

Optimal Power Control for Wireless Queueing Networks

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Abstract

- ▶ Optimally controlling transmit power for jointly minimizing:
 - ▶ Energy expenditure
 - ▶ Buffer overflow
- ▶ Considered System
 - ▶ Wireless Link,
 - ▶ Finite-capacity buffer,
 - ▶ Random packet arrivals,
 - ▶ Choosing from two alternative power levels.
- ▶ Result
 - ▶ Optimal policy is of threshold type.

Motivation

- ▶ Energy Efficiency
 - ▶ A key concern in wireless networks,
 - ▶ Limited and non renewable power supplies.
- ▶ Power Control, Traditional Work:
 - ▶ Mitigating effects of interference,
 - ▶ Satisfying some QoS constraints,
 - ▶ SINR, BER constraints (Foschini (93), Yates (95))

Energy Efficiency

- ▶ Bambos (96): Maximizing battery lifetime
 - ▶ Minimizing average power subject to average success rate constraint.
 - ▶ Interference has a known distribution.
 - ▶ No error control technique is specified.
- ▶ Zorzi (97): Trade-off between instantaneous performance and energy efficiency:
 - ▶ ARQ error control,
 - ▶ Gilbert-Elliott Channel,
 - ▶ Decreasing power in favor of increased retransmissions can be a more energy-efficient.

Random Arrivals

- ▶ Previous work generally considered variance of interference and noise.
- ▶ Random arrival of data is not considered generally.
- ▶ In a limited buffer system with random arrivals
 - ▶ An arrival burst can occur when queue is long.
 - ▶ Buffer overflow can be avoided by increasing power.

System Model

- ▶ Wireless Link
 - ▶ Finite buffer capacity (length= K)
 - ▶ Bernoulli packet arrivals (rate= λ)
 - ▶ Equal length packets (length= 1 bit)
 - ▶ Two alternative power Levels ($P_1 < P_2$)
 - ▶ BPSK modulation
 - ▶ ARQ error control
 - ▶ Power decisions are made at every time slot.
 - ▶ Success probability = $1 - Q\left(\sqrt{\frac{PT}{N_0}}\right)$

State Transitions



MDP

- ▶ State, x_t : Buffer occupancy in the t^{th} slot ($x_t \in \mathcal{X} = \{0, 1, \dots, K\}$)
- ▶ Control: u_t : Power control decision ($u_t \in \{0, 1, 2\}$)
- ▶ u_t is allowed only when $x_t = 0$.
- ▶ Find the policy that minimizes the discounted cost:

$$V^{\alpha} = E_x \left[\sum_{t=0}^{\infty} \alpha^t g(x_t, u_t) \mid x_0 = x, u_0 \in \{0, 1, 2\} \right]$$
- ▶ Overflow cost:

$$O(x, u) = \begin{cases} x & x \leq K \\ \infty & x > K \end{cases}$$
- ▶ Energy Expenditure cost:

$$E(x, u) = \begin{cases} P_1 & u = 1 \\ P_2 & u = 2 \\ 0 & u = 0 \end{cases}$$
- ▶ Single Stage Cost:

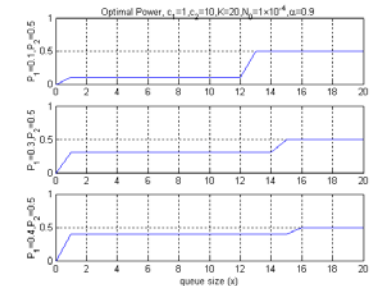
$$g(x, u) = E(x, u) + \alpha O(x, u) + \alpha V^{\alpha}(x)$$
- ▶ Dynamic Programming Equations:

$$V^{\alpha}(x) = \min_{u \in \mathcal{U}(x)} \{g(x, u)\}$$

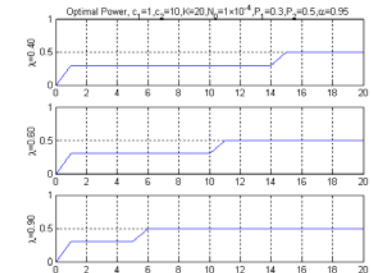
$$V^{\alpha}(x) = \min_{u \in \mathcal{U}(x)} \{g(x, u) + \alpha \sum_{x'} d_{x,x'}(u) V^{\alpha}(x')\}$$

Computational Results

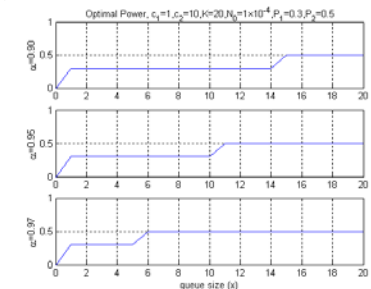
Changing Power Levels



Changing Arrival rate



Changing Discount Factor



Optimality of Threshold Policy

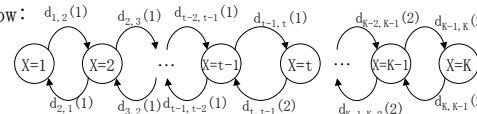
- ▶ For $0 < x < K$ optimal policy chooses P_2 if:

$$c_1(P_2 - P_1) \leq \alpha \left[Q \left(\frac{P_1}{N_0} \right) (V^n(x) - V^n(x+1)) - (Q \left(\frac{P_2}{N_0} \right) (V^n(x) - V^n(x+1))) \right]$$
- ▶ Here $h_x^n = V^n(x) - V^n(x+1)$
- ▶ Theorem: For $P_1 < P_2$ the below inequalities hold in every iteration n :
 - ▶ For x greater than or equal to some x^n : $h_x^n \leq h_{x+1}^n$ (2)
 - ▶ For $x < x^n$: $c_1(P_2 - P_1) \geq \alpha (\mu_1 - \mu_2) (\lambda h_{x+1}^n + (1 - \lambda) h_x^n)$ (3)
 - ▶ For all x : $0 \leq h_x^n \leq c_2$ (4)
- ▶ (2) and (3) are useful for proving that the right hand side of inequality (1) crosses left side at most at one point.

Practical Determination of Thresholds

- ▶ Let $0 \leq t \leq K$ be the threshold value.
- ▶ Queue state probabilities $\pi = [\pi_1, \pi_2, \dots, \pi_K]$ can be found by solving the Markov chain below:
- ▶ Find t^* minimizing the cost below:

$$t^* = \arg \min_t \left[\sum_{i=1}^t Q_0(0,0) + \sum_{i=1}^t Q_1(x,1) + \sum_{i=t+1}^K Q_2(x,2) \right]$$



Results

- ▶ The optimal power control policy of threshold type.
- ▶ Optimal threshold
 - ▶ Increases as P_1 approaches to P_2 ,
 - ▶ Decreases as the arrival rate increases,
 - ▶ Decreases as the discount factor α increases.