Modeling and Simulation of Pursuit Control Laws in Bat Prey Capture

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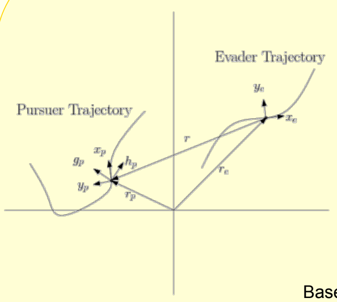
Introduction

- Pursuit is a subject of interest in various contexts. What are different pursuit strategies?
- What control law enables bats to successfully capture prey?
- What in the course of evolution caused bats to behave in the way they do now?

Planar Model Set-up



$$\begin{cases} \dot{r}_p = x_p, \\ \dot{x}_p = u_p y_p, \\ \dot{y}_p = -u_p x_p, \\ \dot{h}_p = u_h g_p, \\ \dot{g}_p = -u_h h_p, \end{cases}$$

$$\begin{cases} \dot{r}_e = \nu_e x_e, \\ \dot{x}_e = \nu_e u_e y_e, \\ \dot{y}_e = -\nu_e u_e x_e, \end{cases}$$

Base line: $r = r_p - r_e.$

Subscript p for pursuer
Subscript e for evader
r is position vector
x is unit vector tangent to its trajectory
y is unit vector normal to x
h is unit vector indicating head direction
g is unit vector normal to h
Pursuer is moving at unit speed, evader speed is v_e <1.

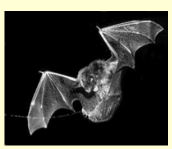


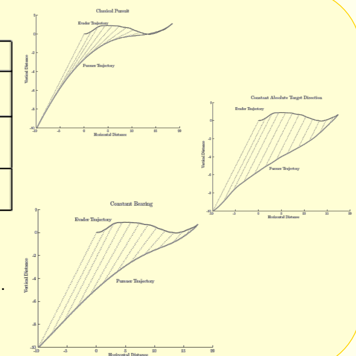
Photo by Sven Dear

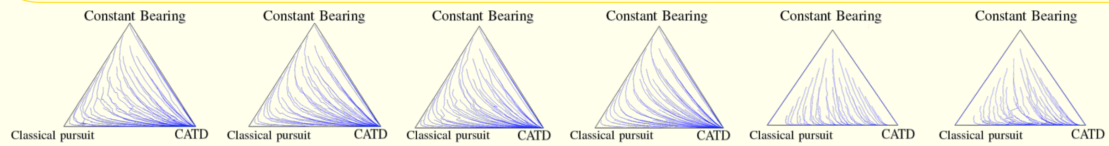
Three Pursuit Manifolds and Controls

THREE CONTROL LAWS

Strategy	Cost Function	Control Laws
Classical pursuit	$\Lambda = \frac{r}{\lvert r\rvert} \cdot x_p$	$u_{pcp} = -\mu \left(\frac{r}{\lvert r\rvert} \cdot y_p\right) - \frac{1}{\lvert r\rvert}\left(\frac{r}{\lvert r\rvert} \cdot \dot{r}^{\perp}\right)$
CATD[1]	$\Gamma = \frac{\frac{d}{dt}\lvert r\rvert}{\lvert\frac{dr}{dt}\rvert} = \frac{r}{\lvert r\rvert} \cdot \frac{\dot{r}}{\lvert\dot{r}\rvert}$	$u_{pcatd} = -\mu\left(\frac{r}{\lvert r\rvert} \cdot \dot{r}^{\perp}\right)$
Constant Bearing	$\Phi = \frac{r}{\lvert r\rvert} \cdot R(\theta) x_p$	$u_{pcb} = -\mu\left(y_p \cdot \frac{R(\theta) r}{\lvert r\rvert}\right) - \frac{1}{\lvert r\rvert}\left(\frac{r}{\lvert r\rvert} \cdot \dot{r}^{\perp}\right)$

Three pursuit manifolds are defined using cost functions, from which control laws are derived.
Classical pursuit: Pursuer's velocity is pointing directly toward the evader.
CATD (Constant Absolute Target Direction): Pursuer keeps the absolute target direction constant, i.e. baselines are parallel.
Constant Bearing: Pursuer keeps the bearing angle (angle between base line and pursuer velocity vector) constant.





Simulation 1 Simulation 2
1 linear path, 1 circular path turning rate 0.5, 1 trial of sinusoid path, 15 trial of random paths

Simulation 3 Simulation 4
Piecewise linear random evader paths, 75 trials per generation

Simulation 5 Simulation 6
Circular evader paths, turning rate uniform random in [0,1], 50 trial per generation.

Evolutionary Game[2]

- Population was broken into three subpopulation groups, which use CP, CATD, CB respectively.
- The three subgroups are given as three proportions, p_i, with
$$\sum_{i=1}^{3} p_i = 1 \qquad 0 \le p_i \le 1, \forall i$$
- The population distribution is evolved as
$$p_i' = \frac{p_i W_i}{\overline{W}} \quad \text{where} \quad \overline{W} = \sum_{i=1}^{3} p_i W_i.$$
p_i' represents the proportion of successive generation to p_i and W_i is payoff for that population
- For simulation 1, 3, 5, the payoffs are defined as (1), for simulation 2, 4, 6 the payoffs are defined as (2)
$$W_i = \frac{1}{n}\sum_{k=1}^{n} \frac{1}{\tau_k} \text{ (1)} \quad , \quad W_i = \frac{1}{\frac{1}{n}\sum_{k=1}^{n} \tau_k} \text{ (2).}$$

Conclusion

- The simulated evolutionary game shows that CATD under various circumstances yields a better payoff when measured by time. This can help explain why bats today use CATD pursuit strategy in prey capture.
- Derived control laws can also be used to analyze head motion of the bat head.

1. E. W. Justh, P. S. Krishnaprasad, Steering laws for motion camouflage, *Proceedings of the Royal Society A*, Jun 27, 2006, pp. 3629-3643.
2. Ermin Wei. Senior Thesis. 2007.