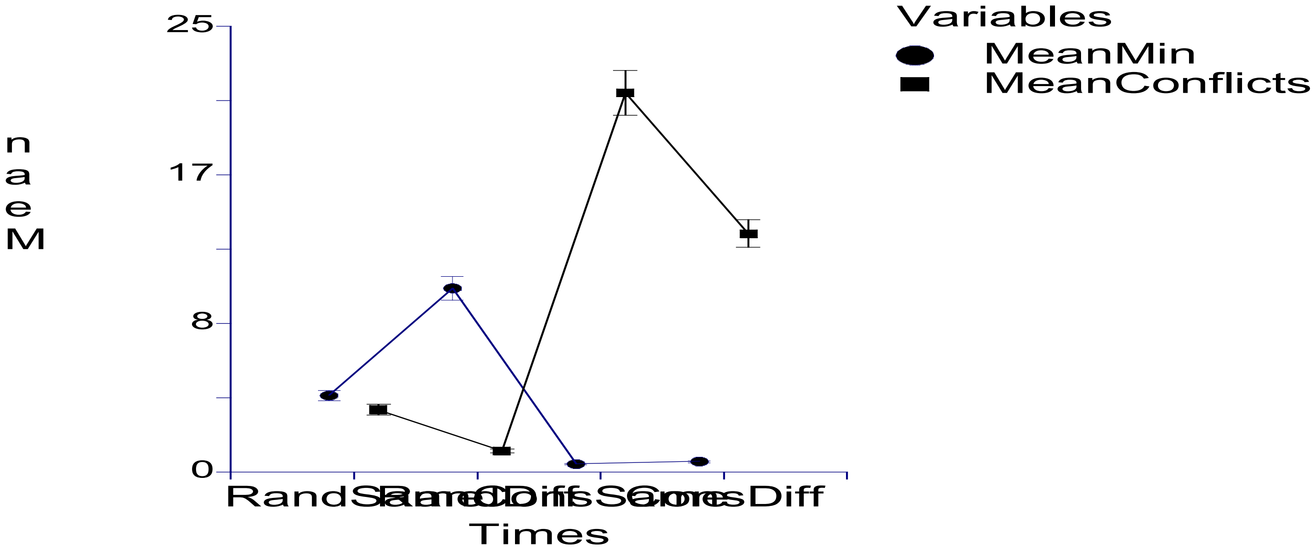


Figure 10: Effect of directions of flight

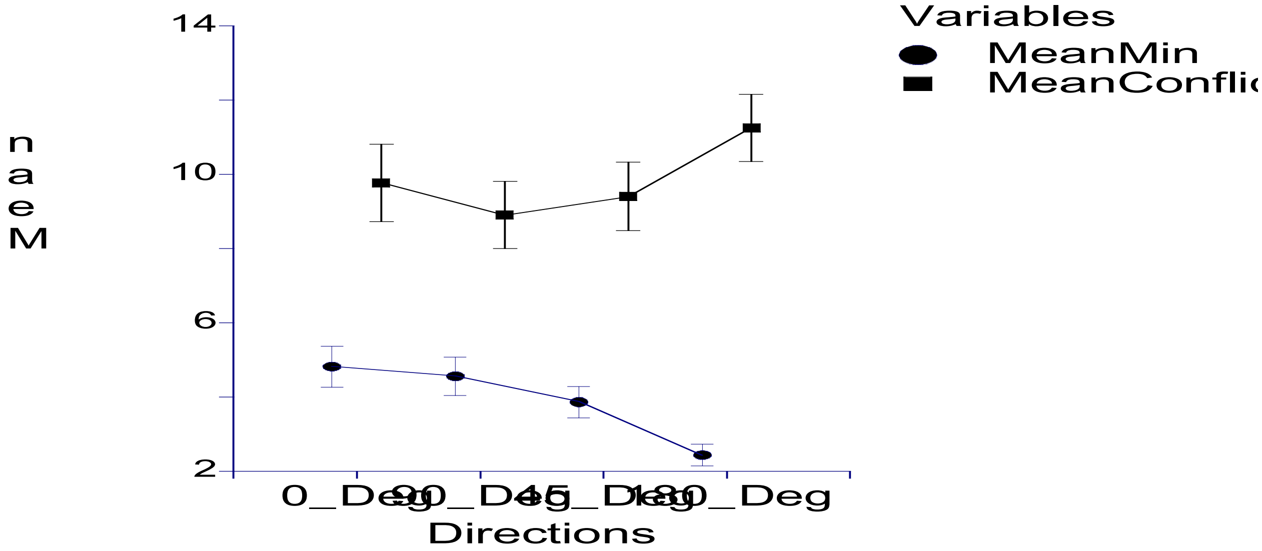


Figure 11: Effect of air speeds

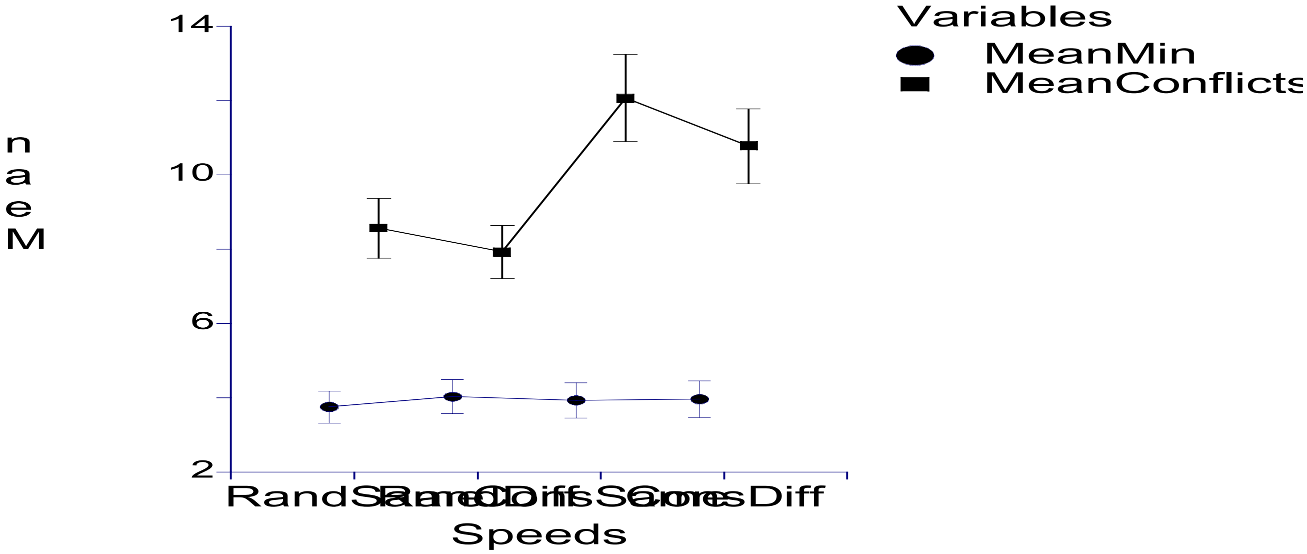


Figure 12: Effect of altitudes

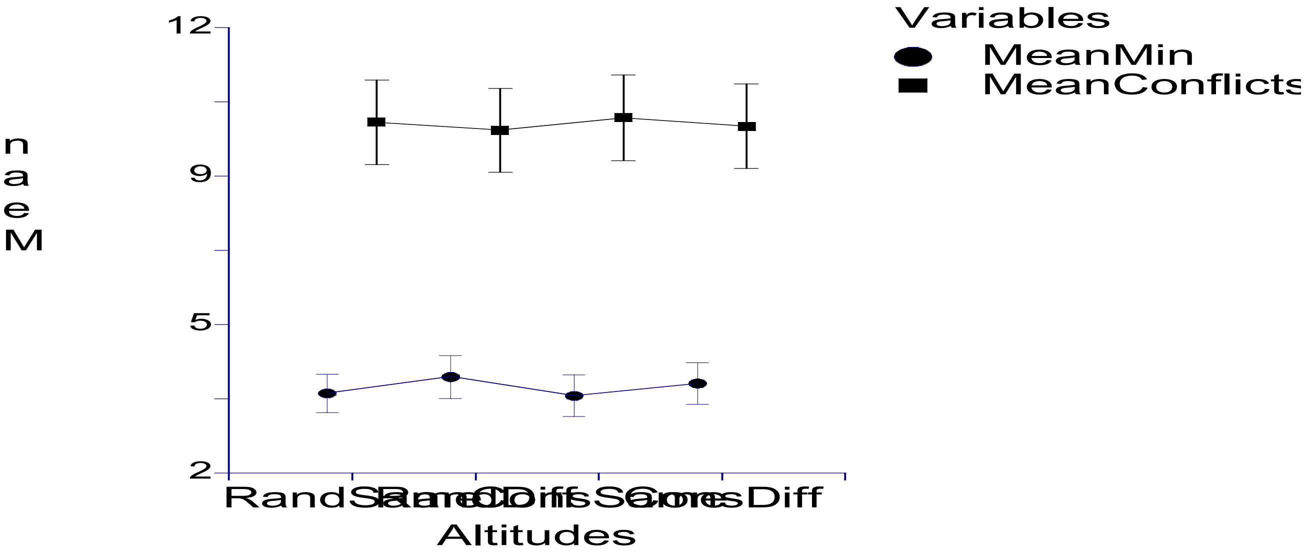
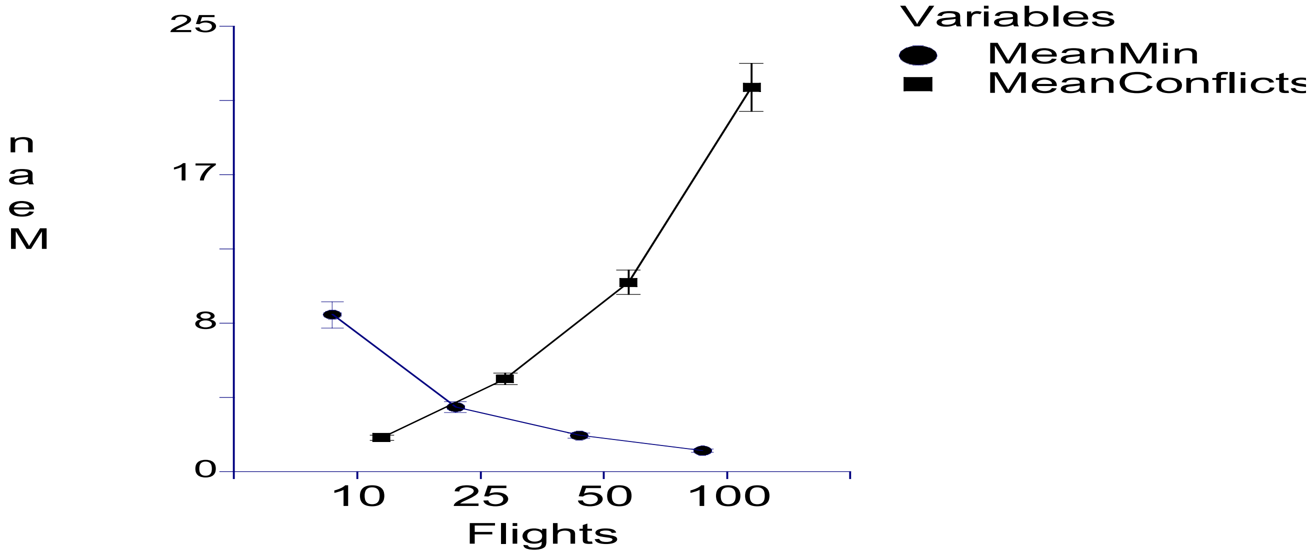


Figure 13: Effect of number of flights



Note: The horizontal axis spacing is not to scale.

Figure 14: Variation across scenarios in estimates of the slopes in log-linear models relating flightwise risk measures to number of aircraft

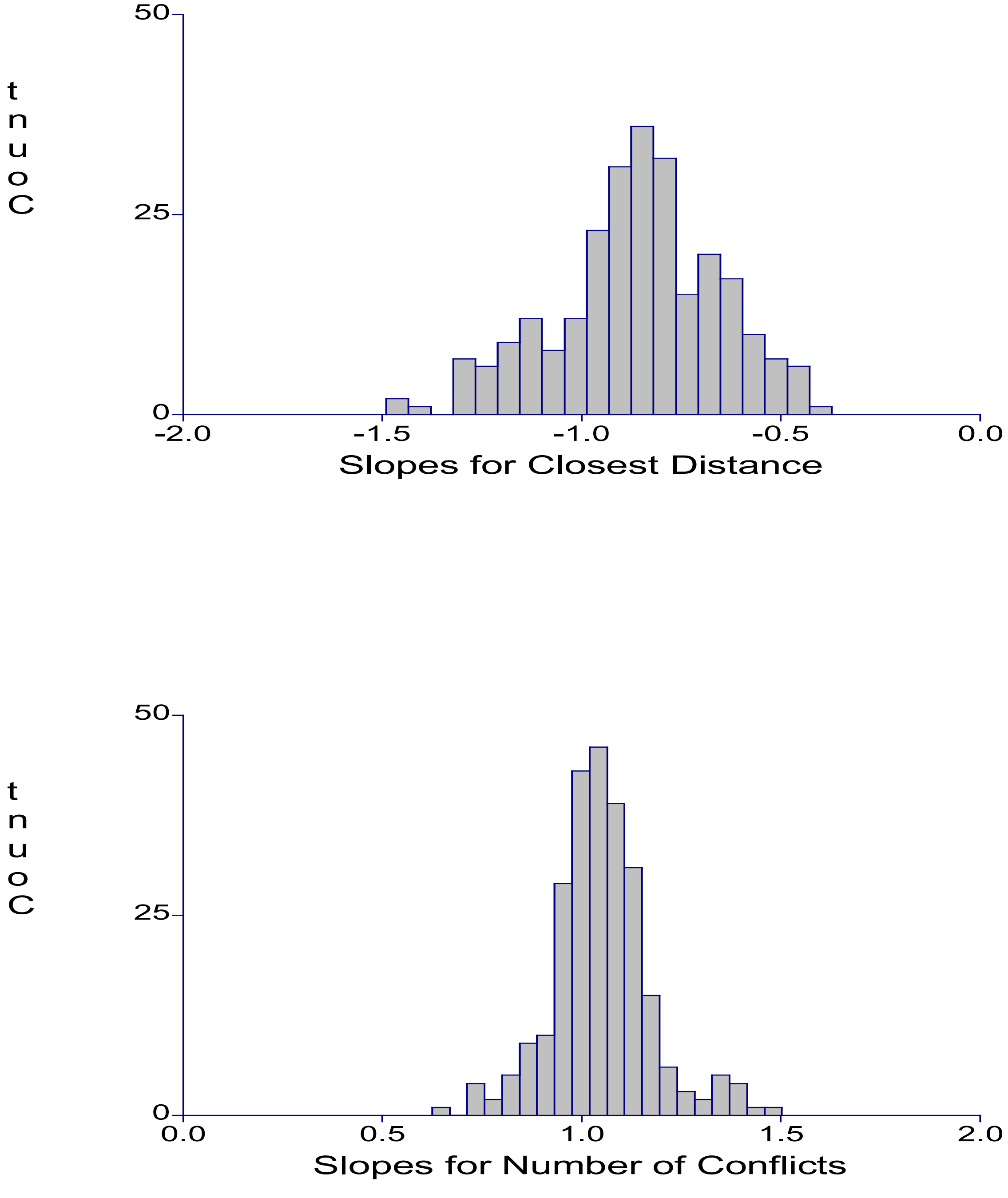


Figure 15: Variation across scenarios in goodness of fit of log-linear models relating flightwise risk measures to number of aircraft

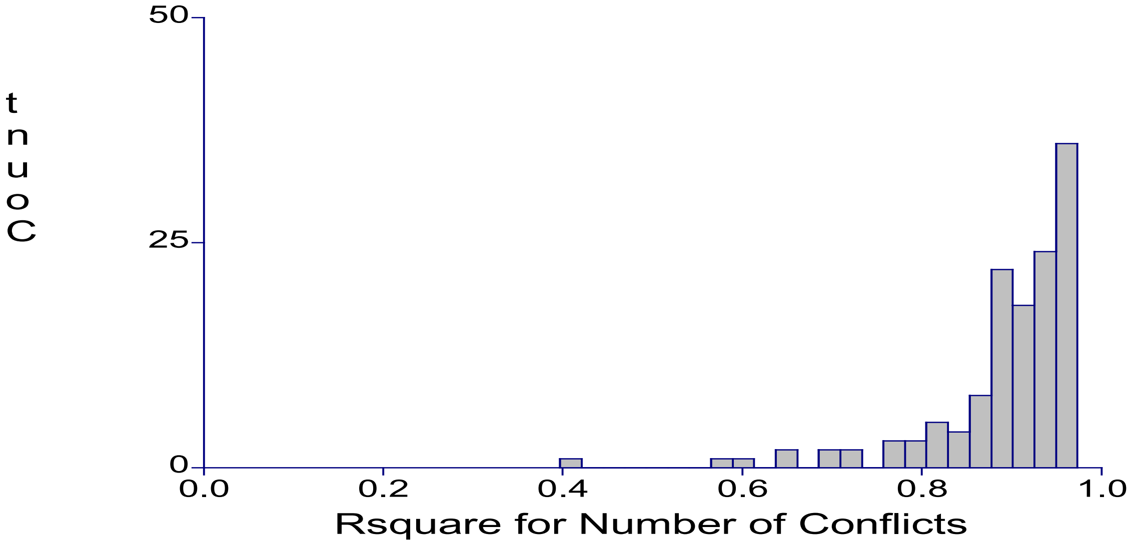


Table 1: Parameters values for experiment with directional clumping

Type of Flow	Random		Clumped	
Group	1	1	2	3
% of Flights	100	60	30	10
Angle In	0 ± 180	200 ± 10	290 ± 10	0 ± 180
Angle Out	0 ± 180	40 ± 10	100 ± 10	0 ± 180
Altitude In	350 ± 50	350 ± 50	350 ± 50	350 ± 50
Altitude Out	350 ± 50	350 ± 50	350 ± 50	350 ± 50
Time In	0.5 ± 0.5	0.5 ± 0.5	0.5 ± 0.5	0.5 ± 0.5
Speed	450 ± 50	450 ± 50	450 ± 50	450 ± 50

Note: Angles specified from 0° along the horizontal, altitudes in 100 s of feet, times in hours, speeds in miles per hour (not using the more conventional knots is immaterial).

Table 2: Parameter values for factorial experiment

Directions	Code	Meaning	Group	InAngle Center	InAngle Range	OutAngle center	OutAngle range	Units
	1	0 Degrees	a	180	10	0	10	Degrees
			b	180	10	0	10	
	2	90 Degrees	a	180	10	0	10	
			b	90	10	270	10	
	3	45 Degrees	a	180	10	0	10	
			b	45	10	225	10	
	4	180 Degrees	a	180	10	0	10	
			b	0	10	180	10	

Times	Code	Meaning	Group	InTime Center	InTime Range	Units
	1	Random, Same	a	0.5	0.25	Hours
			b	0.5	0.25	
	2	Random, Different	a	0.25	0.25	
			b	0.75	0.25	
	3	Constant, Same	a	0.5	0	
			b	0.5	0	
	4	Constant, Different	a	0.25	0	
			b	0.75	0	

Speeds	Code	Meaning	Group	Speed Center	Speed Range	Units
	1	Random, Same	a	500	50	Miles per Hour
			b	500	50	
	2	Random, Different	a	550	50	
			b	450	50	
	3	Constant, Same	a	500	0	
			b	500	0	
	4	Constant, Different	a	550	0	
			b	450	0	

Altitudes	Code	Meaning	Group	InAltitude Center	InAltitude Range	OutAltitude Center	OutAltitude Range	Units
	1	Random, Same	a	35000	5000	35000	5000	Feet
			b	35000	5000	35000	5000	
	2	Random, Different	a	40000	5000	40000	5000	
			b	30000	5000	30000	5000	
	3	Constant, Same	a	35000	0	35000	0	
			b	35000	0	35000	0	
	4	Constant, Different	a	40000	0	40000	0	
			b	30000	0	30000	0	

Note: All distributions are uniform from Center - Range to Center + Range

Table 3: Analysis of variance for logarithm of mean closest approach

Source **Term**	**DF**	**Sum of** **Squares**	**Mean** **Square**	**F-Ratio**	**Prob** **Level**
A (Times)	3	6030.64	2010.21	28289.73	0.00
B (Flights)	3	1639.92	546.64	7692.86	0.00
C (Speeds)	3	240.69	80.23	1129.08	0.00
D (Altitudes)	3	164.47	54.82	771.53	0.00
AC	9	335.32	37.26	524.33	0.00
E (Directions)	3	78.18	26.06	366.74	0.00
AD	9	148.67	16.52	232.47	0.00
AE	9	106.97	11.89	167.27	0.00
BD	9	30.99	3.44	48.45	0.00
CD	9	16.26	1.81	25.43	0.00
ABD	27	28.79	1.07	15.01	0.00
AB	9	6.97	0.77	10.90	0.00
ACD	27	19.42	0.72	10.12	0.00
ADE	27	13.75	0.51	7.17	0.00
DE	9	3.66	0.41	5.72	0.00
ABE	27	7.27	0.27	3.79	0.00
ACE	27	5.77	0.21	3.01	0.00
BE	9	1.62	0.18	2.53	0.01
BDE	27	3.26	0.12	1.70	0.01
BC	9	1.01	0.11	1.58	0.12
ABCD	81	8.29	0.10	1.44	0.01
BCE	27	2.43	0.09	1.26	0.16
ABDE	81	6.72	0.08	1.17	0.15
ACDE	81	6.74	0.08	1.17	0.15
CE	9	0.74	0.08	1.16	0.32
BCD	27	2.19	0.08	1.14	0.28
ABC	27	2.16	0.08	1.13	0.30
ABCDE	243	15.57	0.06	0.90	0.85
BCDE	81	4.82	0.06	0.84	0.85
ABCE	81	4.52	0.06	0.79	0.92
CDE	27	1.47	0.05	0.76	0.80
S	2048	145.53	0.07		
Total (Adjusted)	3071	9084.80			
Total	3072				

Table 4: Analysis of variance for started logarithm of mean number of conflicts

Source **Term**	**DF**	**Sum of** **Squares**	**Mean** **Square**	**F-Ratio**	**Prob** **Level**
A (Times)	3	8045.75	2681.92	1374.08	0.00
B (Flights)	3	5931.07	1977.02	1012.93	0.00
AB	9	1670.44	185.60	95.09	0.00
E (Directions)	3	518.12	172.71	88.49	0.00
AE	9	531.85	59.09	30.28	0.00
BE	9	129.81	14.42	7.39	0.00
AC	9	105.58	11.73	6.01	0.00
C (Speeds)	3	33.69	11.23	5.75	0.00
ABE	27	196.24	7.27	3.72	0.00
BC	9	64.09	7.12	3.65	0.00
ADE	27	122.50	4.54	2.32	0.00
ABDE	81	338.24	4.18	2.14	0.00
ABC	27	101.16	3.75	1.92	0.00
DE	9	25.46	2.83	1.45	0.16
BDE	27	70.52	2.61	1.34	0.11
CE	9	17.49	1.94	1.00	0.44
CD	9	17.27	1.92	0.98	0.45
BCDE	81	152.62	1.88	0.97	0.57
D (Altitudes)	3	5.52	1.84	0.94	0.42
ABCDE	243	444.29	1.83	0.94	0.74
ACDE	81	146.64	1.81	0.93	0.66
CDE	27	48.65	1.80	0.92	0.58
AD	9	14.72	1.64	0.84	0.58
BCD	27	43.48	1.61	0.83	0.72
ACD	27	42.57	1.58	0.81	0.75
BD	9	12.96	1.44	0.74	0.67
ACE	27	38.34	1.42	0.73	0.84
ABD	27	37.98	1.41	0.72	0.85
ABCE	81	112.94	1.39	0.71	0.97
ABCD	81	106.61	1.32	0.67	0.99
BCE	27	27.73	1.03	0.53	0.98
S	2048	3997.27	1.95		
Total (Adjusted)	3071	23151.61			
Total	3072				