Delay-minimal Transmission for Energy Constrained Wireless Communications

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INTRODUCTION

- For many wireless networks, especially for sensor and mobile ad-hoc networks, efficient consumption of energy is a critical issue.
- Our objective is to minimize the overall delay of the packets subject to an energy constraint on the transmitter in a single user system.

SCENARIO I: RANDOM PACKET ARRIVALS

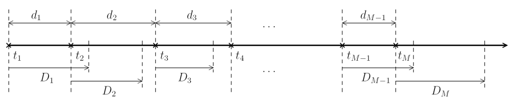

Fig. 1. System model with random packet arrivals.

- Delays for individual packets

$$\begin{aligned} D_1 &= f(e_1) \\ D_2 &= (D_1 - d_1)^+ + f(e_2) \\ D_3 &= (D_2 - d_2)^+ + f(e_3) \\ &\vdots \\ D_M &= (D_{M-1} - d_{M-1})^+ + f(e_M) \end{aligned}$$

- The problem can be formulated as

$$\begin{aligned} \min \quad & \sum_{i=1}^{M} D_i \\ \text{s.t.} \quad & \sum_{i=1}^{M} e_i \leq E \\ & e_i \geq 0, \quad i = 1, 2, \ldots, M \end{aligned}$$

  - The objective function is convex with respect to **e**.
  - The cost function has non-differentiable points.
  - Depending on whether the insides of (.)+ functions are negative or positive, we have 2^M possible cost functions.

Example: three-packets case

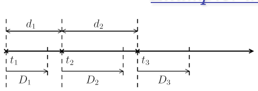

Fig. 2. Case 1.

$$\begin{aligned} \min \quad & f(e_1) + f(e_2) + f(e_3) \\ \text{s.t.} \quad & f(e_1) \leq d_1, \quad f(e_2) \leq d_2 \\ & e_1 + e_2 + e_3 \leq E, \qquad e_1, e_2, e_3 \geq 0 \end{aligned}$$

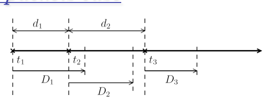

Fig. 3. Case 2.

$$\begin{aligned} \min \quad & 2f(e_1) + f(e_2) + f(e_3) - d_1 \\ \text{s.t.} \quad & f(e_1) > d_1, \quad f(e_1) + f(e_2) \leq d_1 + d_2 \\ & e_1 + e_2 + e_3 \leq E, \qquad e_1, e_2, e_3 \geq 0 \end{aligned}$$

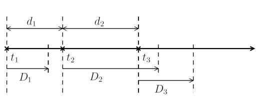

Fig. 4. Case 3.

$$\begin{aligned} \min \quad & f(e_1) + 2f(e_2) + f(e_3) - d_2 \\ \text{s.t.} \quad & f(e_1) \leq d_1, \quad f(e_2) > d_2 \\ & e_1 + e_2 + e_3 \leq E, \qquad e_1, e_2, e_3 \geq 0 \end{aligned}$$

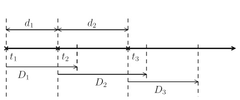

Fig. 5. Case 4.

$$\begin{aligned} \min \quad & 3f(e_1) + 2f(e_2) + f(e_3) - 2d_1 - d_2 \\ \text{s.t.} \quad & f(e_1) > d_1, \quad f(e_1) + f(e_2) > d_1 + d_2 \\ & e_1 + e_2 + e_3 \leq E, \qquad e_1, e_2, e_3 \geq 0 \end{aligned}$$

An Iterative Approach

- Break the global problem into smaller local sub-problems
  - Initially, allocate the total energy E to the first packet.
  - Then, optimize the distribution of the total energy E over first two packets, while keeping the energies allocated to the rest of the packets fixed.
  - Continue this process until the last packet, then return to the first packet, and repeat the procedure.
- The local optimization problem is

$$\begin{aligned} \min \quad & \sum_{j=i}^{M} D_j(e_i^k, e_{i+1}^k) \\ \text{s.t.} \quad & e_i^k + e_{i+1}^k = e_i^{k-1} + e_{i+1}^{k-1}, \quad e_i^k, e_{i+1}^k \geq 0 \end{aligned}$$

- The algorithm may converge to a strictly sub-optimal fixed point. We use "multiple node relaxation method", and "ϵ-relaxation method" to escape sub-optimal fixed points.

Dynamic Programming Approach

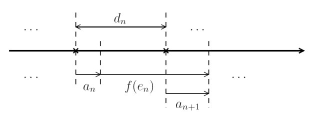

Fig. 6. System model for the dynamic programming approach.

- Define $S_n(e,a)$ as the minimal delay for the last *M-n* packets

$$\begin{aligned} S_n(e,a) = \min_{0 \leq e_n \leq e} \{ & a + f(e_n) \\ & + S_{n+1}\left(e - e_n, (a + f(e_n) - d_n)^+\right)\} \end{aligned}$$

for $n = 1, 2, \ldots, M-1$, and $S_{M+1}(e,a) = 0$.

- Compute $\{S_n(e,a), 0 \leq e \leq E\}$ in a backward recursion and keep track of $\hat{e}_n(e,a)$
- Then, the optimal energy allocation strategy is

$$\begin{aligned} e_1 &= \hat{e}_1(E, 0) \\ a_n &= (a_{n-1} + f(e_{n-1}) - d_{n-1})^+ \\ e_n &= \hat{e}_n\left(E - \sum_{i=1}^{n-1} e_i, a_i\right) \end{aligned}$$

SCENARIO II: PACKETS READY BEFORE TRANSMISSION STARTS

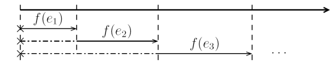

Fig. 7. System model when all packets are ready before the transmission starts.

- Delay for the i-th packet is $D_i = \sum_{k=1}^{i} \tau_k = \sum_{k=1}^{i} f(e_k)$
- The delay optimization problem becomes

$$\begin{aligned} \min \quad & \sum_{i=1}^{M} (M - i + 1) f(e_i) \\ \text{s.t.} \quad & \sum_{i=1}^{M} e_i \leq E \\ & e_i \geq 0, \quad i = 1, \ldots, M \end{aligned}$$

- The cost function has a fixed form, and has a unique global optimal solution satisfying the KKT condition

$$e_i = f'^{-1}\left(\frac{-\lambda}{M - i + 1}\right), \quad i = 1, 2, \ldots, M$$

- Following the iterative approach, the local optimization is

$$\begin{aligned} \min \quad & (M - i + 1) f(e_i^k) + (M - i) f(e_{i+1}^k) \\ \text{s.t.} \quad & e_i^k + e_{i+1}^k = e_i^{k-1} + e_{i+1}^{k-1}, \quad e_i^k, e_{i+1}^k \geq 0 \end{aligned}$$

- The algorithm converges to a fixed point. The convergence point satisfies the KKT condition, and thus is optimal.

SIMULATION RESULTS

- DP based algorithm always converges to the solution that the built-in Matlab function finds.
- Iterative algorithm converges to a fixed point.
- Dimension-3 relaxation and ϵ-perturbation methods escape sub-optimal fixed point.

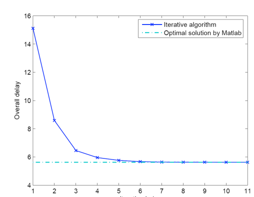

Fig. 8. Overall delay as a function of the iteration index, when $E = 4.8$

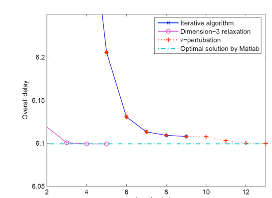

Fig. 9. Overall delay as a function of the iteration index, when $E = 4.5$