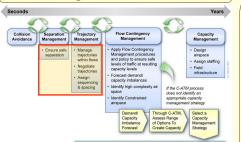


Advanced Stochastic Network Queuing Models of the Impact of 4D Aircraft Trajectory Precision



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Background-Motivation



- delivery of clearances and
- Time-based metering is used in some localities to improve predictability and throughput. Required navigation performance (RNP) operations are used initially to manage complexity and increase

4DTs that specify accurate current as future aircraft position Metering, controlled time of arrival CTA) exchange, and more flightspecific adjustments increase overa Safety, security, and environmental

considerations are integrated in TBO. Flight crew-initiated, dynamic trajectory adjustment is possible with ATM and airport operations center (AOC) collaboration.

Visualization

Interactive Tool to Visually Evaluate the Queue Size and Average Delay for Aircraft in Queue



Queue Size: 0 aircraft in queue:

St. Louis Queue Queue Size: 2 Average delay of aircraft in queue:

St. Louis Queue Queue Size: 5 Average delay of aircraft in queue:

St. Louis Queue Queue Size: 8 aircraft in queue:

Research Objectives

- · Develop Queuing Models that Predict Benefit of Increased Aircraft Trajectory Precision
 - > Reduced inter-arrival time
 - Reduced variation in inter-arrival time
 - Reduced service time.
 - > Reduced variation in service time
 - Increased number of servers
- Develop Modeling and Visualization

Environment to Allow

- > Validation of Queuing Model Results **Against Simulation**
- > Visualization of Benefit Mechanisms
- Validate Proposed Queuing Models
- Apply Validated Models to Next Generation
- Air Transportation System (NGATS) Concepts

Delay Savings

Use of Queuing Models to Estimate Delay Savings From 4D Trajectory Precision



Computed delays with the stochastic and deterministic models for each airport, under each capacity scenario, for various peak and non-peak days



Used different

for 7 airports: Atlanta (ATL)

Boston (BOS) Chicago (ORD)

Dallas (DFW)

New York (LGA)

San Francisco (SFO)

Miami (MIA)

capacity scenarios

Delays at SFO under capacity scenario 3 on a peak day

Stochastic Comparison of stochastic and deterministic models

The comparison of the stochastic and the deterministic models quantifies the resulting decrease of delays due to the Increase of 4DT Trajectory precision: Approximately 10% reduction from stochastic

Modeling the Levels of Aircraft Trajectory Uncertainty

- · Low Precision Case: Stochastic Queuing Models
 - Captures present-day system
 - > Arrivals are time-dependent Poisson process
 - Service times are time-dependent Erlang k process
 - > Assume n servers
 - \triangleright Kendall notation: (M(t)/Ek(t)/n)
 - Employ previously developed DELAYS & Approximate Network Delays (AND) models
- High Precision Case: Deterministic Queuing Models
 - - Arrival schedule (aggregate or disaggregate)
 - Capacity or deterministic minimum headways
 - > Construct cumulative arrival and departure curves to obtain
 - Delay and queue length by time of day
 - Average and total delay
- Intermediate Case: Diffusion Approximation
 - > Dynamics of joint probability density functions are analogous to dynamics of physical flows or other
 - > Continuous approximations using systems of coupled partial differential equations
 - > Because derivatives of probability density functions are modeled, they can be integrated to produce moment estimates
 - Exploit fast numerical solvers

Diffusion Approximation to a Single Airport

Model Development

Necessary assumptions made:

- Continuity; the gueue length measurement at any time needs not be integer
- ·Markovian; the changes of queue lengths are independent of the prior queue states •2nd order approximation; the transition
- density function a can be captured in its first and second moments

Governing Differential Equation

$$\frac{\partial f_i(x,t)}{\partial t} = \frac{1}{2} \frac{\partial^2}{\partial x^2} V_i(x,t) f_i(x,t) - \frac{\partial}{\partial x} M_i(x,t) f_i(x,t)$$

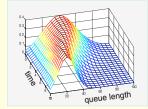
Kolmogorov forward equation

Initial and Boundary Conditions

·Queue starts empty at the beginning and ends up empty at the end of the day •Prevent negative queue lengths with a reflecting barrier:

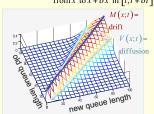
$$f(0;t)M(0;t) - \frac{1}{2} \frac{\partial}{\partial x} f(x;t)V(x;t)\Big|_{x=0} = 0, \quad t > 0$$

 $f_i(x;t)$ = density of length of queue i at time t



Queue Length Probability Density Function

 $g_i(\delta x, x; \delta t, t)$ = probability density of change from x to $x + \delta x$ in $[t, t + \delta t]$



State Transition Probability Function