Generalized Synchronization Trees

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THIRTY YEARS OF SYSTEMS RESEARCH EXCELLENCE

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Past: Algebraic Reasoning about Computing Processes

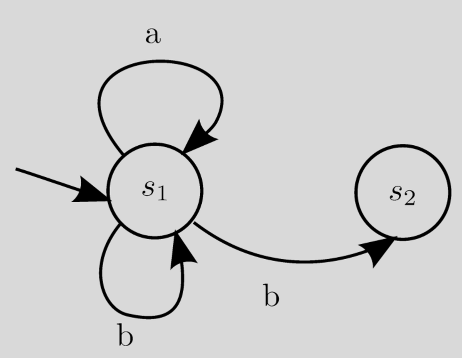
Milner [2] devised synchronization trees for labeled transition systems (Process Algebra):

Definition:

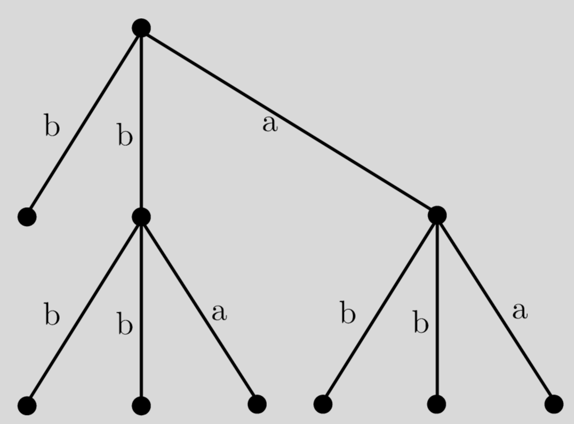
A **Synchronization Tree** (ST) over a set of labels L is a tuple $(V, E, \mathcal{L})$ where:

- (V, E) is an undirected, connected, acyclic graph (V, E) with a specially identified root node r *and*
- $\mathcal{L}$ is a function $\mathcal{L}: E \rightarrow L \cup \{\varepsilon\}$.

- Each path in the tree is an execution of the transition system.
- Nondeterminism: multiple children with the same label.
- In a synchronization tree, every execution leads to another synchronization tree (self-similarity).
- Composition: algebraic operations on synchronization trees.



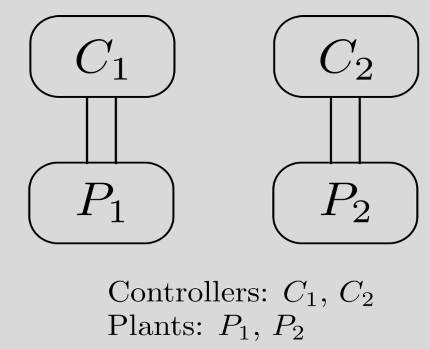
Labeled Transition System

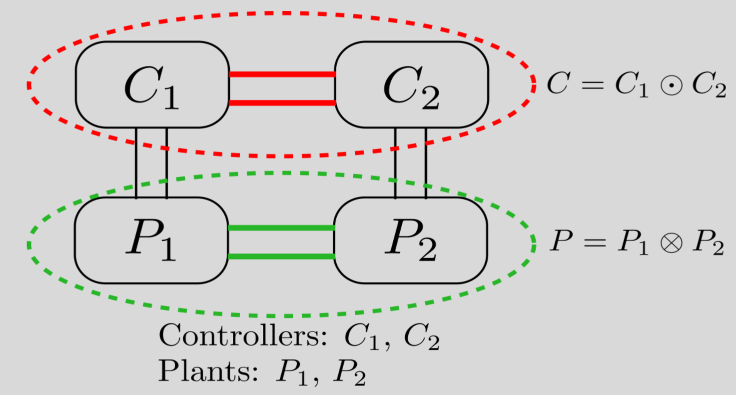


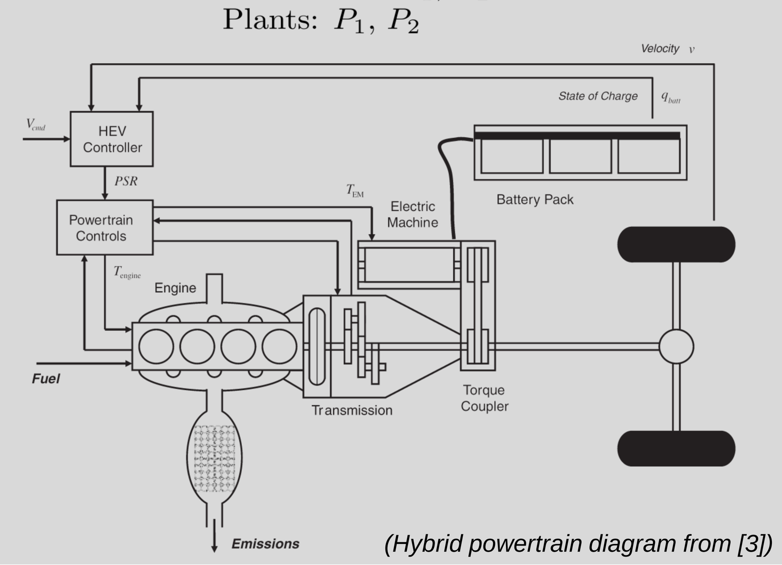
Synchronization Tree

Future: Compositional Design of Control Systems

Reason about complicated systems based on the models/behaviors of components:







(Hybrid powertrain diagram from [3])

Whole-system analysis

Current Research: Generalized Synchronization Trees

Extending Process Algebra

Goal: extend Process Algebra to model cyber-physical systems (CPSs).

Common restrictions in CPS modelling frameworks:

- transitions (choices) are **discretized** *or*
- composition is **synchronous**.

Generalizing the process algebraic approach promises a rich CPS-modelling framework in which:

- choice can be **continuous** *and*
- rich, **asynchronous composition** operations are natural.

This research is a new point of contact between the disciplines of computer science and control theory.

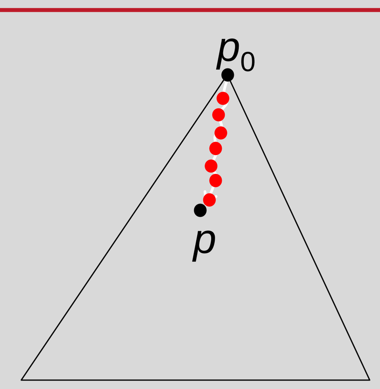
Generalizing Trees

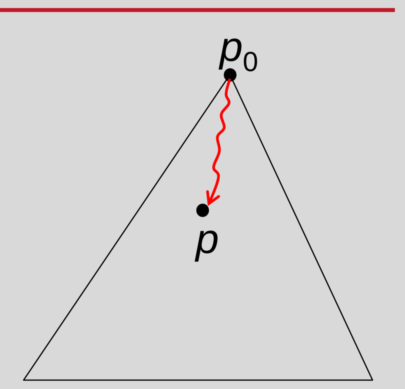
Definition:

A **tree** is a partially ordered set $(P, \leq)$ with the following properties:

1) There is a $p_0 \in P$ such that $p_0 \leq p$ for all $p \in P$. p_0 is the *root* of the tree;
2) For each $p \in P$, the set $\{p' \in P \mid p' \leq p\}$ is linearly ordered by $\leq$.

The nodes in a ST have a natural tree partial order: each node can be associated with a sequence of transitions, and two nodes are ordered if one's sequence of transitions is a prefix of the other's.





Generalized Synchronization Trees (GSTs)

Definition:

A **Generalized Synchronization Tree** [1] over a set of labels L is a tree $(P, \leq)$ along with a labeling function $\mathcal{L}: P \backslash \{p_0\} \rightarrow L$.

- In a synchronization tree, the nodes form a discrete GST tree with the canonical partial order.

Simulation for GSTs

In the following, let $G_P = (P, p_0, \leq_P, \mathcal{L}_P)$ and $G_Q = (Q, q_0, \leq_Q, \mathcal{L}_Q)$ be two GSTs.

Definition:

A **trajectory** (of G_P) starting from $p \in P$ and ending at $p' \in P$ is the set $(p, p'] \stackrel{\text{def}}{=} \{r \in P | p \leq r \leq p'\}$.

Definition:

A trajectory $(p, p']$ of G_P is **order equivalent** to a trajectory $(q, q']$ of G_Q if there exists an order preserving bijection $\lambda: (p, p'] \rightarrow (q, q']$ such that $\mathcal{L}_Q(\lambda(r)) = \mathcal{L}_P(r)$ for all $r \in (p, p']$. λ is called an **order equivalence** between $(p, p']$ and $(q, q']$.

Definition:

G_P **weakly simulates** G_Q if there is a relation $R \subseteq P \times Q$ such that $(p_0, q_0) \in R$ and

- For any $(p, q) \in R$ and $q' \geq q$, there exists a $p' \geq p$ such that $(p', q') \in R$, and there is an order equivalence between $(p, p']$ and $(q, q']$.

Simulation for GSTs (continued)

A new, semantically different kind of simulation for GSTs:

Definition:

G_P **strongly simulates** G_Q if there is a relation $R \subseteq P \times Q$ such that $(p_0, q_0) \in R$ and

- For any $(p, q) \in R$ and $q' \geq q$, there exists a $p' \geq p$ such that $(p', q') \in R$, and there is an order equivalence λ between $(p, p']$ and $(q, q']$ such that $(r, \lambda(r)) \in R$ for each $r \in (p, p']$.

Proposition:

If G_P and G_Q are STs, then G_P strongly simulates G_Q if and only if it weakly simulates G_Q.

Theorem:

There exist GSTs G_P and G_Q such that G_P weakly simulates G_Q but G_P doesn't strongly simulate G_Q.

References and Funding Information

NSF Grant: CNS-1446665. CPS Breakthrough Grant: "Compositional Modeling of Cyber-Physical Systems."

NSF Grant: CCF-0926194. "RECOVERY: Collaborative Research: Next-Generation Model Checking and Abstract Interpretation with a Focus on Embedded Control and Systems Biology." (CMACS)

[1] J. Ferlez, R. Cleaveland, and S. Marcus. *Generalized synchronization trees*. In FOSSACS 2014, volume 8412 of Lecture Notes in Computer Science, pages 304–319, Grenoble, France, 2014. Springer-Verlag.

[2] Robin Milner. *A Calculus of Communicating Systems*. Number 92 in Lecture Notes in Computer Science. Springer-Verlag, 1980.

[3] Edward Dean Tate Jr, Jessy W. Grizzle, and Huei Peng. *Shortest path stochastic control for hybrid electric vehicles*. Int. J. Robust Nonlinear Control, 18(14):1409-1429, December 2007.

NSF Grants CNS-1446665 and CCF-0926194