Motion camouflage in a stochastic setting

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Overview

In this poster, we extend (to the stochastic setting) recent work on 2-dimensional models and steering control laws for motion camouflage, a stealthy pursuit strategy observed in nature. We discuss a family of admissible stochastic evader controls. After presenting the background and deterministic formulation, we discuss the use of a high-gain pursuit law in the presence of sensor noise as well as in the case when the evader's steering is driven by a stochastic process, demonstrating (in the planar setting) that motion camouflage is still accessible (in the mean) in finite time. The work here also is a basis for game-theoretic study of optimal evasion strategies.

What is "motion camouflage"?

Motion camouflage is a stealthy pursuit strategy which relies on ***minimizing the perceived relative motion*** of a pursuer as observed by its prey.

Examples in nature:

- Hoverflies (Srinivasan, Davey, based on Collett & Land : 1995)
- Dragonflies (Mizutani, Chahl, & Srinivasan: 2003)
- Bats (Ghose, Horiuchi, Krishnaprasad, & Moss: 2006)

(Bats use Constant Absolute Target Direction (CATD), which is geometrically indistinguishable from MC "at infinity")

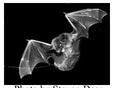
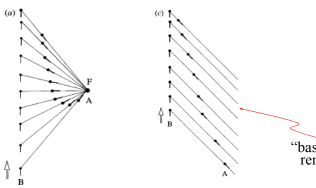


Photo by Steven Dear



Motion camouflage wrt a fixed point — Motion camouflage "at infinity"

Figures from M.V. Srinivasan and M. Davey, *Proc. Roy. Soc. Lond. B*, Vol. 259, No. 1354, pp. 19-25, 1995.

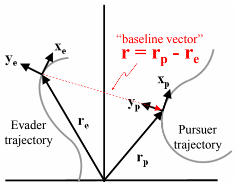
Modeling the Pursuer-Evader System

$$\dot{\mathbf{r}}_p = \mathbf{x}_p, \quad \dot{\mathbf{x}}_p = \mathbf{y}_p u_p, \quad \dot{\mathbf{y}}_p = -\mathbf{x}_p u_p \qquad (1)$$

$$\dot{\mathbf{r}}_e = \nu \mathbf{x}_e, \quad \dot{\mathbf{x}}_e = \nu \mathbf{y}_e u_e, \quad \dot{\mathbf{y}}_e = -\nu \mathbf{x}_e u_e \qquad (2)$$

$$0 < \nu < 1$$

$$|\mathbf{r}| \neq 0$$



Transverse component of relative velocity

$$\mathbf{w} = \dot{\mathbf{r}} - \left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}\right) \frac{\mathbf{r}}{|\mathbf{r}|}$$

Motion camouflage (at infinity) iff:

$$\mathbf{r}_p = \mathbf{r}_e + \lambda \mathbf{r}_\infty$$

λ a time-dependent scalar
$\mathbf{r}_\infty$ a fixed vector

$$\Leftrightarrow$$

$$\mathbf{w} = 0$$

Cost Function & Control Law

Cost function

$$\Gamma(t) = \frac{\frac{d}{dt}|\mathbf{r}|}{\left|\frac{d\mathbf{r}}{dt}\right|} \qquad (3)$$

rate of change of baseline length

absolute rate of change of baseline vector

$$\Gamma \in [-1, +1]: \begin{cases} \Gamma = +1 & \text{Pure lengthening of baseline} \\ \Gamma = 0 & \text{Pure rotation of baseline} \\ \Gamma = -1 & \text{Pure shortening of baseline} \end{cases}$$

Motion camouflage control law

$$u_p = -\mu\left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}^\perp\right), \quad \mu > 0 \qquad (4)$$

($\perp$ indicates ccw rotation by $\pi/2$)

Against a slower evader with deterministic u_e, this control law ensures attainment of motion camouflage in finite time

E.W. Justh and P.S. Krishnaprasad, *Proc. R. Soc. A*, Vol. 462, pp.3629-3643, 2006

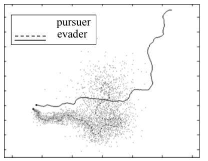
Extending to the Stochastic Setting

In extending to the stochastic setting, two questions naturally arise:

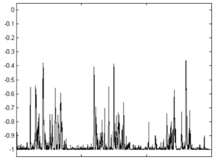
- *How will sensor noise affect the pursuer's ability to attain motion camouflage?*
- *Can a stochastically steering evader thwart the pursuer's attainment of motion camouflage?*

Sensor Noise

In this simulation, we corrupt the pursuer's measurements of $\mathbf{r}$ and $\dot{\mathbf{r}}$ with zero-mean $\mathbb{R}^2$-valued Gaussian noise processes. These noisy measurements are then used by the pursuer to calculate (4).



Pursuer and evader trajectories with noisy measurements superimposed. The noise processes have covariance matrices of the form $\mathrm{diag}(\sigma^2, \sigma^2)$ where $\sigma = .15|\mathbf{r}|$ for one noise process and $\sigma = .15|\dot{\mathbf{r}}|$ for the other.



Plot of Γ vs. time, which denotes deviation from motion camouflage state. Deviations are caused by measurement noise as well as evader maneuvering (since μ is finite)

Figures from E.W. Justh and P.S. Krishnaprasad, *Proc. R. Soc. A*, Vol. 462, pp.3629-3643, 2006

Stochastically Steering Evader

Possible stochastic processes to use for evader steering control:

$$dz = \alpha(z,t)dt + \beta(z,t)dW, \quad z(0) = z_0, \qquad (5)$$
$$u_e = z,$$

$W(\cdot)$ is standard Brownian motion

$$dz = \alpha(z,t)dt + \sum_{i=1}^{m} \beta_i(z,t)dN_i, \quad z(0) = z_0, \qquad (6)$$
$$u_e = z,$$

$N_i,\ i = 1, 2, \ldots, m$ are Poisson counters with rates λ_i

A Biological Example of a Stochastic Control Process

The "run and tumble" motion of bacterial chemotaxis can be modeled as a stochastic control process as follows:

$$dz = \frac{1}{2}z(z-1)dN_1 - \frac{1}{2}z(z+1)dN_2 + (z^2-1)dN_3 - (z^2-1)dN_4, \quad z(0) = z_0, \qquad (7)$$
$$u_e = z,$$

$$\lambda_1 = \lambda_2 = \lambda_H$$
$$\lambda_3 = \lambda_4 = \lambda_L$$
$$\lambda_H >> \lambda_L$$

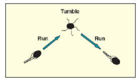
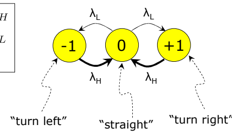


Figure from http://www.ccbi.cam.ac.uk/iGEM2005/index.php/Chemotactic_Patterning



Admissible Stochastic Controls

Let $X = [x_e \ \ y_e] \in SO(2)$; then X evolves according to the stochastic differential equation

$$dX_t = X_t \hat{A} u_e dt, \quad \hat{A} = \begin{pmatrix} 0 & -\nu \\ \nu & 0 \end{pmatrix}$$

A stochastic process u_e is an admissible control if:

1. u_e is such that $X = [x_e \ \ y_e]$ evolves on the prescribed manifold (i.e. SO(2))
2. u_e has piecewise continuous sample paths and bounded first and second moments

Note: (5) and (6) satisfy these requirements (with the exception of the case where $dz = dW$)

Proposition: Accessibility of Motion Camouflage

Proposition: Consider the system (1) – (2), with control law (4), and Γ defined by (3), with the following hypotheses:

(A1) $0 < \nu < 1$ (and ν is constant)

(A2) u_e is an admissible stochastic control process (as defined above)

(A3) $E[1-\Gamma_0^2] > 0$, where $\Gamma_0 = \Gamma(0)$, and

(A4) $|\mathbf{r}(0)| > 0$.

Then motion camouflage is accessible in the mean in finite time using high-gain feedback (i.e., given any $\epsilon > 0$, the gain $\mu > 0$ can be chosen such that there exists a time t_1 for which $E[1-\Gamma^2(t_1)] \leq \epsilon$.)

Proof:

Analyzing the stochastic differential equation (SDE)

Taking time derivative of gamma along trajectories of (1)-(2) and substituting (4) gives the following SDE:

$$d\Gamma = -\left[\frac{\mu}{|\dot{\mathbf{r}}|}(1 - \nu(\mathbf{x}_p \cdot \mathbf{x}_e)) - \frac{|\dot{\mathbf{r}}|}{|\mathbf{r}|}\right]\left[\frac{1}{|\dot{\mathbf{r}}|^2}\left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}^\perp\right)^2\right]dt + \frac{1}{|\dot{\mathbf{r}}|}\left[\frac{1}{|\dot{\mathbf{r}}|^2}\left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}^\perp\right)\right](\nu - (\mathbf{x}_p \cdot \mathbf{x}_e))\nu^2 u_e dt.$$

Noting that

$$\frac{1}{|\dot{\mathbf{r}}|^2}\left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}^\perp\right)^2 = 1 - \Gamma^2 \geq 0,$$

and using bounds on $|\mathbf{x}_p \cdot \mathbf{x}_e|$ and $|\dot{\mathbf{r}}|$, we have the following inequality:

$$d\Gamma \leq -(1-\Gamma^2)\left[\mu\left(\frac{1-\nu}{1+\nu}\right) - \frac{1+\nu}{|\mathbf{r}|}\right]dt + \frac{\nu^2(1+\nu)}{(1-\nu)^2}\left(\sqrt{1-\Gamma^2}\right)|u_e|dt.$$

Now, for any $\mu > 0$, we can define constants $r_0 > 0$ and $c_0 > 0$ such that

$$\mu = \left(\frac{1+\nu}{1-\nu}\right)\left(\frac{1+\nu}{r_0} + c_0\right),$$

which provides us with the simplified SDE

$$d\Gamma \leq -(1-\Gamma^2)c_0 dt + \frac{\nu^2(1+\nu)}{(1-\nu)^2}\left(\sqrt{1-\Gamma^2}\right)|u_e|dt,$$

for all $|\mathbf{r}| \geq r_0$.

Bounds for E[Γ]

Taking the expectation of both sides and applying the Cauchy-Schwartz Inequality yields

$$\frac{d}{dt}E[\Gamma] \leq -c_0 E\left[1-\Gamma^2\right] + c_1\sqrt{E\left[1-\Gamma^2\right]}, \quad |\mathbf{r}| > r_0 \qquad (8)$$

where we have used the assumption $E[u_e^2] \leq u_{max}^2$ for some $u_{max} > 0$ and we've defined

$$c_1 = \frac{\nu^2(1+\nu)}{(1-\nu)^2}u_{max} > 0.$$

Given $0 < \epsilon << 1$, choose $c_0 > c_1/\sqrt{\epsilon}$ (thereby choosing μ) and define $c_2 = c_0 - c_1/\sqrt{\epsilon} > 0$. Then we have

$$\frac{d}{dt}E[\Gamma] \leq -E\left[1-\Gamma^2\right]\left(c_0 - \frac{c_1}{\sqrt{E\left[1-\Gamma^2\right]}}\right) \leq -c_2\epsilon, \qquad (9)$$

$$\Longrightarrow \quad E[\Gamma] \leq -c_2\epsilon t + E[\Gamma_0], \qquad (10)$$

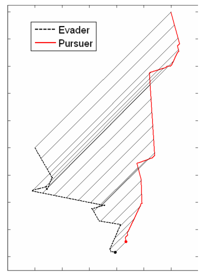
provided that $|\mathbf{r}| > r_0$ **and** $\boxed{E[1-\Gamma^2] > \epsilon}$

Conclusion of the Proof

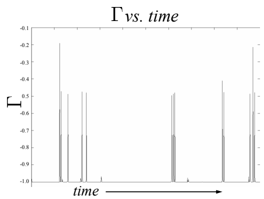
Since $E[\Gamma]$ is bounded below by -1 and (10) is monotone decreasing, a contradiction argument shows that the condition $E[1-\Gamma^2] > \epsilon$ must be violated at some finite time.

- There exists a finite time t_1 such that $E[1-\Gamma^2(t_1)] \leq \epsilon$ i.e. motion camouflage is accessible in the mean in finite time.

Simulation



Plot of pursuer and evader trajectories; pursuer using control law (4) with gain $\mu = 1$ and evader using "run-and-tumble" stochastic control (7) with $\lambda_L = .07\lambda_H$ and $\nu = 0.9$.



Corresponding plot of the cost function Γ. Deviations from motion camouflage state correspond to abrupt evader maneuvers.

Ongoing/Future Work

- Studies in (possibly stochastic) evader speed variation (i.e. relaxing assumption A1)
- Game-theoretic analysis of pursuer-evader interaction
  - Evader maximizes "pursuer visibility" (i.e. push Γ towards 0), while Pursuer minimizes visibility (i.e. push Γ towards -1)

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