Sparse Estimation of Self-Exciting Point Processes with Application to LGN Neural Modeling

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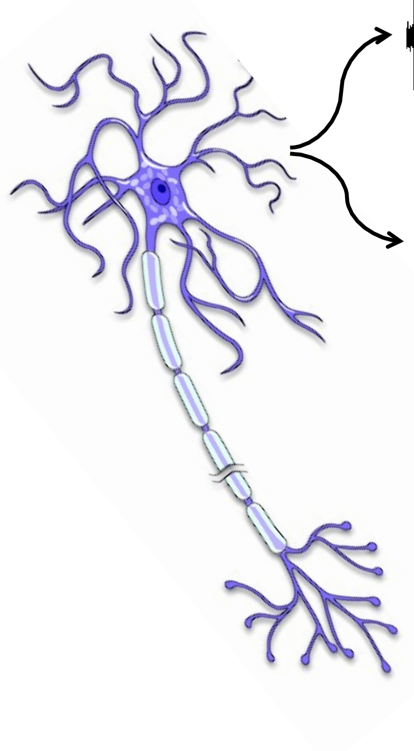
Introduction



Sparse Spontaneous Activity

Dependent Data

- **Goal:**
  - ∗ Estimation of the sparse dependence
- **Requirements:**
  - ∗ Parametric model
  - ∗ Principled inference framework
  - ∗ Consistent with neurophysiology
- **Application:**
  - ∗ Real-time processing of biophysical data, neural prosthetics etc.

Formulation

- **Self-Exciting Point Processes**
  - Future evolution dependent on History
- **Generalized Linear Model (GLM):**

$$x_i \sim \text{Poisson}(\lambda_{i|H_i}) \approx \text{Bernoulli}(\lambda_{i|H_i})$$
$$\log(\lambda_{i|H_i}) = \mu + \underline{\theta}'\underline{x} = \mu + \sum_{j=1}^{p} \theta_j x_{i-j} \ll 1$$

- **Conditional Intensity Function:**

$$\lambda_{i|H_i} = \mathbb{P}(x_i = 1|H_i) \triangleq \lim_{\Delta\to 0} \frac{\mathbb{P}(X(i+\Delta)-X(i)=1|H_i)}{\Delta}$$

- **Negative log-likelihood:**

$$\mathcal{L}(\underline{\theta}, n) \triangleq -\log \mathbb{P}(\{x_i\}_{i=1}^n) \approx -\frac{1}{n}\sum_{i=1}^{n} x_i \log \lambda_{i|H_i} - \lambda_{i|H_i}$$

- **ML Estimation:**

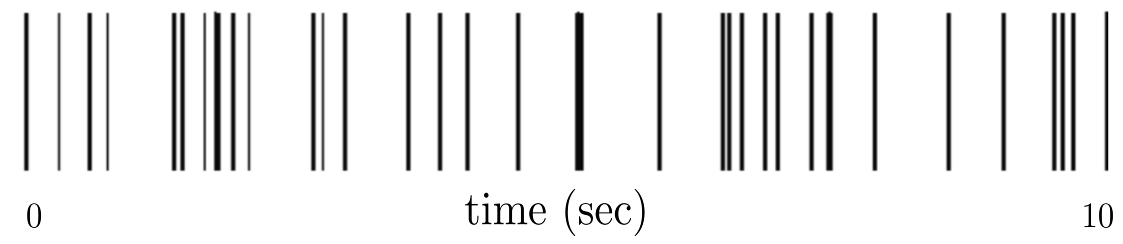
$$\hat{\underline{\theta}}_{\text{ML}} = \arg\min_{\theta\in\mathbb{R}^p} \mathcal{L}(\underline{\theta}, n)$$

- **Sparse Estimation:**

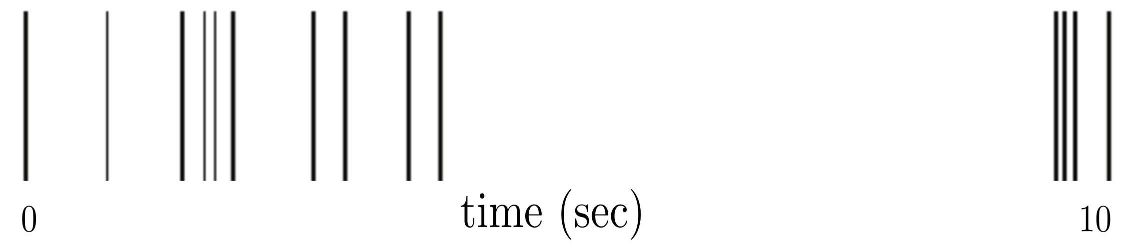
$$\boxed{\hat{\underline{\theta}}_{\text{sp}} = \arg\min_{\theta\in\mathbb{R}^p} \mathcal{L}(\underline{\theta}, n) + \lambda_n \|\underline{\theta}\|_1}$$

Methodology

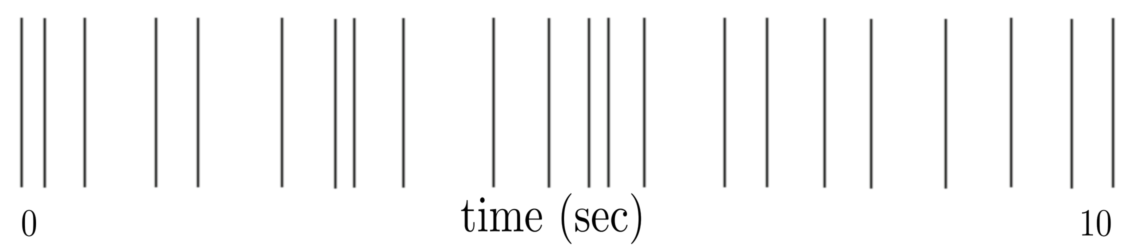
- Binary Data (Homogenous)



- Thinning

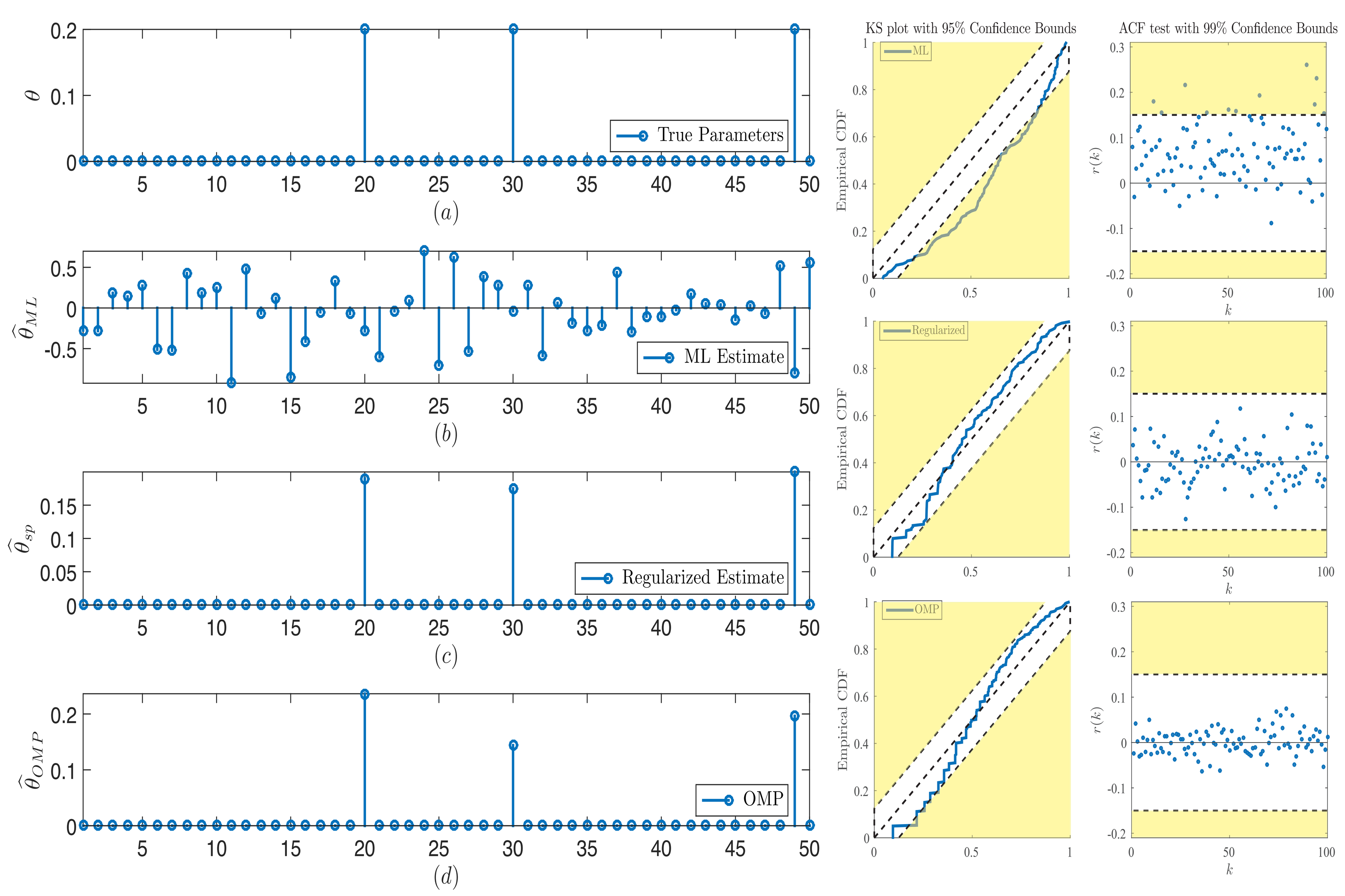


- Model selection via AIC and estimation
- Time-Rescaling



- Test Goodness-of-fit: **KS** plots, **QQ** plots and **ACF** test
- **Given:** a point process and an estimated CIF
- **Goal:** Find a statistical measure of goodness of estimate
- Rescaled process should look like a homogenous Poisson process

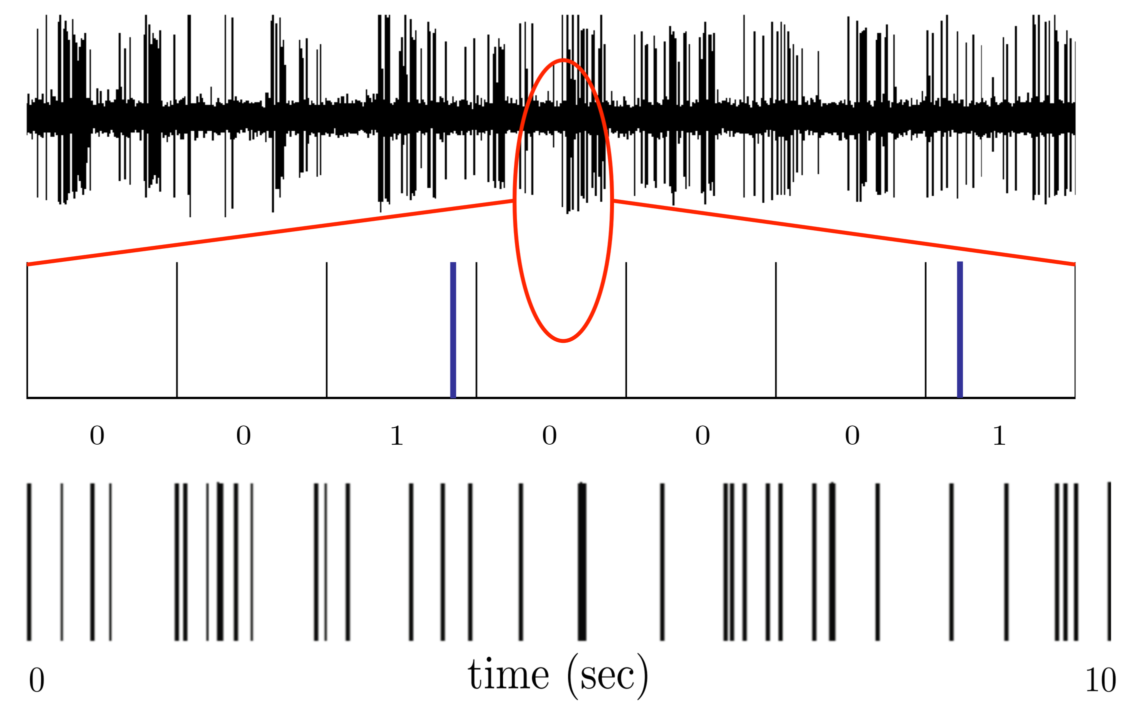
Application to Synthetic Data

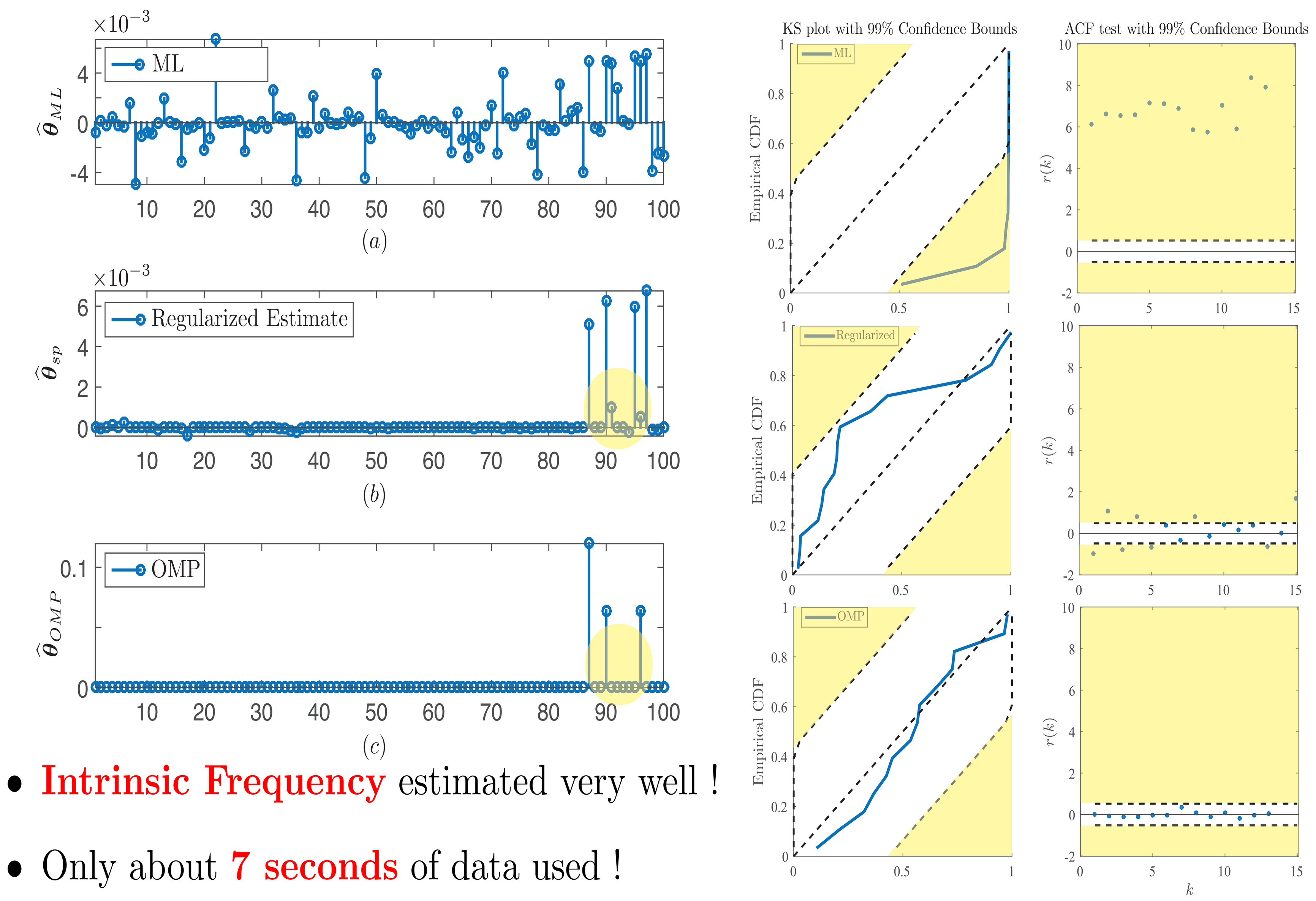


- Sparse estimtion significantly outperforms ML estimation
- Up to 2 orders of magnitude reduction in used data !
- **Orthogonal Matching Pursuit** performs good sparse estimation
- Need $O(s^2 \log p)$ samples for ℓ_1 and $O(s^2 \log^2 s \log p)$ for OMP

Application to LGN Neurons

- Spiking responses of 72 neurons in rat **Lateral Geniculate Nucleus**
- **Intrinsic frequency** $\sim 11 - 11.5$ Hz reported using **Two-Photon Microscopy** (Borowska, 2011)
- Raw Data
- Discretization
- Binary Representation





- **Intrinsic Frequency** estimated very well !
- Only about **7 seconds** of data used !

Conclusions and Future Work

- **GLM**'s are good candidates to capture the sparsity in neural data
- Sparse estimation overcomes **overfitting** and **Large sample size** compared to ML estimation
- Error can be suitably bounded using sparse estimation
- Extension to multivariate case
- Application in seismology, criminology, gene regulatory networks etc

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