A New Achievability Scheme for the Relay Channel

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Introduction

- Relay channel, as illustrated in Figure 1, is the simplest cooperative channel and has been widely studied in information theory.
  - Three terminals: transmitter X, relay (X_1, Y_1), and receiver Y.
  - Characterized by the transition probability $p(y, y_1|x, x_1)$.
  - Consider an n-length block code with encoders at transmitter and relay, respectively, and a decoder at receiver.
  - Rate R is achievable if $\forall\ 0 < \epsilon < 1$, $\eta > 0$, and sufficiently large n, $\exists\ n$-length block code with average probability of error $P_e \leq \epsilon$ and cardinality of message M, such that $\frac{1}{n}\ln M \geq R - \eta$.
  - The problem of interests is to determine the capacity of the relay channel, defined as the supremum of all achievable rates.

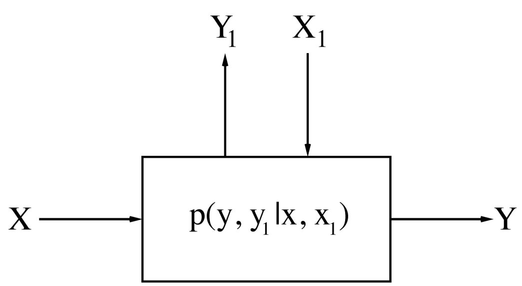

Figure 1: Relay channel.

Existing Achievability Schemes

- Decode-And-Forward (DAF)
  - Relay decodes to form full correlation (common information).
  - Drawback of DAF: full decoding at the relay limits the rate.
- Compress-And-Forward (CAF)
  - Relay does not decode, thus forms only partial correlation.
  - Relay performs compression to remove correlation.
  - Drawback of CAF: relay does not unitize partial correlation efficiently.

Motivation

- Focus on the case where relay does not fully decodes.
  - Transmitter sends X, desired at the receiver.
  - Relay receives Y_1, correlated with X, but not desired at the receiver. So relay plays a role of correlated helper.
  - A MAC channel with correlated helper.
- Ahlswede-Han scheme: achievable rate in MAC with correlated helper is improved by keeping correlation between the channel inputs.
- This observation motivates our new achievability scheme. The major difficulty is to obtain single-letter characterization in block-Markov code.

A New Achievability Scheme

- Codebook generation as shown in Figure 2:
  - Correlation kept between channel inputs (**benefit**).
  - Correlation introduced between blocks (**price**).

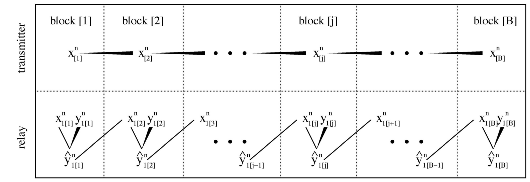

Figure 2: Codebook Generation

- Joint decoding over all B blocks to avoid possible rate loss.

Main Result

- Rate R is achievable if following conditions hold

$$
\begin{aligned}
R &\leq I(Y, \hat{Y}_1; X | X_1, \tilde{\hat{Y}}_1, \tilde{Y}, \tilde{X}_1, \tilde{\tilde{X}}) \\
I(\hat{Y}_1; Y_1 | X_1, X) &< I(Y; \hat{Y}_1, X_1 | X, \tilde{Y}, \tilde{X}, \tilde{X}_1) \\
R + I(\hat{Y}_1; Y_1 | X_1, X) &\leq I(Y; \hat{Y}_1, X_1, X | \tilde{Y}, \tilde{X}_1, \tilde{\tilde{X}})
\end{aligned}
$$

where

$$
\tilde{\tilde{X}} \longrightarrow (\tilde{X}, \tilde{\hat{Y}}_1, \tilde{X}_1, \tilde{Y}, \tilde{Y}_1) \longrightarrow (X, \hat{Y}_1, X_1, Y, Y_1)
$$

$$
p(x, \hat{y}_1, x_1, y, y_1, \tilde{x}) = p(\tilde{x}, \tilde{\hat{y}}_1, \tilde{x}_1, \tilde{y}, \tilde{y}_1, \tilde{\tilde{x}})
$$

$$
\begin{aligned}
&p(x, \hat{y}_1, x_1, y, y_1 | \tilde{x}, \tilde{\hat{y}}_1, \tilde{x}_1, \tilde{y}, \tilde{y}_1) \\
&\qquad = p(x|\tilde{x}) p(x_1|\tilde{\hat{y}}_1) p(y_1, y | x, x_1) p(\hat{y}_1 | y_1, x_1)
\end{aligned}
$$

- This result contains CAF as a special case, and therefore, our scheme is potentially better than CAF.