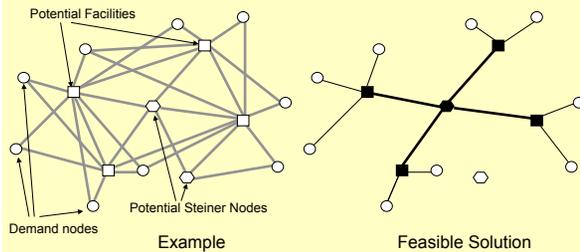


## Problem Description

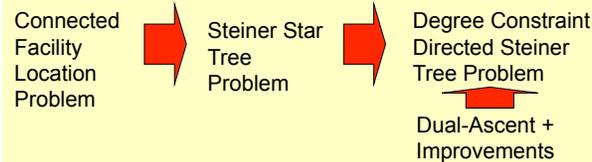
These problems typically arise in the design of telecommunication networks where open facilities need to communicate with each other. For example, in distributed networks demand points must connect to hubs and hubs must communicate with each other to maintain consistent data.



## Objective

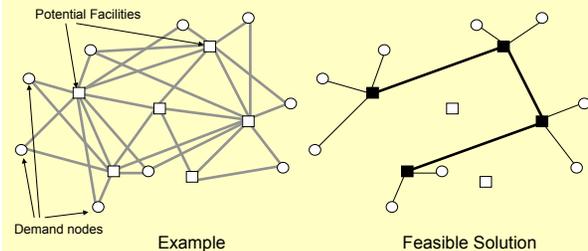
- Develop an effective heuristic that finds high quality solutions rapidly.
- Problem is NP-complete

## Heuristic Approach



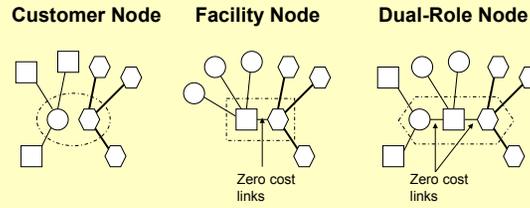
## Steiner Star Tree Problem

Steiner nodes (hubs) incur a cost even if no customers are directly connected to the Steiner node.



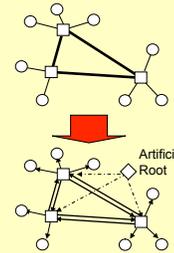
## Transformation into Steiner Star Tree Problem

The transformation is based on node splitting and adding edges accordingly.



## Transformation into Degree Constraint Directed Steiner Tree Problem

- Create an artificial node (root node) with a degree one constraint.
- Create a directed arc from the root node and every facility node with cost equal to the fixed cost of the facility.
- Replace each edge by two directed arcs such that the cost of the arc equals the cost of the edge plus the facility opening cost (if any) at the end node.



## Improvement Approaches

### Sequential Improvements

- Construct a minimum spanning tree on the set of open facilities and selected Steiner nodes
- Eliminate any Steiner node with degree two or less in the Steiner Tree (when triangle inequality is satisfied)
- Remove any open facility that serves only one customer and is a leaf node in the Steiner Tree (when triangle inequality holds)

Using this order ensures that the upper bound cannot get worse.

### Local Search Improvements

- Order open facilities in increasing order of
  - Node degree in the Steiner Tree
  - Number of customers served
- Remove open facilities one at a time in order and compare change in the solution cost
  - If removal results in an improvement, update solution; otherwise, restore open facility into solution

## Computational Experiments

We tested our heuristic approach on randomly generate problems on a 100x100 grid with a total of 100 nodes varying number of demand nodes (and facility nodes), ratio between Steiner tree edges and assignment edges and facility opening costs: 630 instances.

Quality (Optimality)	Speed
Average (Std Dev) Gap 2.52% (2.05%) Maximum Gap 10.06%	Average (Std Dev) Time 3.44s (0.80s) Maximum Time 6.03s

Avg Gap Dual + Improvements (Avg Gap Dual)

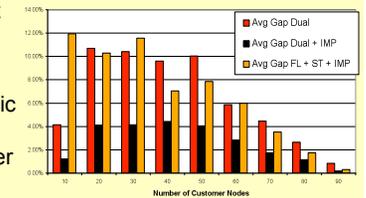
Demand Nodes	Facility Nodes	Opening Facility Cost													
		w=0	w=5	w=10	w=15	w=20	w=25	w=30							
10	90	0.81%	(1.43%)	1.07%	(2.32%)	1.32%	(3.65%)	1.20%	(4.14%)	0.74%	(4.67%)	0.31%	(2.94%)	0.31%	(3.33%)
20	80	2.87%	(6.35%)	3.28%	(8.13%)	3.92%	(9.45%)	4.11%	(10.70%)	4.95%	(13.69%)	4.84%	(15.31%)	4.92%	(17.32%)
30	70	3.35%	(5.87%)	3.98%	(8.31%)	4.44%	(9.40%)	4.18%	(10.43%)	4.85%	(11.26%)	5.35%	(11.81%)	4.86%	(14.75%)
40	60	3.41%	(7.15%)	3.48%	(7.33%)	3.89%	(6.23%)	4.42%	(9.60%)	4.21%	(10.11%)	4.84%	(12.92%)	4.73%	(14.55%)
50	50	2.24%	(5.14%)	2.78%	(8.00%)	3.30%	(7.58%)	4.07%	(10.93%)	4.21%	(10.18%)	4.02%	(10.16%)	4.26%	(12.19%)
60	40	1.88%	(4.56%)	2.04%	(5.64%)	2.27%	(5.70%)	2.83%	(5.85%)	2.94%	(6.79%)	3.00%	(7.33%)	3.05%	(9.17%)
70	30	0.97%	(2.85%)	1.27%	(3.42%)	1.42%	(3.19%)	1.73%	(4.45%)	2.23%	(5.28%)	2.87%	(6.04%)	2.21%	(5.36%)
80	20	0.57%	(1.91%)	0.78%	(2.07%)	1.04%	(2.29%)	1.14%	(2.64%)	0.97%	(1.93%)	1.38%	(2.55%)	1.45%	(2.71%)
90	10	0.14%	(0.96%)	0.17%	(0.82%)	0.25%	(0.92%)	0.18%	(0.83%)	0.17%	(0.74%)	0.25%	(1.05%)	0.30%	(1.28%)

Improvement Steps reduce significantly the upper bound. Then, does Dual-Ascent yield "good" starting solutions?

## Benchmark Solution

- Solve Facility Location Problem to optimality
- Use Dual-Ascent to obtain a Steiner Tree on the open facilities
- Run all improvement approaches

ConFL General Version  
Average % Gap M3 w=15



## Paired t-test

On average the heuristic solutions are between 3.51% and 4.11% closer to optimality.

## Conclusions and Further Research

- For our test problems the heuristic generated solutions that were on average within 3% from optimality, and in the worst case instances within 10% from optimality.
  - Proposed heuristic yields significant better solutions for the ConFL than the benchmark approach.
  - Similar high quality results were obtained for related problems: Rent-or-Buy and Steiner Star Tree Problem.
- We'd like to test this heuristic on problems with different characteristics:
- Random edge costs
  - Sparse networks