Nonlinear Analysis of TCP-RED Instabilities

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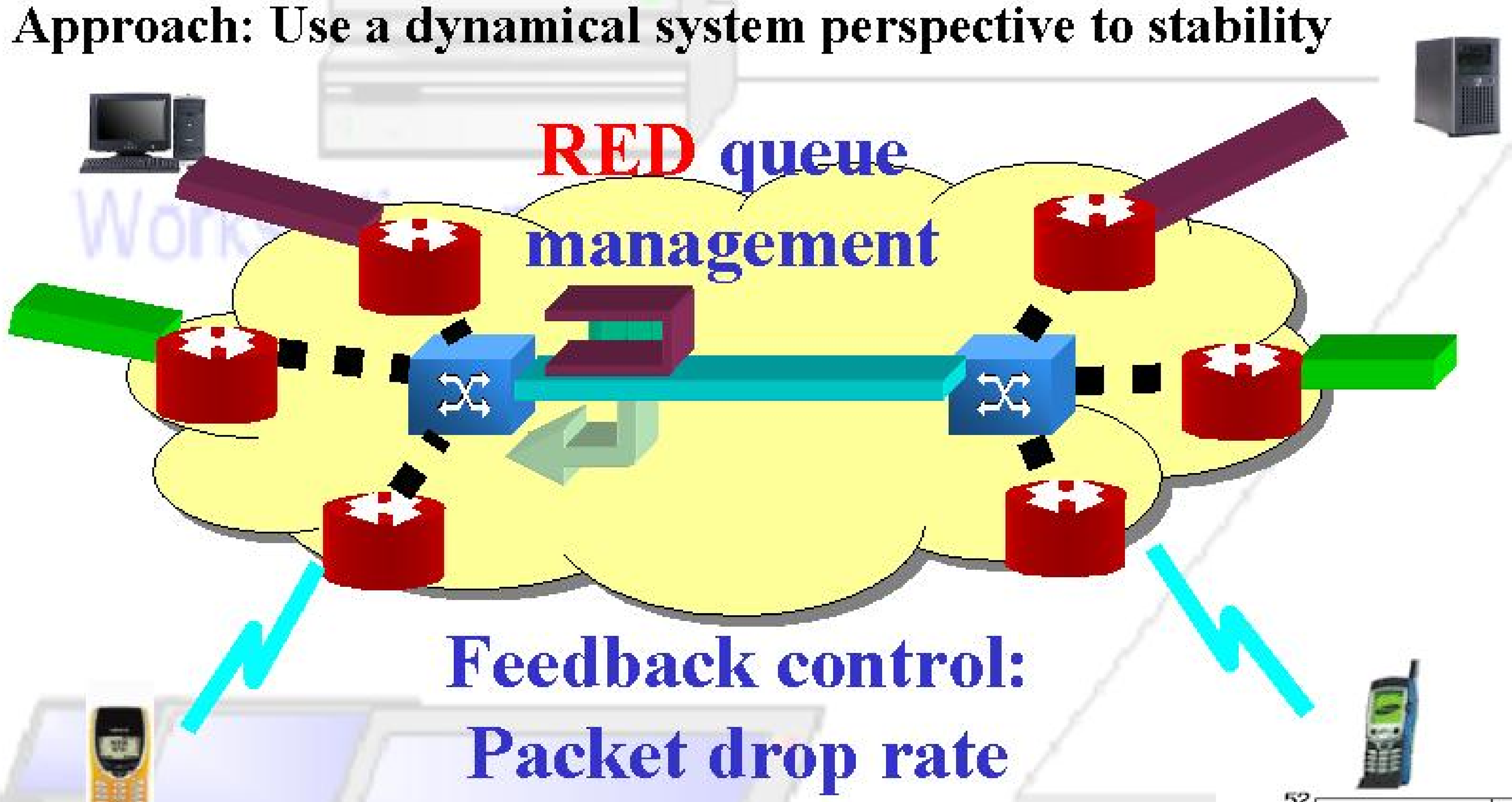
Problem: How to set RED parameters in a robust way?

Approach: Use a dynamical system perspective to stability

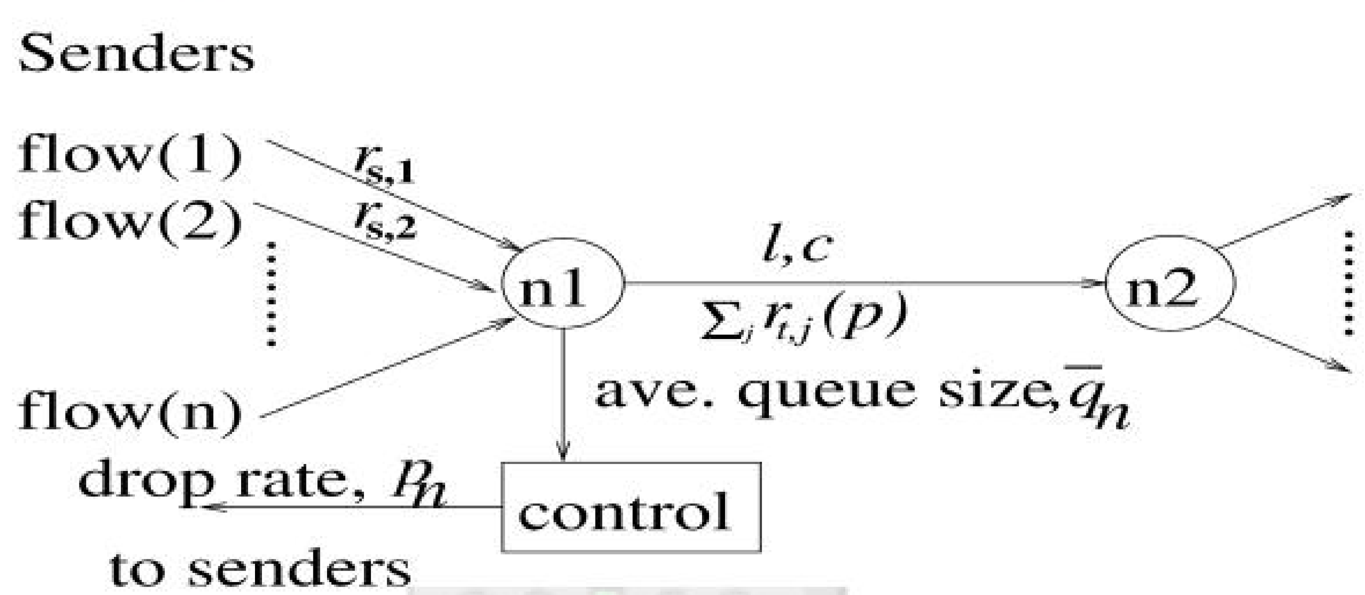


Mathematical Modeling:

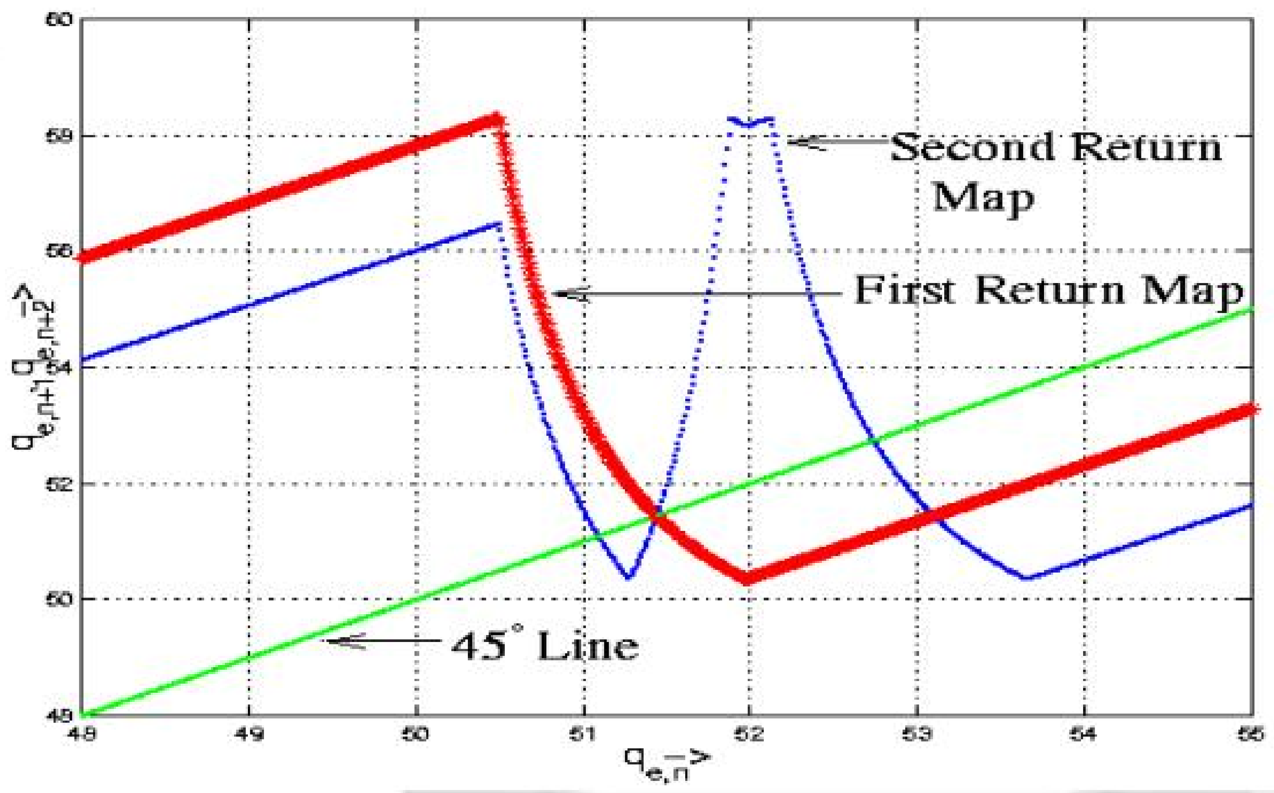
$$q_{k+1} = G(p_k)$$
$$\bar{q}_{e,k+1} = A(\bar{q}_{e,k}, q_{k+1})$$
$$p_{k+1} = H(\bar{q}_{e,k+1})$$

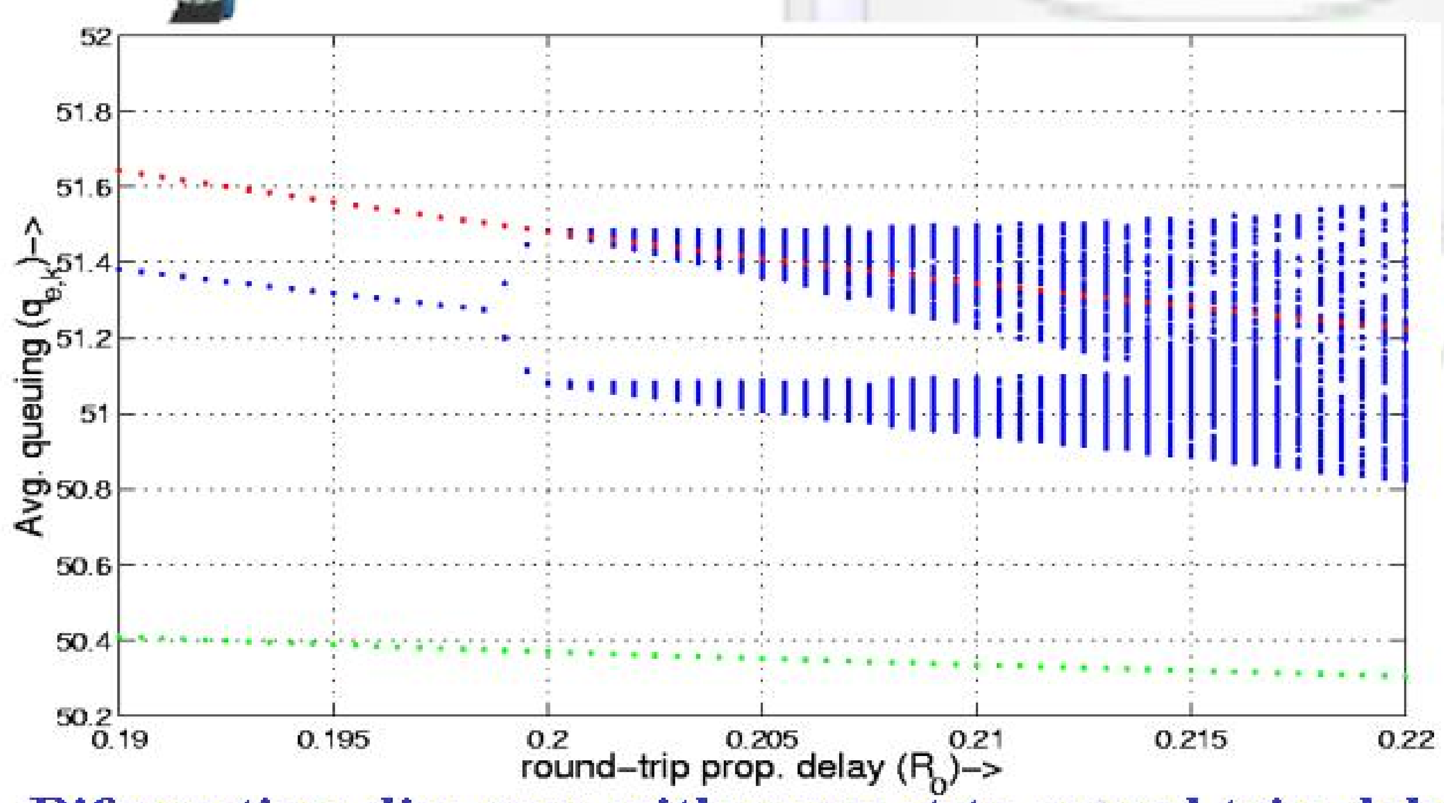
- ✓Essentially nonlinear first order discrete time dynamical system
- ✓Nonlinearity comes from TCP transfer function, square root dependence on drop prob.
- ✓Model as a self clocking system
- ✓Piecewise smooth map

Simlified one hop network model as a feedback system



First and second return map and their intersection with 45 degree line show the existence of a fixed point and period two orbit





Bifurcation diagram with respect to round trip delay R_0.

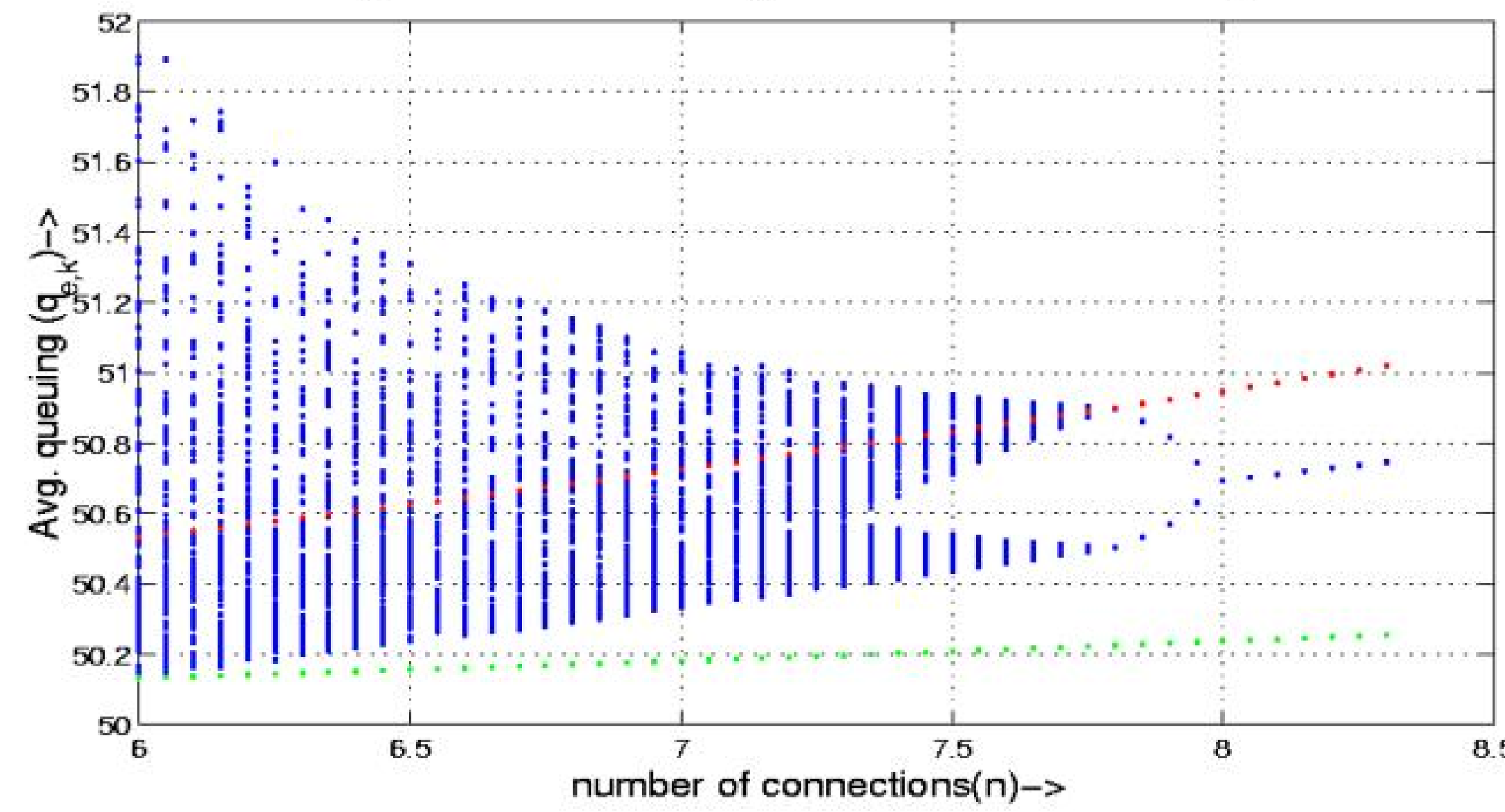
One Dimensional First Order Discrete Time Map:

$$\bar{q}_{e,k+1} = \begin{cases} (1-w)\bar{q}_{e,k} & \text{if } \bar{q}_{e,k} > b_1 \\ (1-w)\bar{q}_{e,k} + wB & \text{if } \bar{q}_{e,k} < b_2 \\ (1-w)\bar{q}_{e,k} + +w\left(\frac{nK}{\sqrt{\frac{p_{max}(\bar{q}_{e,k}-q_{min})}{(q_{max}-q_{min})}}} - \frac{R_0 c}{M}\right) & \text{otherwise} \end{cases}$$
$$:= f(\bar{q}_{e,k}, \rho)$$

where ρ represents the parameter vector in the system.

- n = Number of active TCP connections
- M = Maximum segment size or packet size
- R_0 = Round trip time
- K = Modeling constant between 1 and $\sqrt{8/3}$
- w = Exp. averaging weight
- c = Bottleneck bandwidth

- ✓Stability analysis of period doubling and dynamical stability criteria
- ✓Provides stability criteria for emerging period two orbit
- ✓Uses bifurcation diagrams to investigate the solutions as system parameters like number of connection (n), round trip time (R) and RED parameters like pmax, q_min, q_max and exponentially averaging weight (w) vary.
- ✓Existence of primary bifurcation in the form of period doubling
- ✓Secondary bifurcations like border collision
- ✓Sequence of bifurcations leading to Chaos
- ✓Sensitivity observed in practice can be reproduced from



Bifurcation diagram with respect to number of TCP connections (n).

Conclusion:

Model successfully captures the dynamical phenomena of TCP networks with RED AQM and provides useful information about setting control parameters. It also gives insight to formulate new control mechanisms.

Reference: Nonlinear Instabilities of TCP-RED by P. Ranjan, E. H. Abed and R. La, Accepted for INFOCOM 2002.