A New Upper Bound for a Multiple Access Channel with Feedback

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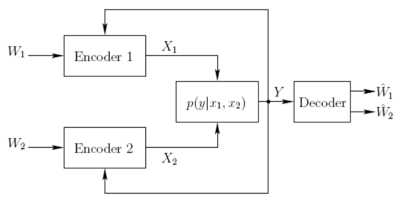
Multiple Access Channel with Feedback

Introduction



- Channel described by $p(y|x_1,x_2)$.
- For a MAC, **feedback increases capacity** unlike a single user channel.
- Feedback capacity region not known in general.
- Only known for special cases, e.g., for a class of channels: $X_1=f(X_2,Y)$.
- Capacity region also known for the AWGN MAC with feedback.

Cut-Set Outer Bound

$$\mathcal{CS} = \Big\{(R_1,R_2) : R_1 \le I(X_1;Y|X_2)$$
$$R_2 \le I(X_2;Y|X_1)$$
$$R_1 + R_2 \le I(X_1,X_2;Y)\Big\}$$

over all input distributions $p(x_1,x_2)$.

Main idea behind our outer bound:

- Cut-set bound allows all correlations.
- Even with perfect feedback, **full correlation** between X_1 and X_2 **may not be possible** to achieve.
- Restricting the set of allowable $p(x_1,x_2)$ and corresponding rates.

New Outer Bounds

$$\mathcal{DB}^{(1)} = \Big\{(R_1,R_2) : R_1 \le \min(I(X_1;Y|X_2), H(X_1|T))$$
$$R_2 \le I(X_2;Y|X_1,T)$$
$$R_1 + R_2 \le I(X_1,X_2;Y)\Big\}$$

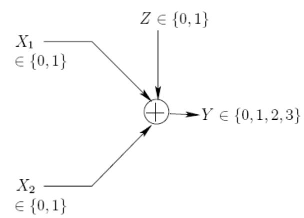
$$\mathcal{DB}^{(2)} = \Big\{(R_1,R_2) : R_1 \le I(X_1;Y|X_2,T)$$
$$R_2 \le \min(I(X_2;Y|X_1), H(X_2|T))$$
$$R_1 + R_2 \le I(X_1,X_2;Y)\Big\}$$

where both bounds are evaluated over all distributions of the conditionally independent form $p(t)p(x_1|t)p(x_2|t)$.

Remarks :

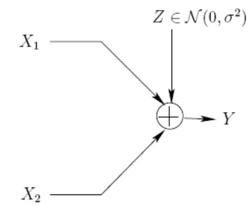
- For the case when $X_1=f(X_2,Y)$, our bounds are tight and yield the feedback capacity region.
- While evaluating the outer bounds, it is sufficient to restrict attention to auxiliary random variables T such that $|T| \le |X_1||X_2| + 3$.
- For binary inputs, we have $|T| \le 7$, making exhaustive search intractable.

Binary Additive Noise MAC



- Channel given by $Y=X_1+X_2+Z$, where Z is binary and uniform on $\{0,1\}$.
- Capacity region not known.
- Our bounds show that the **cut-set bound is not tight.**

AWGN MAC

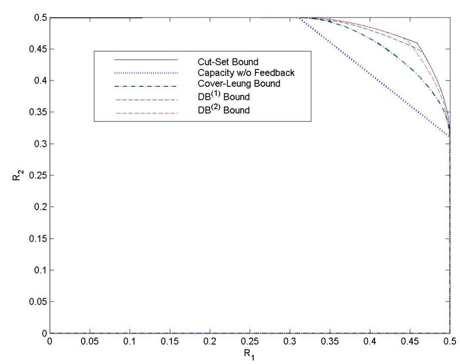


- Capacity region known [Ozarow, 1984]
- **Cut-set bound is tight**.
- An extension of Kailath-Schalkwijk scheme is capacity achieving..

For symmetric-rate point (R,R) in the feedback capacity region:

- Cut-set upper bound: 0.45915 bits/tx.
- Our upper bound: 0.45330 bits/tx.
- Cover-Leung's inner bound: 0.43621 bits/tx.
- Best known inner bound: 0.43879 bits/tx.

Comparison of Our Bounds with the Cut-Set Bound



- $|T|=2$ is sufficient to evaluate our bounds by making use of composite functions.
- Our bounds improve upon the cut-set bound for all points where feedback increases capacity.

Conclusion: *Cut-set bound is in general not tight for the multiple access channel with feedback.*