Nonlinear Analysis of TCP-RED Instabilities

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Problem: How to set RED parameters in a robust way?
Approach: Use a dynamical system perspective to stability

First and second return map and their intersection with 45 degree line show the existence of a fixed point and period two orbit

✓ Stability analysis of period doubling and dynamical stability criteria
✓ Provides stability criteria for emerging period two orbit
✓ Uses bifurcation diagrams to investigate the solutions as system parameters like number of connection (n), round trip time (R) and RED parameters like $p_{max}$, $q_{min}$, $q_{max}$ and exponentially averaging weight ($w$) vary.
✓ Existence of primary bifurcation in the form of period doubling
✓ Secondary bifurcations like border collision
✓ Sequence of bifurcations leading to Chaos
✓ Sensitivity observed in practice can be reproduced from

Mathematical Modeling:

$q_{k+1} = G(p_k)$
$q_{e,k+1} = A(q_{e,k}, q_k)$
$p_{k+1} = H(q_{e,k+1})$

✓ Essentially nonlinear first order discrete time dynamical system
✓ Nonlinearity comes from TCP transfer function, square root dependence on drop prob.
✓ Model as a self clocking system
✓ Piecewise smooth map

Simplied one hop network model as a feedback system

Senders

flow(1)
flow(2)

$E_1$
$E_2$

$\sum e_i(p)$

n1

$\frac{L_{c}}{n}$

n2

$\Sigma e_i(p)$

Average queue size, $q_i$

Drop rate, $R_i$
to senders

Feedback control:
Packet drop rate

$u_{e,k+1} = \begin{cases} (1-w)q_{e,k} + w B & \text{if } q_{e,k} > b_1 \\ (1-w)q_{e,k} + w (\sqrt{\frac{n K}{M}} - R_{c}) & \text{if } q_{e,k} < b_2 \\ \text{otherwise} & \end{cases}$

where $\rho$ represents the parameter vector in the system.

$\begin{align*}
    n &= \text{Number of active TCP connections} \\
    M &= \text{Maximum segment size or packet size} \\
    R_0 &= \text{Round trip time} \\
    K &= \text{Modeling constant between 1 and } \sqrt{8/3} \\
    w &= \text{Exp. averaging weight} \\
    c &= \text{Bottleneck bandwidth}
\end{align*}$

Conclusion:

Model successfully captures the dynamical phenomena of TCP networks with RED AQM and provides useful information about setting control parameters. It also gives insight to formulate new control mechanisms.