Generalized Washout Filters for Control of Nonlinear Systems

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Introduction and Motivation

- A washout filter is a high pass filter that washes out (rejects) steady state inputs, while passing transient inputs.
- Feedback through washout filters does not move the equilibrium points of the open-loop system.
- Washout filters facilitate automatic following of a targeted operating point) control energy drops to zero once stabilization is achieved and steady state is reached.
- Although washout filters have been successfully used in many control applications, there is no systematic way for choosing the constants of the washout filters and the control parameters.
- We introduce generalizations of washout filter-aided feedback that overcome the limitations of washout filters and at the same time maintain their benefits.

Washout Filters

In discrete-time, the transfer function of a typical washout filter takes the form (Abed, Wang & Chen 1994)

$$T(z) = \frac{Y(z)}{X(z)} = \frac{z-1}{z-(1-d)}$$

In the time domain, the dynamics of a washout filter can be written as

$$z(k+1) = x(k) + (1-d)z(k),$$

along with the output equation

$$y(k) = x(k) - dz(k).$$

d is the washout filter constant (d2(0,2) for a stable filter).
The control input is taken to be a function of y.

Linear Feedback through Washout Filters

Consider the nonlinear system

$$x(k+1) = f(x(k), u(k)) \qquad (1)$$

where $x 2 <^{n}$.
Suppose that x_o is an unstable operating condition for system (1).
In a small neighborhood of x_o, system (1) can be rewritten as

$$x(k+1) = Ax(k) + bu(k) + h(x(k), u(k))$$

where x now denotes x-x_o (is the state vector referred to x_o),
u is a scalar input, A is the Jacobian matrix of f evaluated at x_o,
b is the derivative of f with respect to u evaluated at x_o, and
h(.,.) represents higher order terms.
The dynamic equations of the washout filters can be written as

$$z_i(k+1) = (1-d_i)z_i(k) + \sum_{j=1}^{n} c_{ij}x_j(k) \qquad (2)$$

where z_i is the state of the i-th washout filter, $i=1,\cdots,m$, and $m< n$.

There is no systematic way for choosing the constants of the washout filters and the control gain.

Generalization of Discrete-Time Washout Filter-Aided Feedback

- We introduce a generalization of washout filter-aided feedback in which the individual washout filters are coupled through a constant coupling matrix.
- Feedback through generalized washout filters is shown to overcome the limitations of washout filters and at the same time maintain their benefits.

The generalized washout filter-aided feedback results in the closed-loop system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + h(x(k), u(k)) \\ z(k+1) &= Px(k) + (I-P)z(k) \\ u(k) &= G(x(k) - z(k)) \end{aligned}$$

where P is a nonsingular matrix, and G is a feedback gain matrix.
The linearization of the closed-loop system can be written as

$$\begin{pmatrix} x(k+1) \\ z(k+1) \end{pmatrix} = \begin{pmatrix} A+BG & -BG \\ P & I-P \end{pmatrix} \begin{pmatrix} x(k) \\ z(k) \end{pmatrix} =: A_c \begin{pmatrix} x(k) \\ z(k) \end{pmatrix}.$$

Proposition: Consider the closed-loop system with generalized washout filter-aided feedback. Suppose that the matrix I-A is nonsingular. Suppose also that the pair (A,B) is stabilizable. Then there exists a nonsingular P2 $<^{n£ n}$, and a G2 $<^{m£ n}$ such that $(x_o^T, x_o^T)^T$ is an asymptotically stable fixed point of the closed-loop system.

Example 1

Consider the two-dimensional example

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{pmatrix} 1.9 & 1 \\ 0.5 & 0 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} - \begin{pmatrix} x_1^3(k) \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(k)$$

The unforced system (u=0) displays chaotic motion.
The fixed point (0,0) is unstable: the eigenvalues of the linearization at the origin are λ_1=2.1343 and λ_2=-0.2343.
Since λ_1>1, the origin cannot be stabilized using stable washout filters nor by using delayed feedback control.

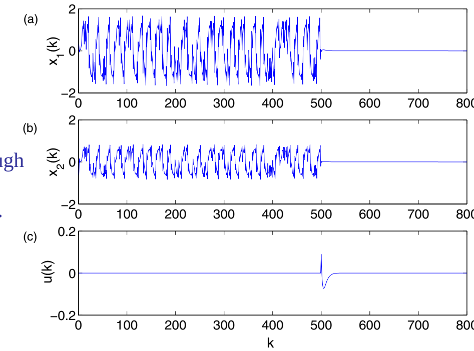
Feedback through generalized washout filters:

$$G = [-1.6343 \;\; -0.7657], \qquad P = 0.1(A-I)^{-1}(A+BG-I) = \begin{pmatrix} -0.01674 & -0.05469 \\ -0.05837 & 0.07265 \end{pmatrix}$$

$$A_c = \begin{pmatrix} A+BG & -BG \\ P & I-P \end{pmatrix} = \begin{pmatrix} 0.2657 & 0.2343 & 1.6343 & 0.7657 \\ 0.5000 & 0 & 0 & 0 \\ -0.0167 & -0.0547 & 1.0167 & 0.0547 \\ -0.0584 & 0.0727 & 0.0584 & 0.9273 \end{pmatrix}.$$

The eigenvalues of A_c are {-0.2343, 0.7277, 0.8164, 0.9}.

Feedback through generalized washout filters.



Time series (with initial condition (0.3,-0.6)) of (a) x_1 (b) x_2 and (c) control input u. The control is applied when the trajectory of the open-loop system enters the neighborhood {x=(x_1,x_2)2$<^2$: ||x||<0.15} of the origin.

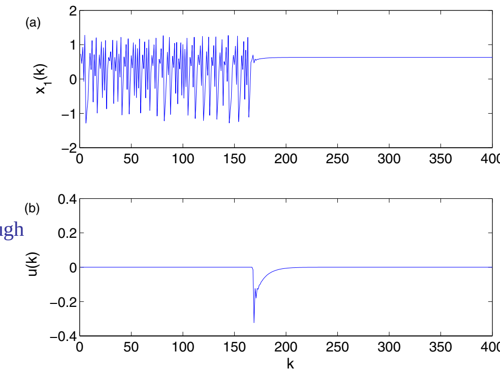
Example 2: Henon Map

Consider the Henon map

$$\begin{aligned} x_1(k+1) &= 1 - 1.4x_1^2(k) + 0.3x_2(k) + u(k) \\ x_2(k+1) &= x_1(k) \end{aligned}$$

The goal is to stabilize the unstable fixed point x_o=(0.6314, 0.6314).

Feedback through generalized washout filters



Time series (with initial condition (0.1; 0.0557)) of (a) x_1 and (b) control input u. The control is applied when the trajectory of the open-loop system enters the neighborhood {x=(x_1,x_2)2 $<^2$: ||x-x_o||<0.1} where x_o=(0.6314, 0.6314).