Optical Alignment for Intersatellite Optical Communication

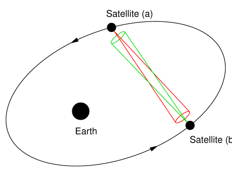
Arash Komaee, P. S. Krishnaprasad and Prakash Narayan

Introduction

- In free-space optical communication, the message is carried by the intensity of a laser beam that propagates through free-space from the transmitter toward the receiver.

Adopted from http://www.jaxa.jp

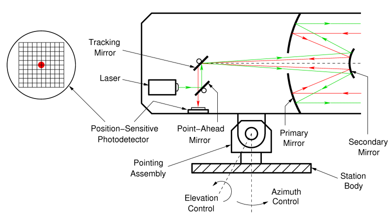
- Owing to the relative motion of the stations, an active mechanism is required to maintain optical alignment.



- Cooperative optical beam tracking is a viable solution in which each station employs the optical beam of the other station as a guide to point its own beam toward the other.

Cooperative Optical Beam Tracking

- Transceiver structure:



- The stations continually measure the arrival direction of their impinging optical beams using a position-sensitive photodetector.
- In short range applications with negligible light propagation delay, the stations transmit their optical beams along this measured direction.
- For a large propagation delay, the optical beams must be transmitted within a certain angle with respect to the instantaneous LOS.

Dynamical Equations

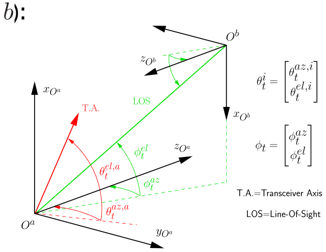
- Tracking error ($i = a, b$):

$$\alpha_t^i = \phi_t - \theta_t^i$$

- Pointing error:

$$\psi_t^i = \phi_{t+t_d} - \theta_t^i$$

t_d: propagation delay



- Dynamics of the tracking and pointing errors:

$$dx_t^i = A_t x_t^i dt + B_t u_t^i dt + D_t dw_t^i \qquad x_t^i = \begin{bmatrix} x_t^{p,i} \\ x_t^d \end{bmatrix} \begin{matrix} \to \theta_t^i \\ \to \phi_t \end{matrix}$$

$$y_t^i = C_t x_t^i$$

$$\psi_t^i = L_t z_t^i \qquad z_t^i = \begin{bmatrix} x_t^{p,i} \\ x_{t+t_d}^d \end{bmatrix}$$

$x_t^i \in \mathbb{R}^n$: state vector, $u_t^i \in \mathbb{R}^4$: control vector,

$w_t^i \in \mathbb{R}^m$: Wiener process, $y_t^i \propto \alpha_t^i$

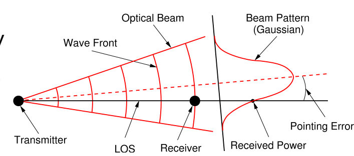
Photodetector Output

- The image of the received optical field on the photodetector surface is a spot of light with an almost Gaussian intensity profile $\gamma(r)$ located at y_t^i.
- The instantaneous optical power:

$\lambda_t^i(r)$: optical intensity

ν_t^i : maximum power

$\bar{\psi}$: beam divergence



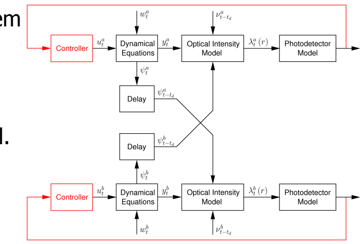
$$\lambda_t^a(r) = \nu_{t-t_d}^a \exp\left(-2\|\psi_{t-t_d}^b\|^2/\bar{\psi}^2\right)\gamma\left(r - y_t^a\right)$$

$$\lambda_t^b(r) = \nu_{t-t_d}^b \exp\left(-2\|\psi_{t-t_d}^a\|^2/\bar{\psi}^2\right)\gamma\left(r - y_t^b\right)$$

- The output of a high resolution photodetector is modeled by a space-time point process $N^i(\mathcal{T} \times \mathcal{S})$ with rate $\lambda_t^i(r)$, given over $\mathbb{R}^2$.
- $\mathscr{B}_t^i$: History of the photodetector output over $[0, t)$.

Control Problem

- The control problem is to find u_t^i in terms of $\mathscr{B}_t^i$ such that:



a) Beam tracking: $\|y_t^i\|$ remains small.

b) Active pointing: keep $\|\psi_t^i\|$ small to maximize the total optical power received by the stations, i.e.,

$$J = \mathrm{E}\left[\int_0^T \Big(\underbrace{\nu_t^a \exp\left(-2\|\psi_t^b\|^2/\bar{\psi}^2\right)}_{\text{power received by station } a} + \underbrace{\nu_t^b \exp\left(-2\|\psi_t^a\|^2/\bar{\psi}^2\right)}_{\text{power received by station } b}\Big) dt\right]$$

Results

- Optimal estimation (Rhodes and Snyder 1977): The conditional expectation $\hat{x}_t^i = \mathrm{E}\left[x_t^i | \mathscr{B}_t^i\right]$ and conditional covariance matrix $\Sigma_t^i = \mathrm{cov}\left(x_t^i | \mathscr{B}_t^i\right)$ are the solutions of:

$$d\hat{x}_t^i = A_t \hat{x}_t^i dt + B_t u_t^i dt + \int_{\mathbb{R}^2} M_t^i \left(r - C_t \hat{x}_t^i\right) N^i(dt \times dr)$$

$$d\Sigma_t^i = A_t \Sigma_t^i dt + \Sigma_t^i A_t^T dt + D_t D_t^T dt - \int M_t^i C_t \Sigma_t^i N^i(dt \times dr)$$

where $M_t^i = \Sigma_t^i C_t^T \left(C_t \Sigma_t^i C_t^T + R\right)^{-1}$

- Optimal estimate of z_t^i is $\hat{z}_t^i = \mathrm{E}\left[z_t^i | \mathscr{B}_t^i\right] = H_t \hat{x}_t^i$, where H_t is a known matrix.
- To satisfy condition (a), set: $C_t \hat{x}_t^a = C_t \hat{x}_t^b = 0$ (*).
- Due to the time delay, it is difficult to directly maximize the objective functional J. As a suboptimal solution, a lower bound on J can be maximized.
- The condition for maximizing this lower bound is:

$$L_t \hat{z}_t^a = L_t \hat{z}_t^b = 0 \qquad (**)$$

- Under some mild assumptions and for known matrices F_t and G_t, (*) and (**) can be achieved using:

$$u_t^i dt = F_t \hat{x}_t^i dt + G_t M_t^i \int_{\mathbb{R}^2} r N^i(dt \times dr)$$

† Reference: A. Komaee, Ph.D. dissertation (2008).