# Source Coding and Parallel Routing


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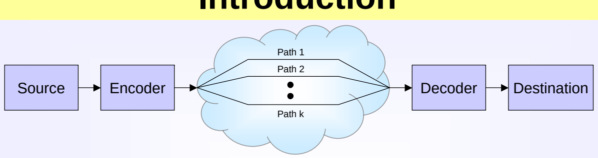
## Introduction




**Problem:** Choose the *routing parameters* together with the *coding parameters* to minimize average *end-to-end distortion*

**Motivation:** Coupling of different network layers

- Parallel routing (**layer 3**): uses diversity but introduces extra delays
- Source coding (**layer 6**): compression

**Trade-off:** Distortion vs. Delay

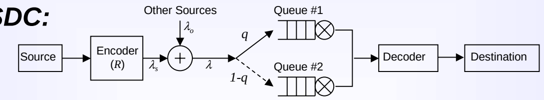
- want less distortion ➔ need longer packets ➔ more delay

## Model

**Source:** Gaussian, Delay-sensitive: packets that arrive later than $\Delta$ sec are useless

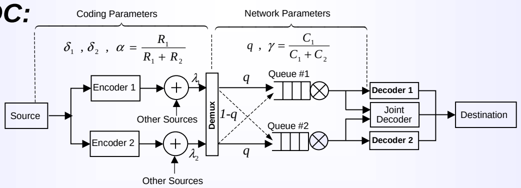
**Encoder:**

- ***SDC:***




- Each source symbol encoded into one packet (description)
- Expected packet length: $R$ bits
- greater $R$ → smaller distortion → longer delay

- ***DDC:***




- Each source symbol encoded into 2 packets
- expected description lengths: $R_1$, $R_2$ < $R$ bits
- Each DDC description carries less info than SDC packet
- If $R_1 + R_2 = R$, both descriptions can jointly carry as much info as an SDC packet
- DDC: more flexibility without adding extra traffic load

## End-to-End Distortion

**SDC:**

$$D_{SDC} = \begin{cases} 2^{-2R} & \text{if } T \le \Delta \\ 1 & \text{if } T > \Delta \end{cases}$$

$$\bar{D}_{SDC} = 2^{-2R}\Pr[T \le \Delta] + \Pr[T > \Delta]$$

- Objective

$$(R^*, q^*) = \arg\min_{(R,q)} \bar{D}_{SDC}$$

**DDC:**

$$D_{DDC} = \begin{cases} d_0 = 2^{-2(R_1+R_2)\frac{1}{1-(\sqrt{\Pi}-\sqrt{\Lambda})^2}} & \text{if } T^1 \le \Delta \ \& \ T^2 \le \Delta \\ d_1 = 2^{-2R_1(1-\delta_1)} & \text{if } T^1 \le \Delta \ \& \ T^2 > \Delta \\ d_2 = 2^{-2R_2(1-\delta_2)} & \text{if } T^1 > \Delta \ \& \ T^2 \le \Delta \\ 1 & \text{if } T^1 > \Delta \ \& \ T^2 > \Delta \end{cases}$$

$$0 \le \delta_1, \delta_2 \le 1 \quad , \quad \Pi = (1-d_1)(1-d_2) \quad \& \quad \Lambda = d_1 d_2 - 2^{-2(R_1+R_2)}$$

$$\bar{D}_{DDC} = d_0 P[T^1 \le \Delta, T^2 \le \Delta] + d_1 P[T^1 \le \Delta, T^2 > \Delta] + d_2 P[T^1 > \Delta, T^2 \le \Delta] + P[T^1 > \Delta, T^2 > \Delta]$$

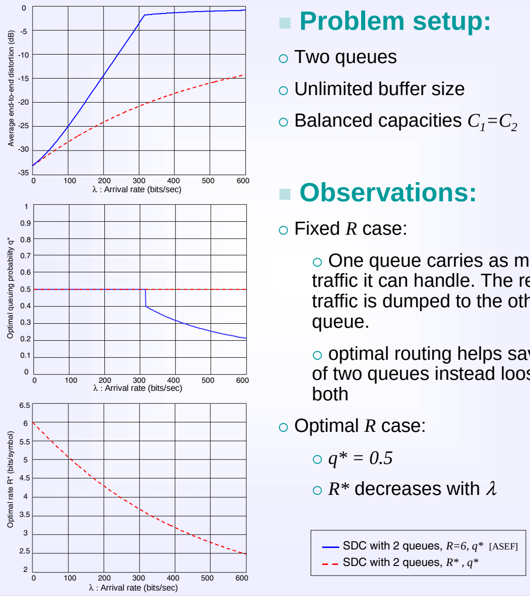
- $\delta_i$ : redundancy of encoder $i$

**small** $\delta$: good individual descriptions that jointly contribute little
**large** $\delta$: not individually good descriptions but jointly can achieve same distortion as SDC

- Objective

$$(R_1^*, R_2^*, q^*, \delta_1^*, \delta_2^*) = \arg\min_{(R_1,R_2,q,\delta_1,\delta_2)} \bar{D}_{DDC}$$

## SDC Results



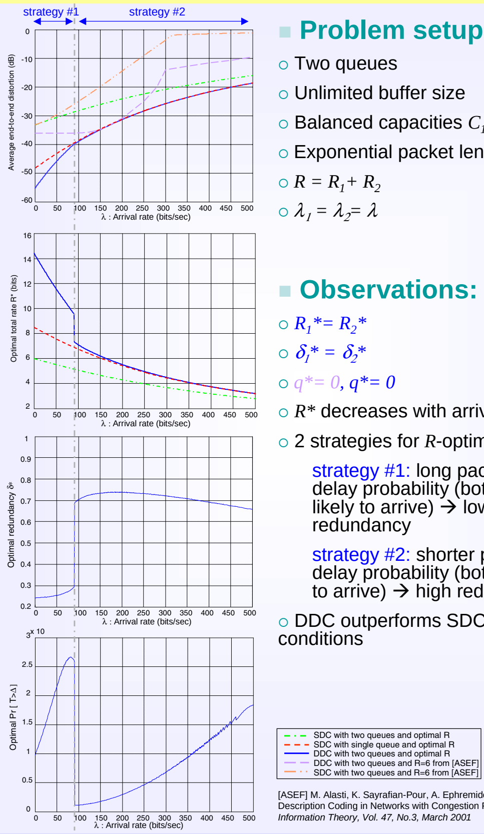

**Problem setup:**

- Two queues
- Unlimited buffer size
- Balanced capacities $C_1 = C_2$

**Observations:**

- Fixed $R$ case:
  - One queue carries as much traffic it can handle. The rest of traffic is dumped to the other queue.
  - optimal routing helps save one of two queues instead loosing both
- Optimal $R$ case:
  - $q^* = 0.5$
  - $R^*$ decreases with $\lambda$

## DDC Results




**Problem setup:**

- Two queues
- Unlimited buffer size
- Balanced capacities $C_1 = C_2 = 1000$ *bits/s*
- Exponential packet length
- $R = R_1 + R_2$
- $\lambda_1 = \lambda_2 = \lambda$

**Observations:**

- $R_1^* = R_2^*$
- $\delta_1^* = \delta_2^*$
- $q^* = 0$, $q^* = 0$
- $R^*$ decreases with arrival rate $\lambda$
- 2 strategies for $R$-optimal DDC

  strategy #1: long packets → high delay probability (both packets less likely to arrive) → low encoder redundancy

  strategy #2: shorter packets → low delay probability (both packets likely to arrive) → high redundancy
- DDC outperforms SDC under similar conditions


[ASEF] M. Alasti, K. Sayrafian-Pour, A. Ephremides, N. Farvardin, "Multiple Description Coding in Networks with Congestion Problem," *IEEE Transaction on Information Theory, Vol. 47, No.3, March 2001*


## Concluding Remarks

- Interesting Observation:
  - Known queuing theory result: *splitting the capacity between 2 links can only increase the delay and in our case increase the distortion.*
  - Known information theoretic result: *MDC does not reduce the distortion compared to SDC.*
  - Our result: *For a delay-sensitive source combining parallel routing and MDC in an optimal way can result in less distortion than an optimal SDC system with a single queue.*
- For a general memory-less source, explicit inner and outer bounds for the multiple description rate-distortion region have been found by Zamir. These bounds maintain the form of the distortion functions we discussed and so we expect our analysis to be applicable for any memory-less source.