# The Mobile Facility Routing Problem

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## Motivation

- Routing mobile facilities such as mobile base stations in cellular networks, such as COLTS and COWs, to provide extra cellular coverage.
- These mobile facilities can operate as a fully functioning cellular tower when at a location, but can quickly be transported from location to location to provide coverage as demand arises.
- These are often used to provided coverage for large gatherings of people (e.g. Sporting Events) and in disaster scenarios

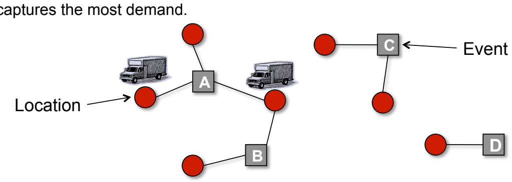
•Other applications: Routing food service trucks, mobile WIFI vehicles, portable traffic cameras, mobile post offices, mobile libraries, Red Cross Blood Mobiles.

## Objective

$E$ - the set of events that generate demand.

$L$ - set of locations where a mobile facility (MF) can capture nearby demand. A mobile facility at a location $l$ may capture demand from some subset of the events $E_l$.

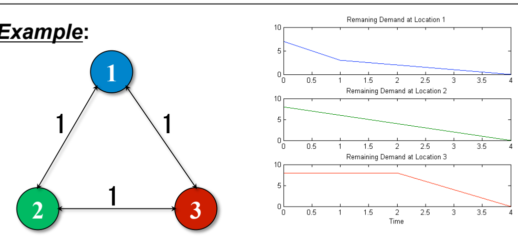
**Objective**: Find a collection of routes for a fleet of mobile facilities that captures the most demand.




## Single Vehicle Example

When using only one mobile facility we know exactly the demand the mobile facility will cover at each location at time $t$. Consequently, we may delete the event nodes from the problem and consider how much demand the mobile facility will be able to capture at each location

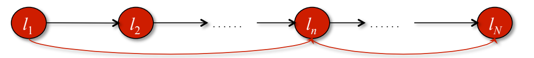
***Example***:




An example with three locations. The travel time between any pair of locations is 1 unit. Each location's remaining demand is graphed. For each location, the height of the graph at time $t$ is the amount of demand the mobile facility will capture by providing service from that location during the interval of time [$t$,4].

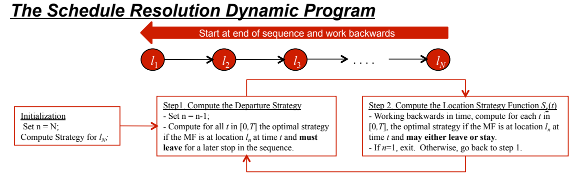
## Scheduling the Route of a Single Mobile Facility over a Sequence

Suppose that we have a sequence of locations, $l_1, l_2, ...., l_N$, and that the MF may visit any subsequence in order according to any feasible schedule.



We have developed the Schedule Resolution Dynamic Program (SRDP) that gives for each location $l_n$ in the sequence a location strategy function that specifies the optimal strategy if the MF is at location $l_n$ at time $t$.

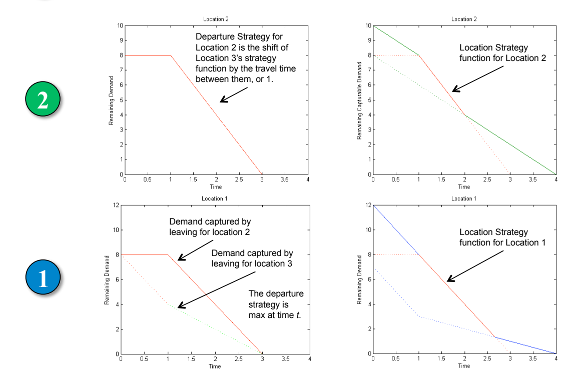
### *The Schedule Resolution Dynamic Program*



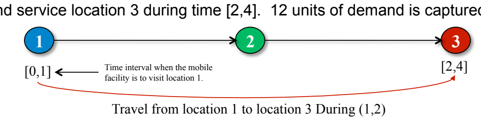

### *Example: Routing over a sequence using the SRDP*

Location | Stage 1 | Stage 2

*Because location 3 is the last location in the sequence, the mobile facility should always stay at location 3 until time 4 if it arrives there at any time t <4.*

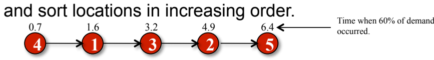



The optimal route is to service location 1 during time [0,1], then travel to location 3 and service location 3 during time [2,4]. 12 units of demand is captured.

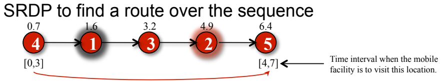



## BoundOpt

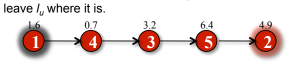
*Step 1:* For each location, compute the time when 60% of the demand has occurred and sort locations in increasing order.




*Step 2:* Run the SRDP to find a route over the sequence




*Step 3:* For each unvisited location $l_1$, if 60% of the demand at location $l_u$ occurs after the departure time of previously visited location in the sequence, $l_1$, move $l_u$ before $l_1$. If 60% of the demand at location $l_u$ occurs after the departure time of previously visited location in the sequence, $l_2$, move $l_u$ after $l_2$. Otherwise leave $l_u$ where it is.

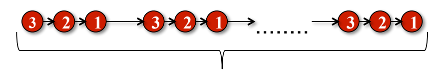


*Step 4:* Run the SRDP on the new sequence to find a route. If improvement is found, go back to *step 2*. Otherwise, exit.

## Finding Optimal Solution

$$K = \frac{T}{\min\{TT_{l,l'} : l, l' \in L, l \neq l'\}}$$

$K$ is an upper bound on the maximum number of locations that can be serviced during the planning horizon $[0,T]$.



Repeat sequence of all locations $K$ times and run SRDP will give an optimal route.

## Preliminary Results

Algorithms tested on two types of data sets.

•Modeled "Realistic" Data Sets: Data sets were created by generating location demands functions to model events that might realistically occur over a planning horizon (e.g. Rush Hour, Sporting Events, etc.) . Scenarios with 25, 50, and 100 locations were tested. BoundOpt ran on average in 0.44 seconds, finding a solution that captured 98.5% of the demand captured by the optimal route. The optimal route was found with the SRDP in 0.88 seconds on average. BoundOpt found the optimal route 80% of the time.

• Mathematically Challenging Data Sets: Data sets were created to be mathematically difficult to solve. Scenarios with 25, 50, and 100 locations were tested. BoundOpt ran on average in 7.22 seconds, finding routes that captured 92.1% of the demand captured by the optimal routes. The optimal routes were found with the SRDP in 85.73 seconds on average. BoundOpt never found the optimal solution on these data sets. This may be caused in part by the inability of BoundOpt to consider visiting a location more times that it occurs in the input sequence.

## Future Research

- Develop solution methods for the MFRP with multiple mobile facilities, utilizing the algorithms presented here for the single mobile facility model.
- Consider extensions to the problem with demand uncertainty.