Optimal rate control policies for proportional fairness in wireless networks

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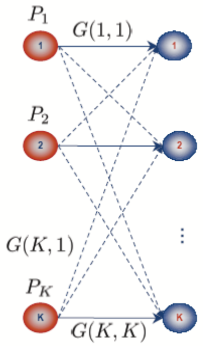
Motivation

- Exploit throughput gains by integrating the physical and MAC layers in wireless networks

Model



- K transmitter/receiver pairs
- Infinite data at transmitters
- Slotted time
- Power of transmitter k : P_k
- Path loss: $0 \leq G(\ell,k) \leq 1$
- Noise power: N
- Transmission success criterion (SINR):

$$\mathrm{SINR}(k) = \frac{P_k G(k,k)}{N + \sum_{i \neq k} P_i G(i,k)} \geq \theta_k$$

Rate Control

- θ_k is an increasing function of the rate
  - More successful concurrent transmissions at lower rates
  - vs. less transmissions at higher rates
- 2^K possible rate allocations to the K pairs
- Restrict attention to $K+1$ rate allocations
  - All K transmitters operate simultaneously
    - Instantaneous rate of transmitter k : r_k^0
  - A single transmitter transmits at any time
    - Instantaneous rate of transmitter k : r_k^k

Optimization: Proportional Fairness

- Probability distribution over rate allocations:

$$\boldsymbol{\pi} = (\pi_0, \pi_1, \ldots, \pi_K)$$

- Find $\boldsymbol{\pi}$ so that the average transmission rates of all transmitters are proportionally fair

$$\max_{\boldsymbol{\pi}} \sum_{k=1}^{K} \log(r_k^0 \pi_0 + r_k^k \pi_k)$$

s.t.

$$\sum_{i=0}^{K} \pi_i = 1, \quad \pi_i \geq 0, \; i \in \{0,1,\ldots,K\}$$

Optimal Policy

- Schedules individually all transmitters in $\mathcal{J}$
- $\mathcal{J}$ contains $|\mathcal{J}|$ transmitters with the lowest rates $\tilde{r}_1^0 \leq \ldots \leq \tilde{r}_{|\mathcal{J}|}^0$, where

$$|\mathcal{J}| = \arg \max_{\ell \in \{1,\ldots,K\}} R(\ell), \quad R(\ell) = \frac{r^{\mathrm{S}} - \sum_{i=1}^{\ell} \tilde{r}_i^0}{K - \ell}$$

- Threshold type policy yields $\boldsymbol{\pi}^*$
- Transmitter k individually transmits with

$$\pi_k^* = \frac{1}{K}\left(1 - \frac{r_k^0}{R(|\mathcal{J}|)}\right)$$

if and only if $r_k^0 \leq R(|\mathcal{J}|)$, otherwise $\pi_k^* = 0$

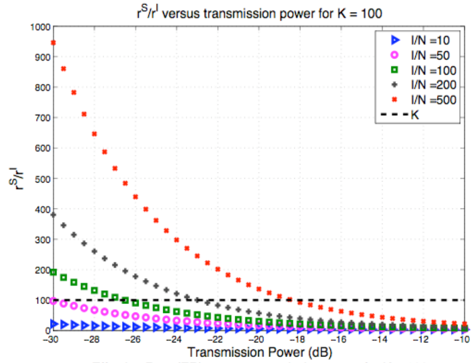
- All transmitters concurrently transmit with

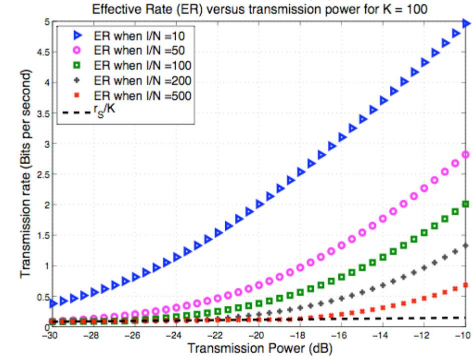
$$\pi_0^* = \frac{1}{K} \frac{r^{\mathrm{S}}}{R(|\mathcal{J}|)}$$

Assumption

- Equal instantaneous rates under individual operation for every transmitter: $r_k^k = r^{\mathrm{S}}, \forall k$

Numerical Analysis





Assumptions

- Equal instantaneous rates under concurrent operation for every transmitter: $r_k^0 = r^{\mathrm{I}}, \forall k$
- Shannon formula for rate:

$$r(\mathrm{SINR}) = W \log(1 + \mathrm{SINR}) \text{ (bits/sec)}$$

- $N = 3.34 \times 10^{-6}$Watts, $G(k,k) = 1$, $W = 1$ Hz

Concluding Remarks

- Coupled MAC with the physical layer
- Considered objective of proportional fairness
- How to generalize to multihop networks?