Swarms in Three Dimensions

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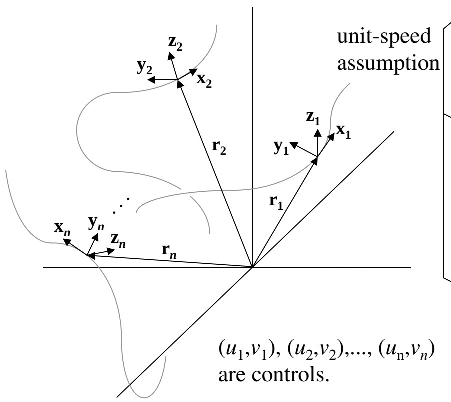
Abstract

- Modeling and analysis of novel control laws for vehicles moving at constant speed.
- Practical motivation: coordinating the flight of meter-scale UAVs (unmanned aerial vehicles). Possible implications for UUV, UGV, or USV swarms, or biological swarming/schooling systems.
- Objective: UAV formation demo in collaboration with NRL.

Model: Unit-Speed Motion



unit-speed assumption

$(u_1,v_1), (u_2,v_2),\ldots, (u_n,v_n)$ are controls.

$$\dot{\mathbf r}_1 = \mathbf x_1,\quad \dot{\mathbf x}_1 = \mathbf y_1 u_1 + \mathbf z_1 v_1,\quad \dot{\mathbf y}_1 = -\mathbf x_1 u_1,\quad \dot{\mathbf z}_1 = -\mathbf x_1 v_1$$

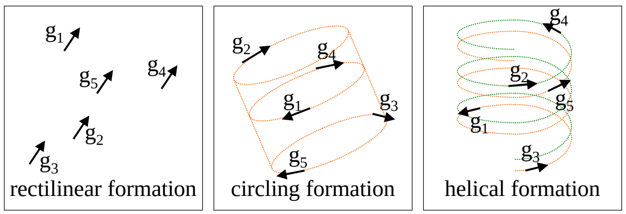
$$\dot{\mathbf r}_2 = \mathbf x_2,\quad \dot{\mathbf x}_2 = \mathbf y_2 u_2 + \mathbf z_2 v_2,\quad \dot{\mathbf y}_2 = -\mathbf x_2 u_2,\quad \dot{\mathbf z}_2 = -\mathbf x_2 v_2$$

$$\vdots$$

$$\dot{\mathbf r}_n = \mathbf x_n,\quad \dot{\mathbf x}_n = \mathbf y_n u_n + \mathbf z_n v_n,\quad \dot{\mathbf y}_n = -\mathbf x_n u_n,\quad \dot{\mathbf z}_n = -\mathbf x_n v_n$$

Equilibrium Shapes

- The control laws are assumed to be invariant under rigid motions in three-dimensional space.
- Therefore, we can define appropriate shape variables (which capture relative distances and angles between vehicles).
- Shape equilibria correspond to steady-state formations.



Rectilinear Control Law

$$u_j = \frac{1}{n}\sum_{k\neq j}\left[-\eta\left(\frac{\mathbf r_{jk}}{|\mathbf r_{jk}|}\cdot\mathbf x_j\right)\left(\frac{\mathbf r_{jk}}{|\mathbf r_{jk}|}\cdot\mathbf y_j\right) + f(|\mathbf r_{jk}|)\left(\frac{\mathbf r_{jk}}{|\mathbf r_{jk}|}\cdot\mathbf y_j\right) + \mu\,\mathbf x_k\cdot\mathbf y_j\right]$$

Align vehicle j perpendicular to the baseline between vehicles j and k. — Steer toward or away from vehicle k to maintain appropriate separation. — Align vehicle j with vehicle k.

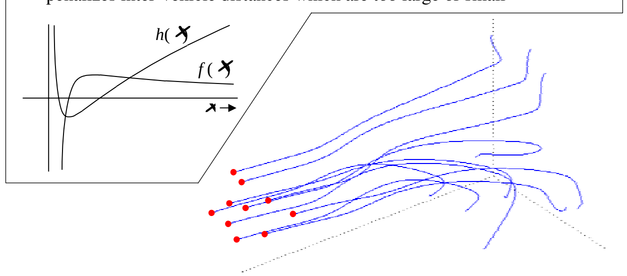
$$v_j = \frac{1}{n}\sum_{k\neq j}\left[-\eta\left(\frac{\mathbf r_{jk}}{|\mathbf r_{jk}|}\cdot\mathbf x_j\right)\left(\frac{\mathbf r_{jk}}{|\mathbf r_{jk}|}\cdot\mathbf z_j\right) + f(|\mathbf r_{jk}|)\left(\frac{\mathbf r_{jk}}{|\mathbf r_{jk}|}\cdot\mathbf z_j\right) + \mu\,\mathbf x_k\cdot\mathbf z_j\right]$$

$$\mathbf r_{jk} = \mathbf r_k - \mathbf r_j,\qquad f(|\mathbf r_{jk}|) = \alpha\left[1-\left(\frac{r_o}{|\mathbf r_{jk}|}\right)^2\right],\qquad \mu > \frac{\eta}{2} > 0,\quad \alpha > 0.$$

Global convergence result for $n = 2$ (Justh, Krishnaprasad 2004) using the Lyapunov function: $V = -\ln(1+\mathbf x_2\cdot\mathbf x_1) + h(|\mathbf r_2 - \mathbf r_1|)$

$-\ln(1+\mathbf x_2\cdot\mathbf x_1)$ penalizes heading-direction differences

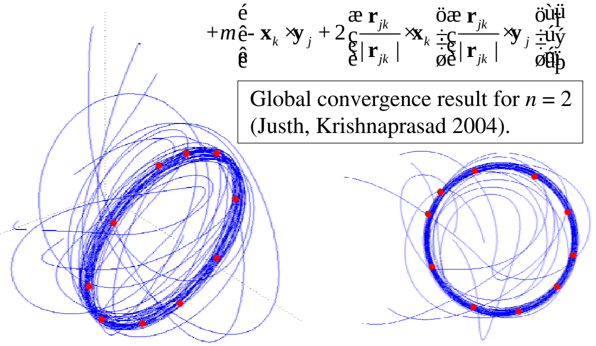
$h(|\mathbf r_2 - \mathbf r_1|)$ penalizes inter-vehicle distances which are too large or small



Circling Control Law

$$u_j = \frac{1}{n}\sum_{k\neq j}\left\{-\eta\left(\frac{\mathbf r_{jk}}{|\mathbf r_{jk}|}\cdot\mathbf x_j\right)\left(\frac{\mathbf r_{jk}}{|\mathbf r_{jk}|}\cdot\mathbf y_j\right) + f(|\mathbf r_{jk}|)\left(\frac{\mathbf r_{jk}}{|\mathbf r_{jk}|}\cdot\mathbf y_j\right) + \mu\left[-\mathbf x_k\cdot\mathbf y_j + 2\left(\frac{\mathbf r_{jk}}{|\mathbf r_{jk}|}\cdot\mathbf x_k\right)\left(\frac{\mathbf r_{jk}}{|\mathbf r_{jk}|}\cdot\mathbf y_j\right)\right]\right\}$$

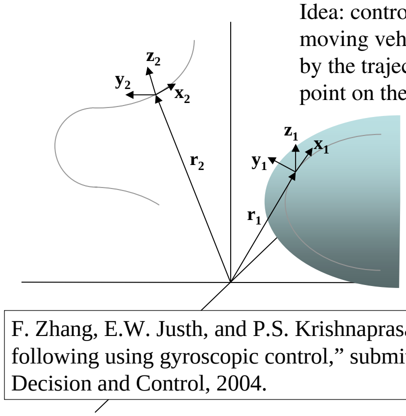
Global convergence result for $n = 2$ (Justh, Krishnaprasad 2004).



Gyroscopic Interaction Laws

- In our formation control model, particles (i.e., vehicles) interact through **gyroscopic forces**.
- Gyroscopic forces do no mechanical work: the kinetic energy (and hence the speed) of each particle remains constant.
- The physically relevant quantities are $\mathbf r_j$ (the position) and $\mathbf x_j$ (the unit tangent vector to the trajectory), $j=1,\ldots,n$, which imposes constraints on the form of (u_j,v_j), $j=1,\ldots,n$.

Future Research: Boundary-Following



Idea: control inputs for the moving vehicle are determined by the trajectory of the closest point on the obstacle surface.

Goals: boundary following and non-collision.

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References

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