Motion camouflage with sensorimotor delay

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Abstract

In recent work, a particular **high-gain feedback law** was shown to drive a **pursuer-evader system** arbitrarily close to a state of **motion camouflage** in **finite time**. However, data collected from bat-insect encounters, in which a strategy akin to motion camouflage is used by the bat to pursue the insect, reveal that a modest feedback gain is used, and significant **sensorimotor delay** is present. We derive constraints among parameters such as feedback gain, sensorimotor delay, and relative speed, for which it is possible to guarantee performance of the feedback law in achieving motion camouflage. Besides helping us to better understand **pursuit in nature**, such pursuit laws have applications in **missile guidance** and **unmanned vehicle control**.

Background

• Hypothesis of Motion Camouflage in **visual insects** first put forward by Srinivasan and Davey (*Proc. R. Soc. B* **259**(1354):19-25, 1995), based on revisiting data on flight behavior of hoverfly *Syritta pipiens,* during mating pursuit, (Collett and Land, *J. Comp. Physiol.*, **99**:1-66, 1975) .

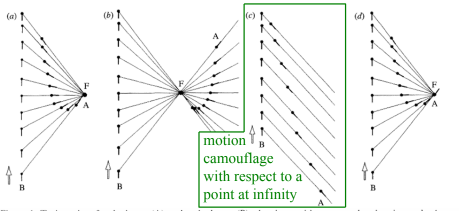

Figure 1. Trajectories of a shadower (A) and a shadowee (B), showing positions at regular time intervals. A can camouflage its motion by emulating a stationary object at a point F which is (*a*) behind it, (*b*) in front of it, (*c*) at infinity, or (*d*) located at A's initial position.

Figure from Srinivasan and Davey, *Proc. R. Soc. B* **259**(1354):19-25, 1995.

• Support for the motion camouflage hypothesis in dragonfly data (Mizutani, Chahl and Srinivasan, *Nature* **423**:604, 2003).

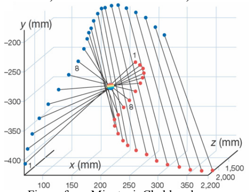

Figure from Mizutani, Chahl and Srinivasan, *Nature* **423**:604, 2003.

• **Pursuit** of mate, rival, or prey is a basic behavior at all scales.
• **Stealth**: *pursuer moves so as to avoid detection by a target.*

Acknowledgements

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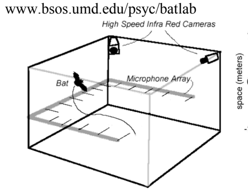
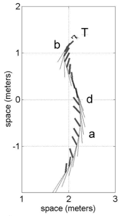
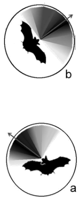
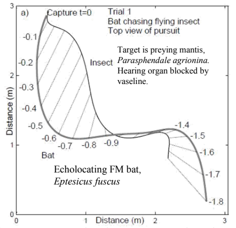
Free-flying Bat-Insect Engagements

Auditory Neuroethology Lab:
www.bsos.umd.edu/psyc/batlab







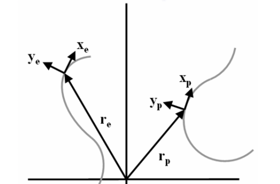


Eptesicus fuscus: the big brown bat

The Constant Absolute Target Direction (CATD) Strategy used by the bat is **geometrically indistinguishable** from **Motion Camouflage.**

• K. Ghose and C.F. Moss, *J. Neuroscience*, 26(6):1704-1710, 2006.
• K. Ghose, T.K. Horiuchi, P.S. Krishnaprasad and C.F. Moss, *PLoS Biology* 4(5): 865-873, 2006.
• K. Ghose, Ph.D. dissertation, University of Maryland, 2006.

Curves and Moving Frames



Frame equations:

$$\dot{\mathbf{r}}_p = \mathbf{x}_p,\quad \dot{\mathbf{x}}_p = \mathbf{y}_p u_p,\quad \dot{\mathbf{y}}_p = -\mathbf{x}_p u_p$$

$$\dot{\mathbf{r}}_e = \nu\mathbf{x}_e,\quad \dot{\mathbf{x}}_e = \nu\mathbf{y}_e u_e,\quad \dot{\mathbf{y}}_e = -\nu\mathbf{x}_e u_e$$

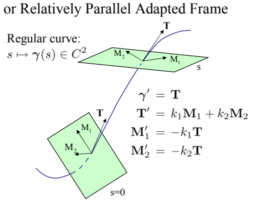
$$0 < \nu < 1$$

Pursuer trajectory: $\mathbf{r}_p$ = pursuer position, $\mathbf{x}_p$ = unit tangent vector, $\mathbf{y}_p$ = unit normal vector, u_p = steering control (or plane curvature).

Evader trajectory: $\mathbf{r}_e$ = evader position, $\mathbf{x}_e$ = unit tangent vector, $\mathbf{y}_e$ = unit normal vector, u_e = steering control (or plane curvature), ν = speed ratio.

Three dimensions: Natural Frenet Frame or Relatively Parallel Adapted Frame

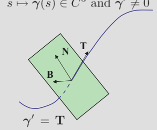
Regular curve: $s \mapsto \gamma(s) \in C^2$



$$\gamma' = \mathbf{T},\quad \mathbf{T}' = k_1\mathbf{M}_1 + k_2\mathbf{M}_2,\quad \mathbf{M}_1' = -k_1\mathbf{T},\quad \mathbf{M}_2' = -k_2\mathbf{T}$$

Frenet-Serret Frame

Regular curve: $s \mapsto \gamma(s) \in C^3$ and $\gamma'' \neq 0$



$$\gamma' = \mathbf{T},\quad \mathbf{T}' = \kappa\mathbf{N},\quad \mathbf{N}' = -\kappa\mathbf{T} + \tau\mathbf{B},\quad \mathbf{B}' = -\tau\mathbf{N}$$

R. L. Bishop, *Amer. Math. Month.* (1975), **82**(3):246-251.
E.W. Justh and P.S. Krishnaprasad, *Proc. IEEE CDC*, 2005.

Delay-Free Planar Pursuit Model

Motion camouflage condition:

$$\mathbf{r} \triangleq \mathbf{r}_p - \mathbf{r}_e = \lambda\mathbf{r}_\infty$$

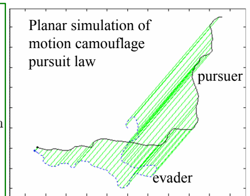
- $\mathbf{r}_\infty$ is a fixed unit vector
- λ is a time-dependent scalar

Equivalent infinitesimal condition (we assume $|\mathbf{r}| \neq 0$):

$$\dot{\mathbf{r}} - \left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\dot{\mathbf{r}}\right)\frac{\mathbf{r}}{|\mathbf{r}|} = 0$$

Transverse component of relative velocity = 0.

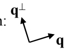
Planar simulation of motion camouflage pursuit law



Distance function: $\Gamma = \dfrac{\frac{d}{dt}|\mathbf{r}|}{|\dot{\mathbf{r}}|} = \dfrac{\mathbf{r}}{|\mathbf{r}|}\cdot\dfrac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}$

Drive Γ to -1 for motion camouflaged pursuit.

Notation:



Pursuer control law: $u_p = -\mu\left(\dfrac{\mathbf{r}}{|\mathbf{r}|}\cdot\dot{\mathbf{r}}^\perp\right)$

(3D law: P.V. Reddy, E.W. Justh and P.S. Krishnaprasad, *Proc. IEEE CDC*, 2006.)

Definition: "motion camouflage is accessible in finite time" if for any $\varepsilon > 0$ there exists a time t_1 such that $\Gamma(t_1) \leq -1 + \varepsilon$.

Proposition: Suppose
(1) $0 < \nu < 1$ (and ν is constant),
(2) u_e is continuous and $|u_e|$ is bounded,
(3) $\Gamma_0 = \Gamma(0) < 1$, and
(4) $|\mathbf{r}(0)| > 0$.
Then motion camouflage is accessible in finite time using high-gain feedback (i.e., by choosing μ sufficiently large).

E.W. Justh and P.S. Krishnaprasad, *Proc. R. Soc. A*, **462**:3629-3643, 2006.

Planar Pursuit Model with Delay

Insert a sensorimotor delay $\tau > 0$ into the pursuer feedback control:

$$\begin{aligned}\dot{\mathbf{r}}_p &= \mathbf{x}_p\\ \dot{\mathbf{x}}_p &= \mathbf{y}_p u_p\\ \dot{\mathbf{y}}_p &= -\mathbf{x}_p u_p\end{aligned} \quad\Longrightarrow\quad \begin{aligned}\dot{\mathbf{r}}_p &= \mathbf{x}_p\\ \dot{\mathbf{x}}_p &= \mathbf{y}_p u_p(t-\tau)\\ \dot{\mathbf{y}}_p &= -\mathbf{x}_p u_p(t-\tau)\end{aligned}$$

Proposition: Suppose
(1) $0 < \nu < 1$ (and ν is constant),
(2) u_e is continuous and $|u_e|$ is bounded,
(3) $\Gamma_0 = \Gamma(0) < 1 - \varepsilon$, and
(4) $|\mathbf{r}(0)| > 0$.
Then motion camouflage is accessible in finite time provided there exists a value of $\mu > 0$ which satisfies:

$$\mu > \frac{(1+\nu)^2}{(1-\nu)r_0}$$

$$c_2 = c_0 - \frac{1}{\sqrt{\epsilon}}\left[c_1 + \tau\mu\left[c_1 + \left(\frac{1+\nu}{(1-\nu)^2}\right)\left(\frac{(1+\nu)^2}{2r_0} + \mu(1+\nu)\right)\right]\right] > \delta$$

$$\delta = \max\left(0, \frac{1}{2}\left(\frac{1+\nu}{|\mathbf{r}(0)| - r_0}\right)\ln\left[\left(\frac{1+\Gamma_0}{1-\Gamma_0}\right)\left(\frac{2-\epsilon}{\epsilon}\right)\right]\right)$$

$$c_0 = \mu\left(\frac{1-\nu}{1+\nu}\right) - \frac{1+\nu}{r_0}, \quad c_1 = \frac{\nu^2(1+\nu)u_{max}}{(1-\nu)^2}$$

Remark: This condition is non-vacuous (P.V. Reddy, M.S. Thesis, University of Maryland, 2007.)

Analysis

Summary of analysis techinque:

- $\epsilon > 0$, $|\mathbf{r}(0)| > 0$, ν, τ, and $u_{max} = \max|u_e|$ are given; choose $0 < r_0 < |\mathbf{r}(0)|$.
- Differentiate the distance function Γ along trajectories of the pursuer-evader system.
- Derive bounds on terms in the expression for $\dot{\Gamma}$ to obtain conditions under which $\dot{\Gamma} \leq 0$, and in particular, $\dot{\Gamma} \leq -(1-\Gamma^2)c_2$ for some $c_2 > 0$.
- From ν, $|\mathbf{r}(0)|$, and r_0, obtain an explicit lower bound on the duration T of the engagement.
- Derive a lower bound on c_2 to ensure that Γ will be driven from $\Gamma(0) = \Gamma_0$ to $\Gamma(t_1) \leq -1 + \epsilon$ for some $t_1 \in [0, T]$.
- Then as long as a feedback gain $\mu > 0$ exists such that c_2 satisfies the necessary lower bound, we may conclude *motion camouflage is accessible in finite time* for the planar pursuit model with delay.

Define $w = -\left(\dfrac{\mathbf{r}}{|\mathbf{r}|}\cdot\dot{\mathbf{r}}^\perp\right)$, so that $u_p = \mu w$

$$\begin{aligned}\dot{\Gamma} &= \left(\frac{\dot{\mathbf{r}}\cdot\dot{\mathbf{r}} + \mathbf{r}\cdot\ddot{\mathbf{r}}}{|\mathbf{r}||\dot{\mathbf{r}}|}\right) - \left(\frac{\mathbf{r}\cdot\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}\right)\left(\frac{\mathbf{r}\cdot\dot{\mathbf{r}}}{|\mathbf{r}|^3}\right) - \left(\frac{\mathbf{r}\cdot\dot{\mathbf{r}}}{|\mathbf{r}|}\right)\left(\frac{\dot{\mathbf{r}}\cdot\ddot{\mathbf{r}}}{|\dot{\mathbf{r}}|^3}\right)\\ &= \frac{|\dot{\mathbf{r}}|}{|\mathbf{r}|}(1-\Gamma^2) + \frac{1}{|\dot{\mathbf{r}}|}\underbrace{\left[\frac{\mathbf{r}}{|\mathbf{r}|} - \left(\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}\right)\frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}\right]}\cdot\ddot{\mathbf{r}}\\ &= \frac{|\dot{\mathbf{r}}|}{|\mathbf{r}|}(1-\Gamma^2) - \frac{1}{|\dot{\mathbf{r}}|}\left[\frac{w}{|\dot{\mathbf{r}}|^2}\dot{\mathbf{r}}^\perp\right]\cdot\ddot{\mathbf{r}}\end{aligned} \qquad = \left[\frac{\mathbf{r}}{|\mathbf{r}|}\cdot\left(\frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}\right)^\perp\right]\left(\frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}\right)^\perp$$

$$\begin{cases}\dot{\mathbf{r}}^\perp = \mathbf{y}_p - \nu\mathbf{y}_e\\ \ddot{\mathbf{r}} = \mathbf{y}_p u_p(t-\tau) - \nu^2\mathbf{y}_e u_e\end{cases}$$

$$\Rightarrow \dot{\mathbf{r}}^\perp\cdot\ddot{\mathbf{r}} = [1 - \nu(\mathbf{x}_p\cdot\mathbf{x}_e)]\,u_p(t-\tau) + \nu^2[\nu - (\mathbf{x}_p\cdot\mathbf{x}_e)]\,u_e$$

$$\dot{\Gamma} = \frac{|\dot{\mathbf{r}}|}{|\mathbf{r}|}(1-\Gamma^2) - \frac{1}{|\dot{\mathbf{r}}|}\left(\frac{w}{|\dot{\mathbf{r}}|^2}\right)[1-\nu(\mathbf{x}_p\cdot\mathbf{x}_e)]\,u_p(t-\tau) \underbrace{- \frac{1}{|\dot{\mathbf{r}}|}\left(\frac{w}{|\dot{\mathbf{r}}|^2}\right)\nu^2[\nu - (\mathbf{x}_p\cdot\mathbf{x}_e)]\,u_e}_{\leq \frac{|w|}{|\dot{\mathbf{r}}|}\left[\frac{\nu^2(1+\nu)}{(1-\nu)^2}\right]u_{max}}$$

$$u_p(t-\tau) = u_p(t) + \underbrace{[u_p(t-\tau) - u_p(t)]}_{\text{want to bound}}$$

Mean Value Theorem $\Rightarrow \exists t^* \in (t-\tau, t)$ s.t. $u_p(t-\tau) - u_p(t) = -\tau\dot{u}_p(t^*) = -\tau\mu\dot{w}(t^*)$

$$|\dot{w}| \leq \frac{(1+\nu)^2}{2r_0} + \mu(1+\nu) + \nu^2 u_{max}, \text{ for } |\mathbf{r}| > r_0$$

$$\Rightarrow |u_p(t-\tau) - u_p(t)| \leq \tau\mu\left[\frac{(1+\nu)^2}{2r_0} + \mu(1+\nu) + \nu^2 u_{max}\right], \text{ for } |\mathbf{r}| > r_0$$

$$\begin{aligned}\dot{\Gamma} \leq& -(1-\Gamma^2)\left[\frac{\mu}{|\dot{\mathbf{r}}|}[1-\nu(\mathbf{x}_p\cdot\mathbf{x}_e)] - \frac{|\dot{\mathbf{r}}|}{|\mathbf{r}|}\right] + \frac{|w|}{|\dot{\mathbf{r}}|}\left[\frac{\nu^2(1+\nu)}{(1-\nu)^2}u_{max}\right.\\ &\left.+ \frac{1}{|\dot{\mathbf{r}}|^2}[1-\nu(\mathbf{x}_p\cdot\mathbf{x}_e)]\,\tau\mu\left(\frac{(1+\nu)^2}{2r_0} + \mu(1+\nu) + \nu^2 u_{max}\right)\right]\\ \leq& -(1-\Gamma^2)\,c_0 + \sqrt{1-\Gamma^2}\left[c_1 + \tau\mu\left(c_1 + \frac{(1+\nu)}{(1-\nu)^2}\left(\frac{(1+\nu)^2}{2r_0} + \mu(1+\nu)\right)\right)\right],\end{aligned}$$

for $|\mathbf{r}| > r_0$

Given $0 < \epsilon << 1$, $|\mathbf{r}| > r_0$, and $(1-\Gamma^2) > \epsilon$,

$$\Rightarrow \dot{\Gamma} \leq -(1-\Gamma^2)c_2 \quad\Rightarrow\quad \Gamma(t) \leq \tanh(\tanh^{-1}\Gamma_0 - c_2 t)$$

$$T = \frac{|\mathbf{r}(0)| - r_0}{1+\nu} > 0 \quad (T = \text{time interval guaranteeing } |\mathbf{r}| > r_0)$$

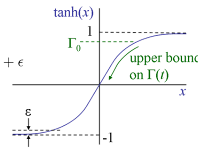
$$x \leq \frac{1}{2}\ln\left(\frac{\epsilon}{2-\epsilon}\right) \Rightarrow \tanh(x) \leq -1 + \epsilon$$

$$\Downarrow$$

$$\tanh^{-1}\Gamma_0 - c_2 T \leq \frac{1}{2}\ln\left(\frac{\epsilon}{2-\epsilon}\right) \Rightarrow \Gamma(T) \leq -1 + \epsilon$$

$$\Downarrow$$

lower bound on c_2



- $c_2 = c_2(\mu, \nu, u_{max}, r_0, \tau, \epsilon, \Gamma_0)$.
- Further calculation needed to derive bounds on μ from bound on c_2
- See P.V. Reddy, M.S. Thesis, University of Maryland, 2007.