Formation Dynamics under a Class of Control Laws

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The Problem

How to establish a satellite formation near a designated elliptic earth orbit is the problem we want to solve. What we want to consider here is the case when the members of the formation have been placed relatively ``far apart''. They have to use their on-board thrusters to get to the desired orbits to form the desired formation..

Shape Space of Elliptic Orbits

In the Kepler two body problem, it is well-known that the angular momentum vector and the Laplace vector are conserved. They are defined as:

angular momentum: $l = q \times p$ Laplace Vector: $A = p \times l - \frac{\mu q}{\|q\|}$

A subset D is defined as:

$$D = \{(l, A) \in R^3 \times R^3 \mid A \cdot l = 0, l \neq 0, \|A\| < m^2 \mu\}$$

This serves as the **shape space of satellite formations**. Every possible elliptic orbit of the satellite corresponds to a unique point on this shape space. The canonical Euclidean metric exists so that we can measure the distance between two orbits. Then we can introduce Lyapunov functions based on this distance measure and obtain control vector fields. By controlling the dynamics on this shape space we are able to control the dynamics of the relative orbits of satellites.

Orbital Transfer of Single Satellite

To control the orbital transfer of a single satellite,we design a Lyapunov function

$$V(q,p) = \frac{1}{2}(\|l - l_d\|^2 + \|A - A_d\|^2)$$

where (l_d, A_d) is the pair of the angular momentum vector and Laplace vector of the target elliptic(circular) orbit. The derivative of V along the integral curve of the system is:

$$\dot{V} = [(l - l_d) \times q + l \times (A - A_d) + (A - A_d) \times p \times q] \cdot u$$

If we let the control u to be:

$$u = -[(l - l_d) \times q + l \times (A - A_d) + (A - A_d) \times p \times q]$$

Then $\dot{V} \leq 0$ along the trajectory of the closed loop system.

A Theorem proved by Chang, Chichka and Marsden claim that the satellite will be controlled towards the target orbit asymptotically

Periodic Satellite Formations

Suppose we have a formation consisting m satellites. Among all the possible formations, we are interested in the formation with periodic shape changes. We define a **periodic formation** to be a formation with all its member satellites have identical length of semi-major axis. This makes sense because they will have the same orbital period. Thus although the shape of the formation is varying, it is varying periodically

On the other hand, given a set of orbits with the identical length of semi-major axis, there are infinitely many possible periodic formations. We need to specify the relative positions of the members on the orbits to determine a specific formation. Notice that the difference of the mean anomalies are constants in this case. So a perio-dic formation can be uniquely determined by specifying the difference of the mean anomalies.

Orbital Transfer of Formations

If we want to set up a periodic formation, we can control each satellite separately to transfer to its target orbit. We are able to achieve any set of orbits this way because we are actually performing orbital transfer for a single satellite m times. However, we are notable to achieve the correct values of the difference of mean anomalies. In order to do that, extra terms involving the differences should be added to the summation of the Lyapunov functions for single satellites. This extension will result in a cooperative orbital transfer of multiple satellites.

If a two satellite formation is considered, our Lyapunov function is:

$$V = V_1 + V_2 + 4\sin^2\left(\frac{\Gamma_1 - \Gamma_2 - \phi}{4}\right)$$

where

$$V_i(q,p) = \frac{1}{2}(\|l_i - l_{di}\|^2 + \|A_i - A_{di}\|^2)$$

$$\Gamma_i = \sqrt{\frac{a_i^3}{a_d^3}} M_i$$

for i=1,2. M_i Are the mean anomalies.

The Control Law

We developed a control law based on this Lyapunov function as:

$$u_i = -[(l_i - l_{di} + \varsigma_i \sin(\frac{\Gamma_1 - \Gamma_2 - \phi}{2}) l_i) \times q_i + l_i \times (A_i - A_{di} + \rho_i \sin(\frac{\Gamma_1 - \Gamma_2 - \phi}{2}) A_i) + (A_i - A_{di} + \rho_i \sin(\frac{\Gamma_1 - \Gamma_2 - \phi}{2}) A_i) \times p_i \times q_i]$$

where ς_i and ρ_i are functions of q and p.

We proved that periodic formations can be established by this control law asymptotically.

Simulation Results

Here we will show a controlled transfer of two satellites from a parking orbit with smaller initial separation to the target orbit with bigger final separation The **relative motion** of the satellites are plotted. The upper figure displays the desired relative motion of the satellites on the target orbit. The lower figure displays the relative motion of the satellites using the control algorithm proposed. As we can see, the desired orbit and separation are achieved asymptotically.