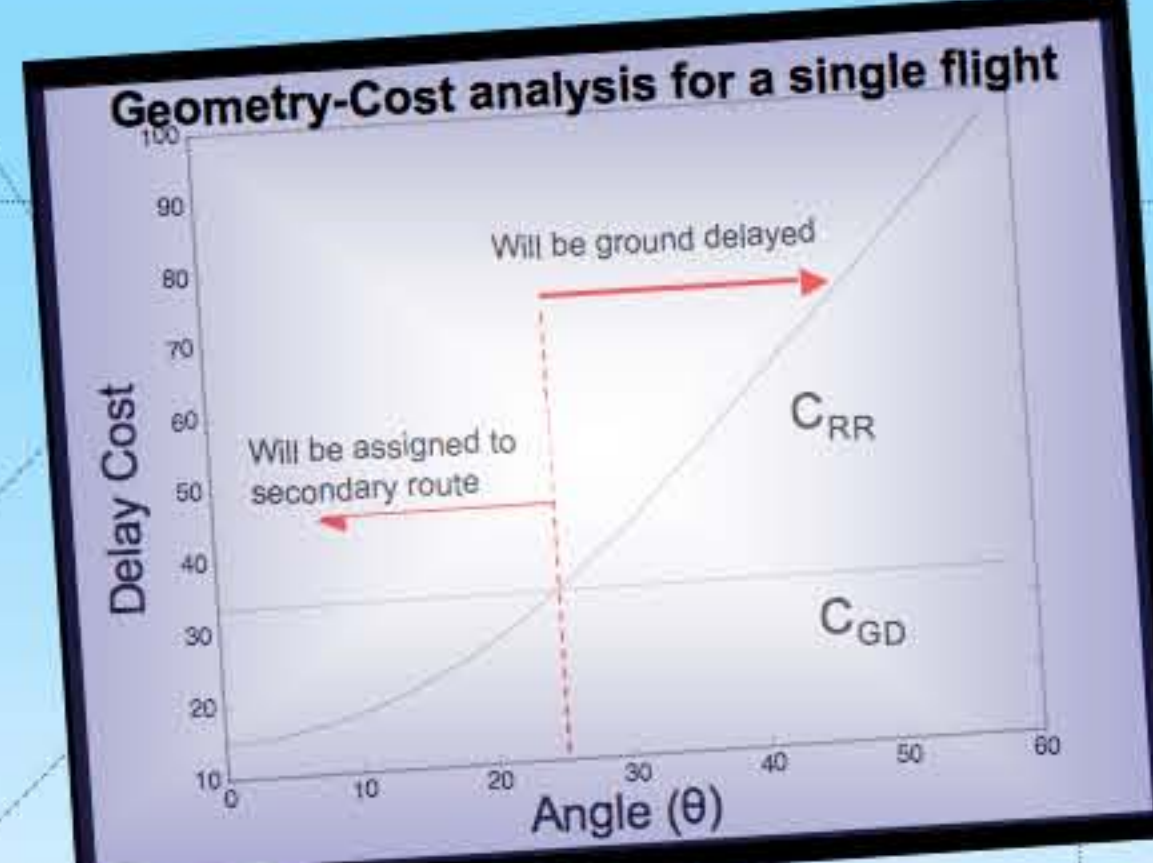
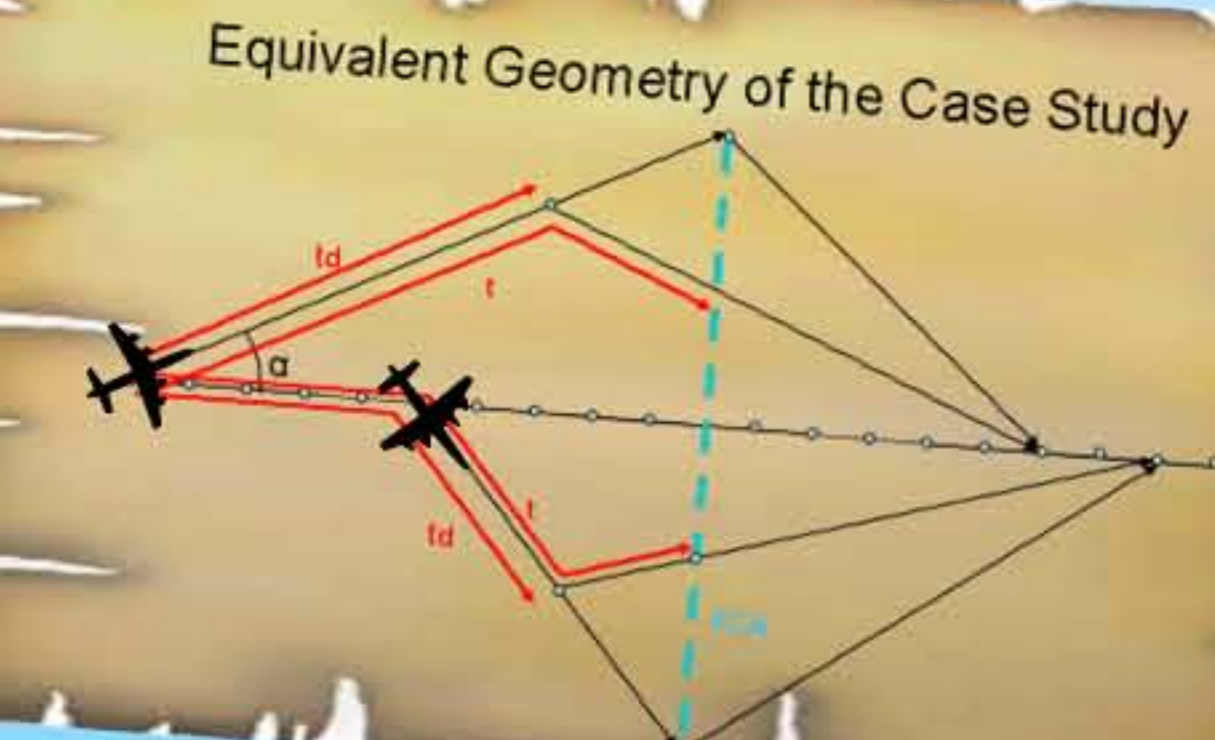
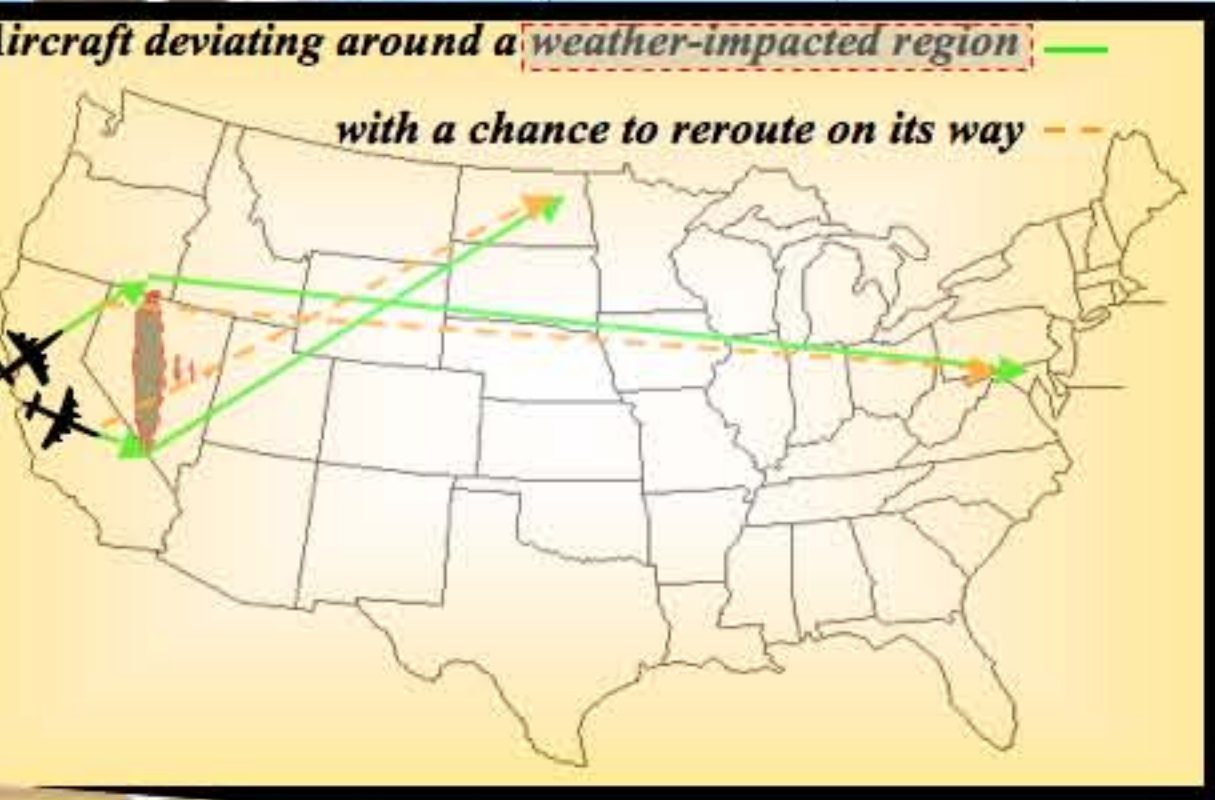


Moein Ganji, David Lovell, Robert Hoffman, Alex Nguyen and Michael Ball

**Abstract**  
 In this project we formulate an optimization problem for the assignment of dispositions to flights whose preferred flight plans pass through a flow-constrained area. For each flight, the disposition can be either to depart as scheduled but via a secondary route that avoids the flow-constrained area, or to use the originally intended route but to depart with a controlled departure time and accompanying ground delay. We anticipate that the capacity through the flow-constrained area will increase at some future time once the weather activity clears. The model is a two-stage stochastic program that represents the time of this capacity windfall as a random variable, and determines expected costs given a second-stage decision, conditioning on that time. The goal is to minimize the expected cost over the entire distribution of possible capacity increase times.

- Model Inputs**
- Location of the FCA (Flow-Constrained Area)
  - Nominal (good weather) capacity of the FCA
  - Reduced FCA (bad weather) capacity of the FCA
  - Start time of the AFP (Airspace Flow Program)
  - Planned end time of the AFP

- Model Outputs**
- An initial plan that designates whether a flight is assigned to its primary route or secondary route;
  - A recourse action for each flight under each possible early clearance time.



- $L$  = length of preferred route (in time)
- $\theta$  = angle of reroute
- $T$  = length of problem time horizon
- $cg$  = cost per unit time of ground delay=1
- $ca$  = cost per unit time of air delay
- $coa$  = fixed cost for planning a reroute
- $CGD$  = expected cost of GD strategy
- $CRR$  = expected cost of RR strategy

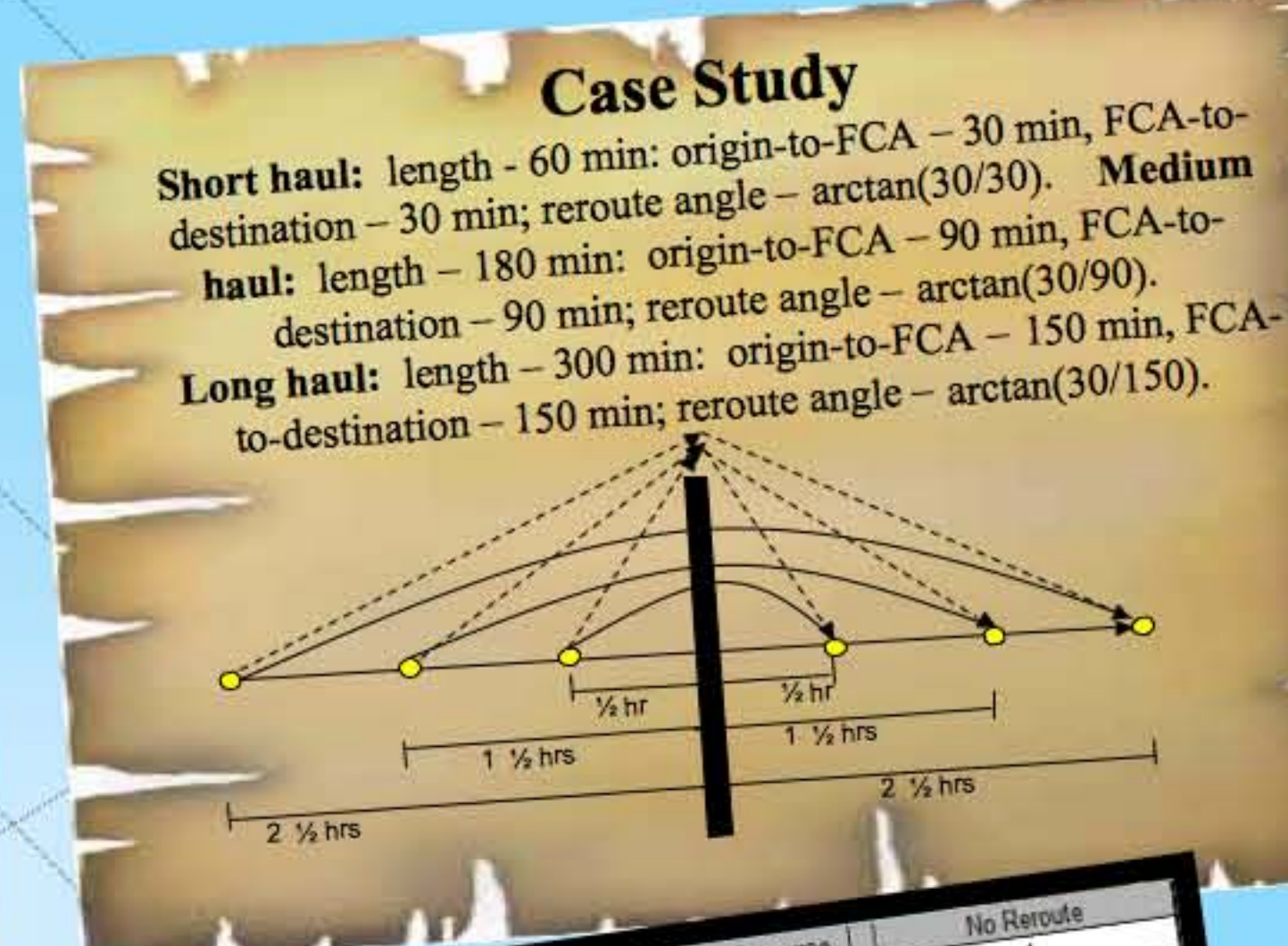
**Decision Variables for IP Model**

$$x_{f,t}^p = \begin{cases} 1, & \text{if flight } f \text{ uses its primary route and} \\ & \text{has an appointment time } t \text{ at the FCA} \\ 0, & \text{otherwise} \end{cases}$$

$$x_{f,t}^s = \begin{cases} 1, & \text{if flight } f \text{ is assigned to its secondary} \\ & \text{route} \\ 0, & \text{otherwise} \end{cases}$$

$$y_{f,t}^p | u = \begin{cases} 1, & \text{if at the time } U = u \text{ of the capacity windfall,} \\ & \text{flight } f \text{ is assigned to its primary route with} \\ & \text{appointment slot } t \text{ at the FCA} \\ 0, & \text{otherwise} \end{cases}$$

$$y_{f,t}^s | u = \begin{cases} 1, & \text{if flight } f \text{ was originally assigned to its} \\ & \text{secondary route, but under capacity} \\ & \text{clearing time } u \text{ has been assigned an} \\ & \text{FCA appointment slot } t \\ 0, & \text{otherwise} \end{cases}$$

$$y_{f,t}^u | u = \begin{cases} 1, & \text{if flight } f \text{ was originally assigned to its} \\ & \text{secondary route, and if, under AFP stop} \\ & \text{time } u, \text{ that decision remains unchanged} \\ 0, & \text{otherwise} \end{cases}$$


	All Options	Reroute but No Recourse	No Reroute
air/ground delay cost	4	2622.8	4
Objective Function	2379.4	455	6808
$C(x)$	187	722.0	11381
$C(x^p)$	764.7		
U Clearance time	3:00 PM   4:20 PM   5:40 PM	3:00 PM   4:20 PM   5:40 PM	3:00 PM   4:20 PM   5:40 PM
Probability of clearance	0.25   0.25   0.25	0.25   0.25   0.25	0.25   0.25   0.25
SV(y <sup>h</sup> )	61   114   0	345   364   110	10117   6247   1928
SV(y <sup>h</sup> ) + SV(x <sup>s</sup> - y <sup>p</sup> )	595.1   207.8   19.7	446.6   68.7   0.0	1264   5134   9453
SV(y <sup>h</sup> ) + SV(x <sup>s</sup> - y <sup>p</sup> )	126   73   187	110   91   345	0.0   0.0   0.0
GD(u)	169.6   556.9   745.0	275.4   653.3   722.0	1264.0   5134.0   9453.0
AD(u)	804.4   2300.6   3166.9	1211.6   2704.0   3232.8	6808.0
C(u)	2379.4	2622.8	6808.0
E(cost)	143.3	593.1	200
E(GD)	559.0	73	200
E(AD)	65	150	84
n(x <sup>p</sup> =1)	148	77	65
n(y <sup>p</sup> =1)	135	0	0
n(x <sup>s</sup> =1)	41	55	10
n(y <sup>s</sup> =1)	11	68	125
n(y <sup>h</sup> =1)	83	12	0
n(x <sup>s</sup> - y <sup>p</sup> )	283	226	428
Running Time (sec)			

**Objective Function**

$$\text{Min} [C(x) - \sum p_s(y_s)]$$

Initial plan cost - possible saving under each scenario U

**Main Constraints**

Under original plan each flight must either be assigned to a secondary route or to its primary routes - possibly with a delay

$$\sum x_{f,t}^p + x_{f,t}^s = 1 \quad \forall f$$

FCA capacity constraints under original plan:

$$\sum x_{f,t}^p \leq C_t^o \quad \forall t$$

under revision, flight can only arrive earlier than  $u + E(f)$  if it has already departed

$$y_{f,t}^p | u = x_{f,t}^p \quad \forall f, u, \quad \forall t \in \{1, \dots, u + E_f\}$$

Under revision, flight originally assigned to its secondary route can go on primary route if it has not departed yet

$$\sum y_{f,t}^p | u \leq 1 - x_{f,t}^s \quad \forall u, \forall f \ni D_f < u$$

Under revision flight on primary route can only arrive earlier at FCA

$$y_{f,t}^p | u \leq \sum_{s=t} x_{f,s}^p + x_{f,t}^s \quad \forall u, f, t$$

Flight on hybrid route cannot arrive at FCA at time  $t$  unless  $u$  plus the revision time is less than  $t$

$$y_{f,t}^p | u = 0 \quad \forall f, u, \quad \forall t | t_{f,t}^p \leq u$$

Flight cannot use a hybrid route unless it was originally assigned to its secondary route

$$y_{f,t}^p | u + y_{f,t}^s | u \leq x_{f,t}^s \quad \forall u, f, t$$

Under all scenarios each flight must either be assigned to a time slot at FCA or complete its secondary route:

$$\sum y_{f,t}^p | u + \sum y_{f,t}^s | u + y_{f,t}^u | u = 1 \quad \forall u, f$$

FCA capacity constraints under scenario  $u$ :

$$\sum y_{f,t}^p | u + \sum y_{f,t}^s | u \leq C_t^u \quad \forall u, t$$

**Conclusions and future work**

We have defined the basics of a stochastic optimization model for simultaneously making ground delay and reroute decisions in response to en route airspace congestion. We have also given the results of an initial computational experiment.

Future steps should include more computational experiments and model refinements aimed at improving the computational performance of the integer program and at exploring the changes in airspace planning the model provides.

We anticipate the need to provide many refinements and extensions to this model to better address practical problem solving.

NEXTOR Research team at UMD

