# Analysis of Optical Beam Propagation Through Turbulence


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## The Random Interface Geometric Optics Model

Models the random medium as random curved interfaces with random refractive index discontinuities across them. Snell's law applied to evaluate the trajectories of the rays crossing the interfaces. Various ray behaviors are Monte-Carlo simulated.

The model contains two simulation routines that:

- Calculate the beam wander of a single pencil thin ray traveling through turbulence
- Calculate the phase wander between two parallel rays traveling through turbulence

## Turbulence Equations

For a plane wave propating in the z-direction, $E = E_0 e^{j(\omega t - kz)}$

For a Laser Beam, $E(r) = E_0 e^{-r^2/w^2}$

### Beam Wander

Beam centroid deviates as it propagates through turbulence.

Mean square beam wander for a Gaussian beam (by Ishimaru),

$$\langle \rho_l^2 \rangle = \frac{W_0^2}{2}\left[(\alpha_1 z)^2 + (1-\alpha_2 z)^2\right] + 2.2 C_n^2 l_0^{-1/3} z^3$$

where $l_0$ is the inner scale, $\alpha_2 = 1/R_0$, where $R_0$ is the radius of equivalent Gaussian wave, $W_0$ is the spot-size, and $\alpha_1 = \lambda/(\pi W_0^2)$

For a plane wave, it simplifies to,

$$\langle \rho_l^2 \rangle = 2.2 C_n^2 l_0^{-1/3} z^3$$

### Phase Wander

Mean square phase difference for a plane wave propagating through weak turbulence found using the phase screen method.

For two narrow collimated beams, $D_p(r)$ becomes,

$$D_p(r) = 0.32 C_n^2 k^2 L r^{5/3}$$

### Aperture Averaging

The intensity variance decreases with the area of the receiver.

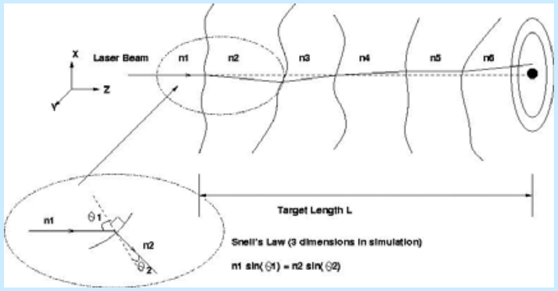
Aperture averaging (by Tatarski),

$$G(D) = \frac{16}{\pi D^2}\int_0^\infty \frac{b_I(\rho)}{b_I(0)} K(\rho)\rho d\rho,$$

where, $K(\rho) = \arccos(\rho/D) - (\rho/D)\left[1-(\rho^2/D^2)\right]^{1/2}$

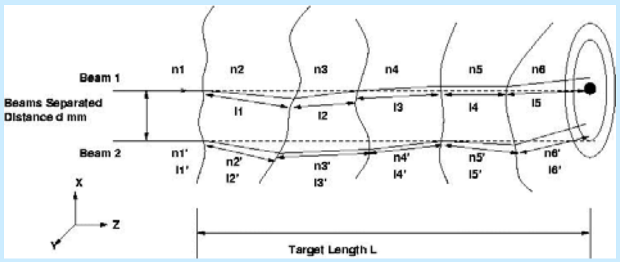
$G(D)$ is the intensity variance of the actual receiver relative to a point receiver $D << \sqrt{\lambda L}$.

## Beam Wander Simulation Model



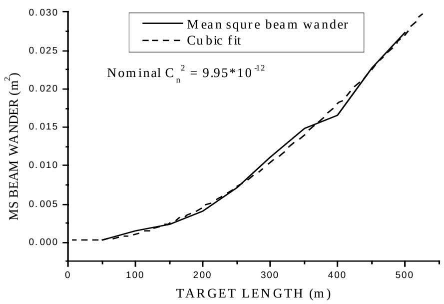

Calculates the 3-D trajectory for a single ray traveling at distance L through a simulated random medium. The mean square beam wander is averaged over each run.

## Phase Wander Simulation Model




The trajectories of two parallel rays propagating through the simulated random medium are computed simultaneously. The difference in the path length traveled $\Delta l$ will yield the phase difference $\Delta\phi$ between the rays $\Rightarrow \Delta\phi = (2\pi/\lambda)\Delta l$

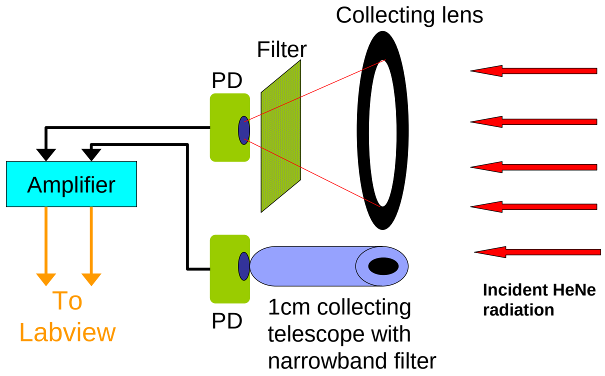
## Beam Wander Simulation Results

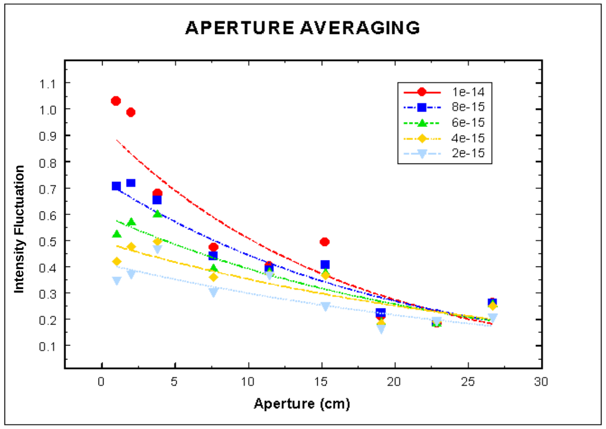



Parameters used: $L$ = 500m with step size = 50m, $\mu_l = 100m$, $c_l = 90m$, $\mu_n = 1.00001$, $c_n = 0.000001$, path length threshold = 2cm, N = 1000.

Results show excellent agreement with the cubic fit described in theory $\langle \rho_l^2 \rangle = 2.2 C_n^2 l_0^{-1/3} z^3$

## Single Pass Aperture Averaging




## Conclusions

- Model to evaluate beam wander, aperture averaging, and phase decorrelations for a Gaussian beam-wave input have been developed
- Model will be verified with measurements