

Dynamic Models for Control on the Internet: Smooth TCP, RED, Stability



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Research Objectives

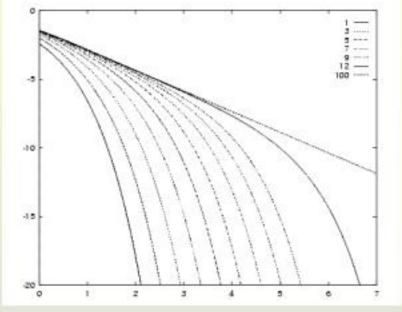
- Establish stochastic models for TCP traffic in the presence of AIMD (Addictive Increase Multiplicative Decrease) end-to-end control and gateway control (RED), emphasized in non-Poisson traffic.
- Explore the performance of RED in terms of TCP congestion window and congestion node queue distribution. Generalized TCP window-based adjustment scheme and optimal ECN marking strategy.
- Develop unified view of congestion node and end node optimization in the problem of Markov Decision Processes when treating non-Poisson nature of TCP traffic as MMPP (Markov Modulated Poisson Process).
- Study the stability and convergence of various flavors of AQM (Active Queueing Management) in the context of stochastic model at equilibrium point when system and users achieve fairness.

Markov Model for Traffic

Truncated-Power-Tail distribution for heavy tail.

$$R(x) = \exp(-xB)\varepsilon', \quad p = \frac{1-\theta}{1-\theta^T} \left[1, \quad \theta, \quad \cdots, \quad \theta^{T-1} \right]$$

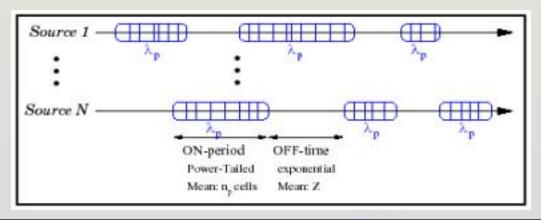
$$B = \mu \operatorname{diag}\left(1, \quad \gamma, \quad \cdots, \quad \gamma^{T-1}\right), \quad 0 < \theta < 1, \quad \gamma = \left(\frac{1}{\theta}\right)^{1/\alpha}$$



Truncated-power-tail distribution with $\alpha = 1.5$ and $\theta = 0.5$ with

 $T = \{1, 2, \dots, 9, 12, 100\}$

Self-similar: Aggregate of ON/OFF process.



Background Knowledge

- TCP, a window-based best-effort flow control protocol, creaqtes self-similar traffic; drastically different from Poisson traffic in most conventional network modeling formulations.
 - Definition of self-similar traffic.

$$X_k^{(m)} \doteq \frac{1}{m} \sum_{i=1}^m X_{km-m+i} \stackrel{d}{=} m^{H-1} X_k$$

$$\rho_X(k) \approx k^{2H-2} L_1(k)$$
, as $k \to \infty$

- Impact of self-similar traffic: Distribution of queue length at gateway decays more slowly for LDR input, which causes large buffer overflow and frequent retransmission of TCP traffic.
- Existing treatment of network-centric and user-centric optimization proves prevailing end-to-end and gateway controls of TCP traffic will stabilize the system and let the system converge to equilibrium point where the fairness (maxmin, proportional, etc) is achieved. It remains a question to see whether it is true in the more realistic stochastic model.

MDP Optimal Control

Queueing dynamics and end-to-end window dynamics.

$$q_{n+1} = (q_n + \sum_{m=1}^{M} r_{mn} - \mu)^+, \quad \mathbf{r}_{n+1} = A_n(a_n)\mathbf{r}_n + B_n$$

$$\mathbf{r}_{n} \triangleq \begin{bmatrix} r_{1n} & r_{2n} & \cdots & r_{mn} \end{bmatrix}^{T}, \quad A_{n}(a_{n}) \triangleq \operatorname{diag} \begin{pmatrix} c_{1n} & \cdots & c_{mn} \end{pmatrix}$$

$$c_{m} \triangleq (1 - 1_{0}(x_{jn}) - a_{jn}/2), \quad B_{n} \triangleq \begin{bmatrix} (1 - a_{1n})/d_{1n}^{2} & \cdots & (1 - a_{mn})/d_{mn}^{2} \end{bmatrix}$$

Cost function.

$$v_n = T_n - aQ_n - bD_n - cV_n$$

 T_n is the throughput at time n.

 Q_n is the queue length (delay) at time n.

 D_n is the drop rate at time n.

 V_n is the fairness index at time n.

$$J_{opt} = \sup \lim_{T \to \infty} \frac{1}{T} E\left(\sum_{t=0}^{T} v_n\right)$$

TCP Traffic with Queueing Loss

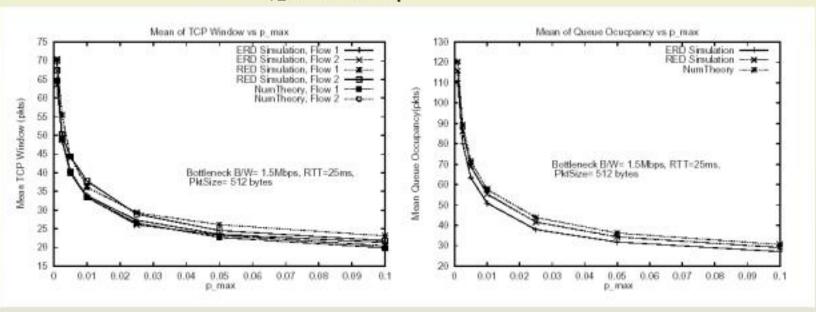
Congestion avoidance TCP model and window evolution with state dependent losses (in virtual time scaled by instance between random losses).

$$\begin{cases} P\{W_{n+1} = w + \frac{1}{w} | W_n = w\} = 1 - p(w) \\ P\{W_{n+1} = \frac{w}{2} | W_n = w\} = p(w) \end{cases}$$

$$\frac{dW}{dt} = \frac{p_{\text{max}}}{p\left(\frac{W}{\sqrt{p_{\text{max}}}}\right)W}, \quad W(\tau^+) = \frac{1}{2}W(\tau^-), \quad \tau \text{ is the loss time.}$$

Fixed-point equation, mean window sizes and mean queue occupancy.

$$W = \frac{1}{\frac{M_i}{C_i}}$$
, and $W = \sqrt{\frac{2}{p(Q)}}$
$$\sum_{i=1}^{N} \frac{c_i}{O + C \cdot RTT}$$



- Generalized TCP adaptation and ECN marking policy.
 - The marking probabilities in an ECN-capable router should be an exponential function of the buffer occupancy. In M/M/1 model, it is shown that.

$$p(Q) = 0$$
 if $Q < \min_{fh}$
= $1 - e^{-\zeta Q^2}$ if $\min_{fh} \le Q$

■ It is proved that if the TCP window update algorithm is modified to include an estimate of the round-trip time without change of sub-additive-increase-multiplicative-decrease, the stationary allocation of rates achieves the potential delay fairness objective.