Experimental Study of Curvature-based Control Laws for Obstacle Avoidance

Fumin Zhang, Alan O'Connor, Derek Luebke, and P.S. Krishnaprasad

Steering Control Law

Our goal is for an autonomous robot to steer itself to a goal in an environment inhabited by obstacles. The goal can always be reached if the robot follows the boundary curve of those obstacles until a certain condition is met.

In order to achieve curve following, a steering control law was designed which maintains constant distance from the obstacle boundary curve.

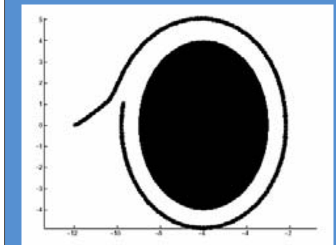
The robot moves with constant forward velocity v to simulate a flying vehicle, so there is only one control, the angular velocity u.

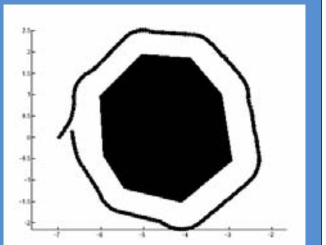
$$\begin{aligned} \dot{x} &= v\cos(\theta) \\ \dot{y} &= v\sin(\theta) \\ \dot{\theta} &= u\,, \end{aligned}$$

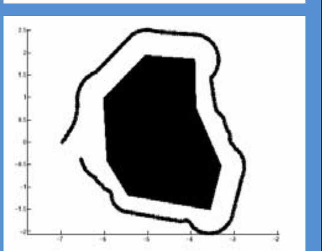
The steering law depends on the tangent vector of and the distance to the boundary curve of the obstacle and incorporates the curvature of the boundary curve. The steering law is derived from the following control Lyapunov function:

$$V = 4\sin^2\left(\frac{\theta-\phi_0}{4}\right) + 4\sin^2\left(\frac{\phi_0-\phi_{-\frac{\pi}{2}}}{4}\right) + g(r_{-\frac{\pi}{2}})\,,$$

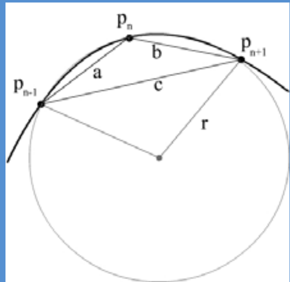
where $g(z)=\int_0^z f(\sigma)d\sigma\,.$ and $\begin{aligned} f(z) &= 0 \text{ when } z=r_c \\ f(z) &< 0 \text{ when } 0\le z<r_c \\ f(z) &> 0 \text{ when } z>r_c\,. \end{aligned}$







These three figures show Matlab simulations of the control law. In each simulation there is one obstacle. The robot is initially on a collision course with the obstacle, but under the control of the curvature-based steering law, it follows the trajectories shown. The three figures show examples of an elliptical, convex polygonal and non-convex polygonal obstacles.



This diagram illustrates the geometric approximation of plane curvature for a series of discrete points. P_k are the datapoints measured by the ladar. The curvature at P_n is approximated by the inverse of the radius r of the circle circumscribing the triangle $\Delta P_{n-1}P_nP_{n+1}$.

Curvature Estimation from Discrete Data

A geometric approximation of the plane curvature based on Heron's formula was used.

$$|\hat{\kappa}(s)| = 4\frac{\sqrt{s(s-a)(s-b)(s-c)}}{abc}\,.$$

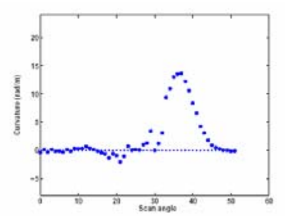
where s is the semiperimeter of the triangle with sides a,b,and c. In order to reduce the effects of noise, the curvature estimate at the point P_n used the triangle $\Delta P_{n-w}P_nP_{n+w}$ where w is called the window size.

In addition, the curvature estimates for a single point obtained using different window sizes were averaged together. The estimates in the figures below were made using the following combination of window sizes:

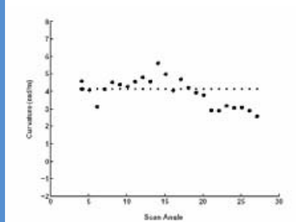
$$\bar{\kappa}(n) = \frac{1}{3}\sum_{w=7}^{9}\hat{\kappa}(P_{n-w},P_n,P_{n+w})$$

F. Zhang, A. O'Connor, D. Luebke, and P.S. Krishnaprasad, "Experimental Study of Curvature-based Control Laws for Obstacle Avoidance." Proc. IEEE International Conference on Robotics and Automation, 2004.

Measured Curvature of a Cabinet

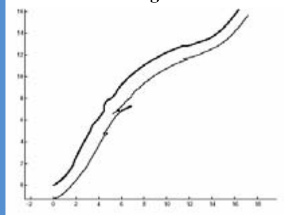


Measured Curvature of a Bin

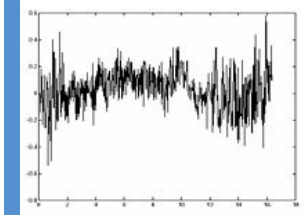


The two figures show the curvature estimates made from ladar data. The geometric formula with window sizes 7-9 was used. On the left is a scan of a cabinet with one sharp corner toward the robot. On the right, is a scan of a cylindrical trash bin. In both figures, the x-axis corresponds to the angle relative to the ladar, and the y-axes shows curvature in rad/s. The lighter dotted lines show the expected curvature, except at the cabinet corner, where a sharp spike is expected.

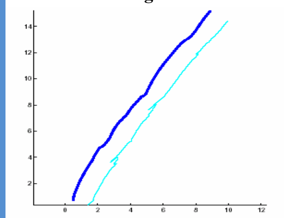
Robot Following Garden Wall



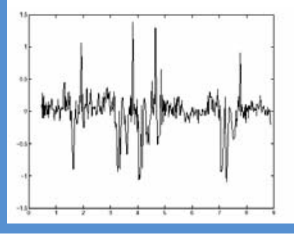
Measured Garden Wall Curvature



Robot Following Corridor Wall



Measured Corridor Wall Curvature

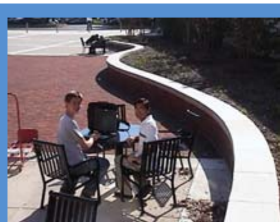


These figures show the results of two experiments. On the left are the robot trajectories (dark) along with the measured location of the obstacle (light). On the right are plots of the estimated boundary curvatures in rad/s as a function of time. The first experiment (top) had the robot follow a curved garden wall. In the second experiment (bottom), the robot followed the wall in a corridor in the A.V.Williams building. The spikes in that curvature plot correspond to the jambs of closed doors along the hall.

Robot Testing

The steering law was implemented as one word of the MDLe robot control language vocabulary. The curvature estimation was implemented as a module of that system.

The steering control law was tested on a mobile robot equipped with a laser rangefinder (ladar). Experiments were performed in a corridor in A.V. Williams as well as along a garden wall outside the Computer Science Instructional Center.



On the left is the Pioneer2 robot with the blue SICK laser rangefinder which was used to test the control law. On the right is the curved garden wall in front of the CSIC which the robot followed in the first experiment.