Adaptive Synchronization and Trajectory Tracking for Mechanical Systems

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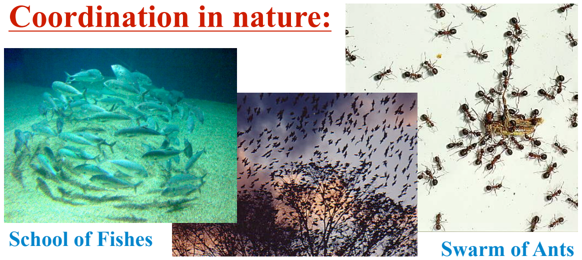
Motivation

Coordination in nature:



School of Fishes

Flocking of Birds

Swarm of Ants

Introduction

a) The problem of coordination and control of multi-agent systems is important in several applications such as sensor networks, unmanned air vehicles, and robot networks.

b) Trajectory tracking algorithms, such as the Slotine-Li controller (Slotine and Li, 87), when applied to a group of mechanical systems, guarantee tracking for individual systems, but do not exploit the cohesive structure of the group.

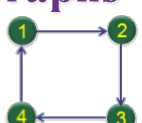
c) The output synchronization results in (Chopra and Spong, 06) are used to ensure trajectory tracking and state synchronization for interconnected mechanical systems.

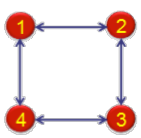
d) This problem has been addressed using contraction theory in (Chung and Slotine, 07).

Interconnection Graphs

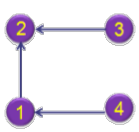
Balanced graph

The vertex of a directed graph is balanced if its in-degree and out-degree are equal.

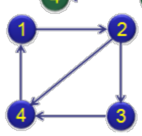




Undirected graph



Weakly connected



Strongly connected

Controlled Synchronization

Passivity-Based Adaptive Motion Control

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)=u \quad \text{(Euler-Lagrange Dynamics)}$$

$$u=Y(q,\dot{q},q^d,\dot{q}^d)\hat{\theta}-Ks+\tau \quad \text{(Slotine and Li, 87)}$$

$$\dot{\tilde{\theta}}=-\Gamma^{-1}Y^Ts \qquad \dot{\tilde{q}}=-\Lambda\tilde{q}+s$$

$$\dot{s}=M(q)^{-1}(-(C(\dot{q},q)+K)s+Y\tilde{\theta}+\tau)$$

$$x(t)=[\tilde{\theta}(t)\ \ \tilde{q}(t)\ \ s(t)]^T$$

$$\tilde{q}(t)=q(t)-q^d(t)$$

$$\tilde{\theta}(t)=\hat{\theta}(t)-\theta$$

where $q\in\mathbb{R}^n, Y\in\mathbb{R}^{n\times p}, \tilde{\theta}\in\mathbb{R}^{p\times 1}, \Gamma,\Lambda>0$

τ is the synchronizing control term

The above system is passive with a positive definite storage function

$$V(x)=\frac{1}{2}\left(s^TMs+\tilde{q}^TP\tilde{q}+\tilde{\theta}^T\Gamma\tilde{\theta}\right)$$

$$\Rightarrow \dot{V}(x)=-\dot{\tilde{q}}^TK\dot{\tilde{q}}-\tilde{q}^T\Lambda K\Lambda\tilde{q}+\tau^Ts$$

State Synchronization and Trajectory Tracking

The desired goal is to guarantee

$$\lim_{t\to\infty}||q_i(t)-q^d(t)||=\lim_{t\to\infty}||\dot{q}_i(t)-\dot{q}^d(t)||=0$$

$$\lim_{t\to\infty}||q_i(t)-q_j(t)||=\lim_{t\to\infty}||\dot{q}_i(t)-\dot{q}_j(t)||=0 \quad \forall i,j$$

Key Idea: Output synchronization of passive systems with a positive definite storage function (Chopra and Spong, 06)

Assume N agents, interconnected using a balanced, strongly connected communication graph. Let the output coupling control be given as

$$\tau_i(t)=\sum_{j\in\mathcal{N}_i}(s_j(t)-s_i(t)) \quad i=1,\ldots,N$$

Consider the positive definite storage function,

$$V(X)=2(V_1(x_1)+\cdots+V_N(x_N))$$

It is then possible to show that

$$\dot{V}(X)=\underbrace{-2\sum_{i=1}^{N}(\dot{\tilde{q}}_i^TK\dot{\tilde{q}}_i+\tilde{q}_i^T\Lambda K\Lambda\tilde{q}_i)}_{\text{Tracking}}-\underbrace{\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}||s_j-s_i||^2}_{\text{Synchronization}}\le 0$$

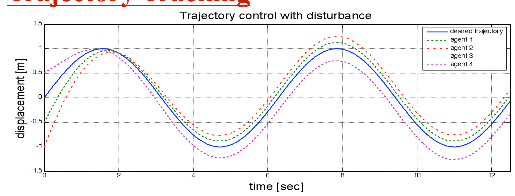
$\lim_{t\to\infty}||s_j(t)-s_i(t)||=0 \ \ \forall i,j$ implies state synchronization

Time delays can also be addressed in this framework.

Simulation Results

The desired trajectory is $q_d=\sin(t)$, a sinusoidal trajectory.

Trajectory Tracking

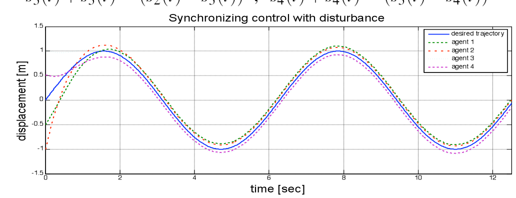


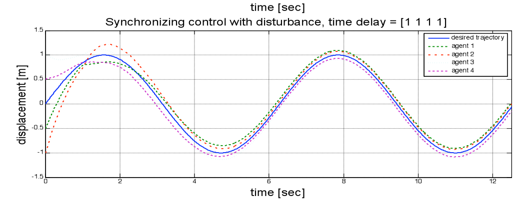
Synchronizing Control

The agents are interconnected using a ring topology.

$$\dot{s}_1(t)+s_1(t)=(s_4(t)-s_1(t)) \ ; \ \dot{s}_2(t)+s_2(t)=(s_1(t)-s_2(t))$$

$$\dot{s}_3(t)+s_3(t)=(s_2(t)-s_3(t)) \ ; \ \dot{s}_4(t)+s_4(t)=(s_3(t)-s_4(t))$$





The synchronizing control improved position tracking in the presence of disturbances and was robust to time delays in communication.

Conclusions

Synchronization and trajectory tracking can be achieved simultaneously in nonlinear mechanical systems. Moreover, it is also demonstrated and shown via simulations that the synchronizing control provides robustness to time delays and disturbances.

In our future work, multiple human operators will be included in the proposed framework. Additional theoretical issues, such as collision avoidance and nonholonomic constraints, will also be studied.