Network coding of packets for multicast over a time-varying communication channel

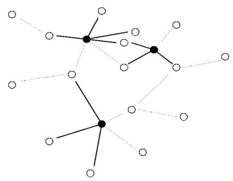
Brooke Shrader (UMD), Randy Cogill (UVA), and Anthony Ephremides (UMD)

Introduction and Motivation

Alternatives to overcome channel errors in multicast transmission:

•repeatedly send packets (ARQ)

•network coding



Previous work: network coding outperforms ARQ for time-invariant channels.

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Contribution of this work

We provide a lower bound on the multicast throughput for random linear coding over a time-varying channel.

•Wireless channels are time-varying due to mobility and fading.

•Random linear network coding naturally adapts coding rate to variations in the channel.

•Need to incorporate *overhead* used to transmit coefficients of random code.

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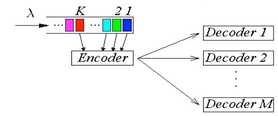
Random linear coding for multicast



•One source node, *M* destination nodes

•Form random linear combination of first *K* packets in queue.

•For packets $s_1, s_2, \ldots, s_K$ form $\sum_{i=1}^{K} a_i s_i$ where a_i are generated randomly and uniformly from *u*-ary alphabet and Σ is modulo-*u*.

•Transmit coefficients a_i in packet header

•Decode: solve a system of linear equations in s_i

•Channel to each destination is independent, identically distributed erasure channel

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Multicast throughput

Each packet consists of *n* *u*-ary symbols.
Let *N* denote the number of random linear combinations needed to decode, where $N \geq K$.
Let T_m denote the number of slots needed for destination *m* to collect *N* random linear combinations.
The multicast throughput is:

$$\lambda^* = \underbrace{\frac{K}{E[\max_m T_m]}}_{\text{rate of random linear code}} \underbrace{\frac{n}{n+K}}_{\text{ratio of information to information+overhead}} \quad \frac{\text{packets}}{\text{transmission}}$$

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Lower bound on throughput

The channels to the M destinations are identically distributed.

For random variables $X_1, X_2, \ldots, X_M$ identically distributed and correlated and for any $t > 0$,

$$E[\max(X_1, X_2, \ldots, X_M)] \leq \frac{1}{t}\left(\ln M + \ln E[e^{tX_1}]\right)$$

Then the multicast throughput is lower bounded as

$$\lambda^* \geq \frac{Kt}{\ln M + \ln E[e^{tT_1}]} \frac{n}{n+K} \quad \frac{\text{packets}}{\text{transmission}}$$

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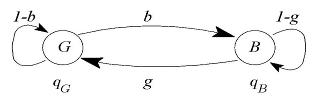
Time-varying packet erasure channel

The channel to each destination node evolves as a Markov chain with "Good" and "Bad" states.



q_G: Probability that a transmitted packet is received in "Good" state

q_B: Probability that a transmitted packet is received in "Bad" state

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Packet erasure probability

Assume that there is no channel coding within packets. For packet to be received, every symbol must be received.

$$q = (1 - P_u)^n$$

P_u: *u*-ary symbol error probability for modulation scheme, depends on SNR, channel model (e.g., AWGN, fading channel)

We assume QAM modulation over AWGN channel with SNR/bit 3.5 dB in "Good" state and -∞ dB in "Bad" state.

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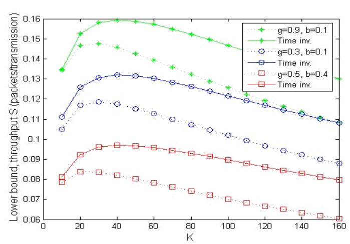
Throughput versus K



M=10, n=250, u=8

Compare to time-invariant channel with probability of reception

$$q = \frac{g}{g+b} q_G + \frac{b}{g+b} q_B$$

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Conclusions and future work

- We quantified the extent to which the time-varying nature of the channel degrades throughput.
- Due to overhead, the throughput approaches zero as K approaches ∞ and there is an *optimum* value of K that maximizes throughput.
- As for time-invariant channel, coding outperforms ARQ for multicast.
- Future work: incorporate channel coding within packet, study how to allocate coding within and among packets.

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