Optimal power and rate control for proportional fairness in time-varying wireless networks

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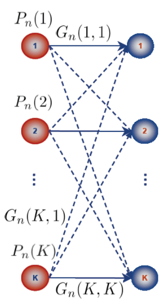
Motivation

- Multiple access problem in wireless networks decides
  - Who transmits when
  - At which transmission power and rate
    - Success of a transmission depends on the power and transmission rates
- Pure scheduling is not always optimal
  - Concurrent operation may be beneficial
- Proportional Fairness
  - Good compromise between efficiency and fairness

Model

- K transmitter/receiver pairs
- Infinite data supply at transmitters
- Slotted time
- Stationary channel process $\{\mathbf{G}_n\}_{n=0}^{\infty}$
  - Channel effect due to: path loss, shadowing, fading

$$\mathbf{G}_n = \{\mathbf{G}_n(i,j), \quad i,j\}$$



- Power of transmitter k at slot n: $P_n(k)$
- Vector of powers belongs in a compact set: $\mathbf{P}_n \in \mathcal{P}$
- Noise power: N
- Transmission success criterion at rate r(SINR):

$$\frac{P_n(k)G_n(k,k)}{N+\sum_{j=1,j\neq k}^{K} P_n(j)G_n(j,k)} \geq \theta_{n,k}(r)$$

- $R^{\pi}_{n,k}$ is the rate allocated to k by policy π at time slot n
- $\mathcal{R}_n$ is the set of all feasible rate allocations at slot n
- Effective rate of transmitter k under policy π at slot $n+1$

$$\alpha^{\pi}_{n+1,k} = \frac{1}{n+1}\sum_{\nu=1}^{n+1} R^{\pi}_{\nu,k}$$

Objective: Proportional Fairness

- A rate vector $\mathbf{r}^{\star}$ is proportionally fair if

$$\sum_{k=1}^{K} \frac{\mathbf{r}_k - \mathbf{r}^{\star}_k}{\mathbf{r}^{\star}_k} \leq 0$$

- Equivalent to maximizing the logarithmic utility of the users
- The objective is to maximize

$$U(\boldsymbol{\alpha}^{\pi}_n) = \sum_{k=1}^{K} \log(\alpha^{\pi}_{n,k})$$

with $\boldsymbol{\alpha}^{\pi}_n$ being the vector of effective rates

Optimal proportionally fair rate and power control

- The optimal policy $\pi^{\star}$ allocates the rates according to

$$\mathbf{R}^{\pi^{\star}}_{n+1}(\boldsymbol{\alpha}^{\pi^{\star}}_n) = \arg\max_{\mathbf{r}\in\mathcal{R}_{n+1}} \sum_{k=1}^{K} \frac{r_k}{\alpha^{\pi^{\star}}_{n,k} + d_k}$$

where $d_k > 0$ is a small constant

- The optimal policy $\pi^{\star}$ allocates the powers according to

$$\mathbf{P}^{\pi^{\star}}_{n+1}(\boldsymbol{\alpha}^{\pi^{\star}}_n) = \left\{ \min_{\mathbf{P}\in\mathcal{P}} \mathbf{P} : \quad \forall\ k = 1,\ldots,K \quad \frac{P_{n+1}(k)G_{n+1}(k,k)}{N+\sum_{j=1,j\neq k}^{K} P_{n+1}(j)G_{n+1}(j,k)} \geq \theta_{n+1,k}(\mathbf{R}^{\pi^{\star}}_{n+1}(\boldsymbol{\alpha}^{\pi^{\star}}_n)) \right\}$$

Simulation Results (K=3)

- Path loss matrix

$$\mathbf{S} = \begin{bmatrix} 0.9 & \beta\,0.9 & \beta\,0.9 \\ \beta\,0.9 & 0.9 & \beta\,0.9 \\ \beta\,0.9 & \beta\,0.9 & 0.9 \end{bmatrix}, \quad \beta \in [0,1]$$

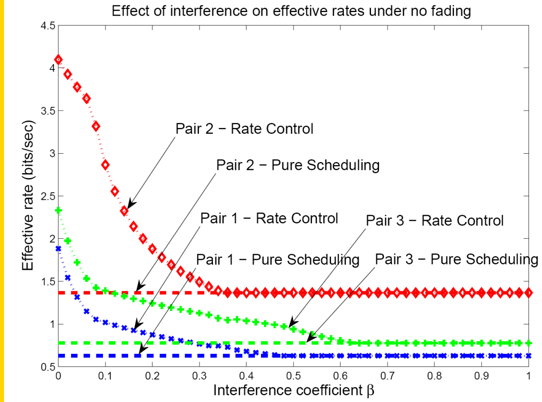
- Rayleigh Fading: Signal power ~ exponential with mean $\bar{P^r}(i,j) = P(i)S(i,j)$ and $G_n(i,j) = P^r_n(i,j)/P(i)$
- Rate/SINR formula: $r(\mathrm{SINR}) = \log_2(1+\mathrm{SINR})$ (bits/sec)
- Powers:

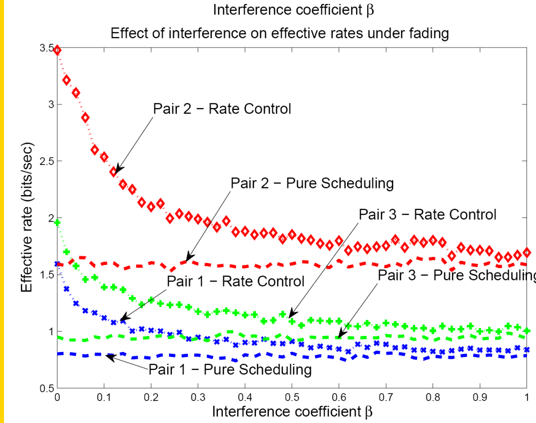
$P(1) = 1.0*10^{-5}$ Watts $\quad P(3) = 1.5*10^{-5}$ Watts

$P(2) = 6.0*10^{-4}$ Watts $\quad N = 3.34*10^{-6}$ Watts

Simulation Results

- Rate control for 3 transmitters with constant powers





Concluding Remarks

- Obtained a proportionally fair rate and power control policy for time-varying, single-hop, wireless networks
- Employed the theory of stochastic approximation to show optimality
- Possible extensions
  - Investigate the stabilizing properties under queuing
  - Generalize results for multi-hop networks
  - Obtain decentralized algorithms