Standard and Quasi-standard Stochastic Power Control Algorithms

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Background and Motivation

Power is an important and limited resource in wireless communications. Power control algorithms that minimize the transmission power while ensuring the quality of service have been widely studied in the literature. Among them, the standard power control algorithms are especially popular. However, the original standard power control algorithms require the perfect knowledge of the interference power, which can only be estimated from noisy observations in practical systems. When the perfect interference information is not available, the convergence of the standard power control algorithms becomes a question mark.

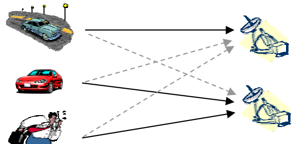
Abstract

We propose a general framework for the stochastic power control (PC) algorithms. Two types of stochastic PC algorithms are proposed: standard stochastic PC algorithms where the interference estimator is unbiased, and quasi-standard stochastic PC algorithms where the interference estimator is only asymptotically unbiased. It is shown that both algorithms converge to the optimal power vector. The stochastic versions of several well-known standard PC algorithms are proposed. They are shown to be either standard or quasi-standard. The algorithms are now ready for practical implementation.

Standard Power Control Algorithms

The SINR requirement can be expressed by

$$p \geq I(p)$$



Where $I(p)$ is a standard interference function that satisfies

1. Positivity. $I(p)>0$
2. Monotonicity. If $p_1 \geqslant p_2$, then $I(p_1) \geqslant I(p_2)$
3. Scalability. $\forall \beta>1$, $\beta I(p)>I(\beta p)$

One needs to estimate $I(p)$

Standard power control algorithm

$p(n+1)=(1-\alpha)p(n)+\alpha I(p(n))$ $\quad \alpha \leqslant 1 \quad p^*=I(p^*)$

Standard Stochastic Power Control Algorithms

Standard stochastic interference function

1. Mean condition. $E\left[\tilde{I}(p,v) \mid p\right]=I(p)$, $I(p)$ is standard.
2. Lipschitz condition. $\exists K_1>0, \forall p_1, p_2, \left\|I(p_1)-I(p_2)\right\|^2 \leq K_1\left\|p_1-p_2\right\|^2$
3. Growing condition. $\exists K_2>0, E\left[\left\|\tilde{I}(p,v)-I(p)\right\|^2 \middle| p\right] \leq K_2\left(1+\|p\|^2\right)$

constraint on the estimation noise

Standard stochastic power control algorithm

$$p(n+1)=(1-\alpha(n))p(n)+\alpha(n)\tilde{I}(n)$$

← Standard stochastic interference

Comparing the deterministic standard power control algorithm with the stochastic standard power control algorithm

$$p(n+1)=p(n)-\alpha(n)(p(n)-I(n))+\alpha(n)(\tilde{I}(n)-I(n)) \quad \text{Stochastic}$$

$$p(n+1)=p(n)-\alpha(n)(p(n)-I(n)) \quad \text{Deterministic}$$

Estimation noise: $\alpha(n)(\tilde{I}(n)-I(n))$

Quasi-standard Stochastic Power Control Algorithms

Quasi-standard stochastic interference function

1. Mean condition. $E\left[\tilde{I}(p,v) \mid p\right]=I(p)+g(p)$, $I(p)$ is standard, $g(p)$ is the bias.
2. Bias condition. $\exists K_3>0, \beta(n)\geq 0, \left\|g(p(n))\right\| \leq \beta(n)K_3\left(1+\|p(n)\|\right)$
3. Lipschitz condition. $\exists K_1>0, \forall p_1, p_2, \left\|I(p_1)-I(p_2)\right\|^2 \leq K_1\left\|p_1-p_2\right\|^2$
4. Growing condition. $\exists K_2>0, E\left[\left\|\tilde{I}(p,v)-I(p)-g(p)\right\|^2 \middle| p\right] \leq K_2\left(1+\|p\|^2\right)$

constraint on the estimation noise

Quasi-standard stochastic power control algorithm

$$p(n+1)=(1-\alpha(n))p(n)+\alpha(n)\tilde{I}(n)$$

← Quasi-standard stochastic interference

Probability one convergence:

Proposition 1:

If $\sum_{n=0}^{\infty}\alpha(n)=\infty, \ \sum_{n=0}^{\infty}\alpha(n)^2<\infty$, then the standard stochastic power control Algorithm converges to p^*, given by $p^*=I(p^*)$, with probability one.

Proposition 2:

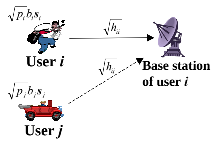
If $\sum_{n=0}^{\infty}\alpha(n)=\infty, \ \sum_{n=0}^{\infty}\alpha(n)^2<\infty, \ \sum_{n=0}^{\infty}\alpha(n)\beta(n)<\infty$, then quasi-standard stochastic power control algorithm converges to p^*, given by $p^*=I(p^*)$, with probability one.

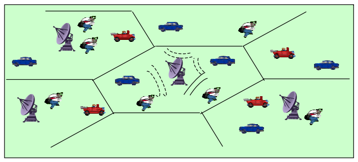
Convergence in probability:

Proposition 5:

If $\exists \alpha^* \geq 0, \ N>0$, such that $\alpha(n)\leq\alpha^*, \ \beta(n)\leq\sqrt{\alpha^*}, \ \forall n\geq N$ then $\forall \varepsilon>0$ $\exists K_8(\varepsilon)>0$, such that in the standard and quasi-standard stochastic power control algorithm $\limsup_{n\to\infty} P\left(\left\|p(n)-p^*\right\|\geq\varepsilon\right)\leq K_8\alpha^*$

Example: Joint Stochastic PC and blind MMSE





Deterministic version :

matched filter output : $z_i=\sum_{j=1}^{K}\sqrt{p_j}\sqrt{h_{ij}}b_js_j+v_i$

$$c_i^Tz_i=\sum_{j=1}^{K}\sqrt{p_j}\sqrt{h_{ij}}b_jc_i^Ts_j+c_i^Tv_i \qquad p_i\geq I_i\left(p,c_i^*\right), \quad c_i^*=\arg\min_{c_i} I_i(p,c_i)$$

Stochastic version :

Stochastic PC $\quad p(n+1)=(1-\alpha(n))p(n)+\alpha(n)\tilde{I}(n)$

Blind MMSE, start from arbitrary c_i and ensure

$c_i(n)=c_i(n)^*+w_i(n), \quad E\left[\left\|w_i(n)\right\|^2 \mid p(n)\right]\leq K_5\alpha(n)$

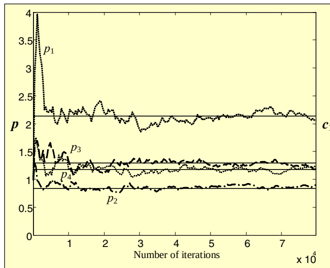
Extended version :

Stochastic PC

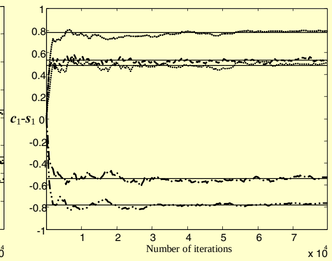
↓

blind MMSE, start from $c_i(n\text{-}1)$

Simulation Results



Example 1: Single base station, 4 users, 5-length random signature, random SIR target. h_{ij}=1, $\alpha(n)$=10/(10000+n)



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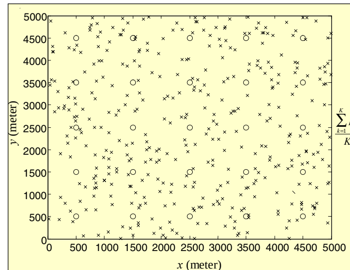
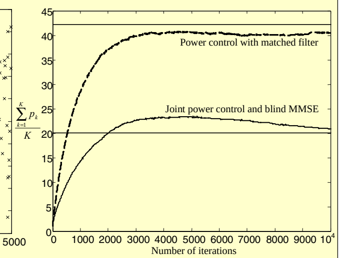


Example 2: 25 base stations (o), 400 users, 150-length random signature, equal SIR target. $h_{ij}=(100/d_{ij})^2$, $\alpha(n)$=10/(10000+n)



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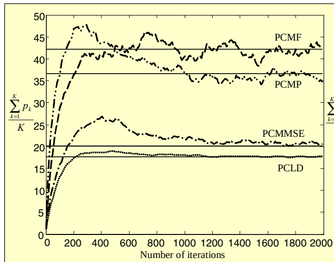
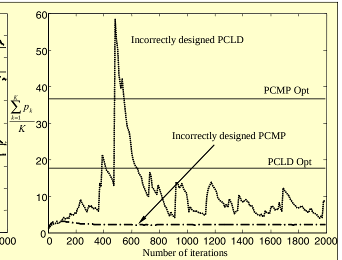


Example 2: Performance of other quasi-standard stochastic power control algorithms in example 2.



Example 2: Example of incorrect stochastic implementations.

Convergence of the deterministic version does not even guarantee a correct behavior of the stochastic version.

Conclusion

We propose a general framework for standard and quasi-standard stochastic power control algorithms. It is shown that, under certain mild conditions, both algorithms converge to the optimal solutions. Different types of convergence are shown under different assumptions on the iteration step size sequence. Several existing stochastic power control algorithms are studied and several new stochastic power control algorithms are proposed. We show that these algorithms are either standard or quasi-standard. In the examples of quasi-standard stochastic power control algorithm, we further extend the algorithms so that the stochastic power control and the parameter optimization can be carried out in parallel. Convergence of the modified algorithms are verified by computer simulations.