Random Graphs for Engineering

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Random graphs are used to model information flows, (inter-)dependencies, communication networks, biological networks, social relationships, etc., e.g.,

- Social networks
- Wireless networks
- Interdependent security

A simple generative random graph model often captures key features of interest, e.g.,

- Wireless networks – random geometric graphs
- Social networks – small world graphs
- Random key predistribution – random key graphs

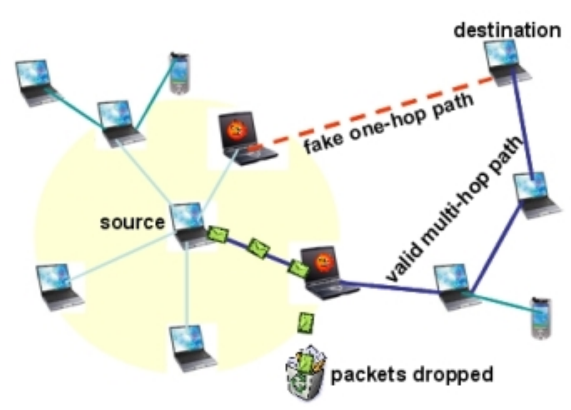
Many such simple generative random graph models have been studied by different research communities

- Statistical behavior or properties reasonably well understood in many (but not all) cases.
- Yet, classical or simple random graph models are often *insufficient* for formulating and studying many of the issues arising in important engineering systems and new classes of models are needed

Connectivity in wireless networks

- Traditional random geometric graphs assume independent and identically distributed locations
- In many wireless networks, node locations *correlated* or *heterogeneous* (e.g., group mobility)
- ***Strong thresholds*** for communication range were identified under **correlated** node placement or **heterogeneous** mobility

source: grin.com



source: http://www.ece.ncsu.edu/netwis/

Wireless networks with security constraints

- Each node is randomly assigned a subset of existing keys
- Two nodes "trust" each other only if they share a common key, in which case a secure communication link can be established between them when a physical link is available.
- Connectivity between nodes can be modeled as the *intersection* of a random geometric graph (communication graph) and a random key graph ("security" graph)
- **Scaling laws** for connectivity and absence of isolated nodes
- Rich class of models to study the impact of a given key distribution scheme and the communication model

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Future Directions:

1. Dynamics of graphs vs. dynamics on graphs

- In many cases of interest, the underlying graph/network evolves over time, shaping dynamical processes on networks, e.g., information/infection propagation in social network and computer networks

2. Composition theory of random graphs

- In some cases, relevant graph models can be constructed in *a component-like manner* by means of ***simple*** operations involving one or more component random graphs, e.g., Intersection, union, thinning, etc.
- A calculus over networks – How do the properties of individual components shape the properties of the composite graph?

3. Local structures of networks and their effects on strategic agents in networks

- Choices of ***strategic*** agents depend on local structures of network, e.g., interdependent security

4. Sampling large networks

- What does the sample reveal about the properties of the network?

Strategic network formation

- Nodes often choose their neighbors strategically, e.g., trust relation, distributed coordination, etc.
- More natural to assign attributes to *edges* (than to nodes) in order to capture the ***importance/value*** of the implied relationships
- New class of random graphs – Each node asks the other end nodes of k highest-value edges to which the node is incident to be *potential* neighbors
  - **Mutual consent** required for an edge to be present
- (i) **Scaling laws of k for network connectivity** (as a function of the number of nodes in the network) and (ii) **Network diameter**
- Intriguing connection to classical Erdos-Renyi graphs

source: California PATH project

source: National Geographic