Biologically-Inspired Optimal Control

C. Shao and D. Hristu-Varsakelis,
Dept. of Mechanical Engineering and Institute for Systems Research

Motivation from Biological Systems

Many biological systems exhibit highly self-organized patterns in spite of the limited intelligence of individuals. As it is shown in figure 1, ants could form a tail along the shortest path between two points while foraging.

Figure 1: Ants trail formation

Motivation:

1. Insects can execute relatively "complicated'' tasks compared to their individual abilities.
2. Most insects possess limited capability for sensing, communicating and computing.
3. Relevance for teams of inexpensive robots.

Problems Addressed

Take a number of "copies" of a dynamical system, i.e. for k= 0, 1, 2 . . .

$$\dot{x}_k = f(x_k, u_k) \qquad x_k \in R^n, u_k \in \Omega \subset R^m$$

Find a trajectory $x(t)$ that minimizes the cost function

$$J(\dot{x}, x, t_0, T) = \int_{t_0}^{t_0+T} g(x(t), \dot{x}(t), t)dt$$

with $x(t_0) = x_0, x(t_0 + T) = x_f$. T is fixed.

Proposed Approach:
Develop appropriate rules which mimic the behavior of natural insects.

Two Local Pursuit Algorithms

1. Sampled Local Pursuit: Choose $0<\delta<\Delta \ll T$. Then let the k^{th} agent evolve according to:

1) For $k=0,1,2\ldots$, let the $t_k = k\Delta$ be the starting time of the k^{th} agent.
2) When $t = t_k + i\delta, i = 0,1,\ldots$, calculate $u_k(\tau,t) = u^*(\tau,t)$ where $u^*(\tau,t)$ is the optimal control to steer $x(\tau)$ from $x_k(t)$ to $x_{k-1}(t)$ and
$$\tau \in \begin{cases} [t, t+\Delta] & if \quad \Delta + i\delta < T \\ [t, t_k + T] & otherwise \end{cases}$$
3) Execute $u_k(\tau,t)$ for the k^{th} agent during $\tau \in [t_k + i\delta, t_k + (i+1)\delta]$.

Repeat step 2 and step 3 until the k^{th} agent reaches x_f .

2. Continuous Local Pursuit: Let $\delta \to 0$, then follow the same rules of Algorithm 1.

Main Results

Theorem: The trajectories of a group of agents evolving under Local Pursuit (Algorithm1&2) converge to a local minimum.

Benefits

1. Complicated problems could be solved by a group of cost-effective agents.
2. Agents make decisions using local information only.
3. Agents only need to calculate the local optimum within a small region, requiring small amounts of computing power and memory.

Selected Simulations

1. **Minimum-Time Control:**
Local Pursuit can be extended to free final time problems. Figure 2 illustrates the solution of a classical minimum time problem ($\ddot{x} = u$) by local pursuit.

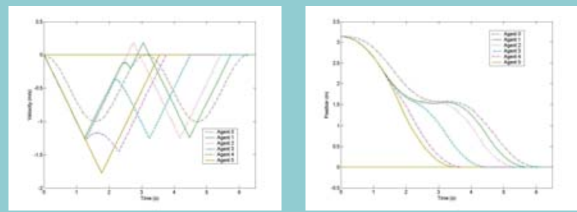

Figure 2: Simulation of Minimum-Time Control, Δ =0.3 π

2. **Finding geodesics by local pursuit:**
The terrain consists of two cones and a plane. The convergence is really rapid with Δ =0.2T.

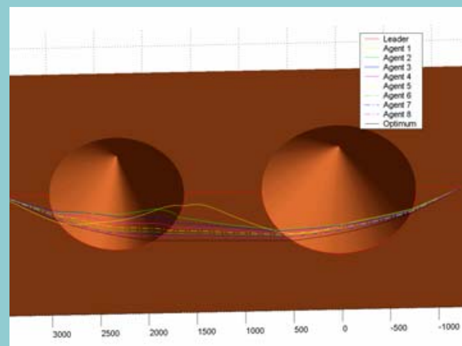

Figure 3: Geodesic discovery on an uneven terrain

Future Work

1. Analytical methods for determining whether the limit trajectory will converge to the global optimum.
2. Investigating local pursuit under noisy observations.
3. Applications of local pursuit in numerical computation of optimal controls.