Sensor Network Platform Positioning using Mutual Motion Camouflage

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Introduction

• Animal formations in nature (bird flocks, fish schools) are extremely fascinating and still not well understood.

• Formations of autonomous vehicles could be effectively used to create mobile and reconfigurable sensor networks.

• Decentralization and simplicity are desired characteristics for the control laws used to create formations, along with richness in the formation patterns achievable; animal formations provide a good source of inspiration.

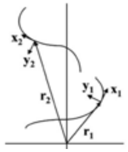
• Pursuit control laws as building blocks for formations: we explore the possibility of achieving formations with desired characteristics, by local application of pursuit laws between members of formation.

• In this work, we analyze two-unit formations achieved by mutual application of the Motion Camouflage pursuit law. We refer to this scenario as Mutual Motion Camouflage, or with the acronym MMC, in the following.

Mathematical Setup

• Each unit is modeled as a point-mass particle traveling at constant speed, using the Frenet-Serret equations of motion, with the curvature (steering) as control variable:

$$\begin{aligned} \dot{\mathbf{r}}_1 &= \mathbf{x}_1 \\ \dot{\mathbf{x}}_1 &= \mathbf{y}_1 \cdot u_1 \\ \dot{\mathbf{y}}_1 &= -\mathbf{x}_1 \cdot u_1 \\ \dot{\mathbf{r}}_2 &= \nu \cdot \mathbf{x}_2 \\ \dot{\mathbf{x}}_2 &= \mathbf{y}_2 \cdot \nu \cdot u_2 \\ \dot{\mathbf{y}}_2 &= -\mathbf{x}_2 \cdot \nu \cdot u_2 \end{aligned}$$



Justh, Krishnaprasad (2004)

- *u = curvature*
- *ν = speed of 'particle' 2; 'particle' 1 has unit speed*

• **Motion Camouflage pursuit law**: seeks to maintain constant the direction of the baseline connecting the positions of particle 1 and its target particle 2, with a Motion Camouflage strategy:

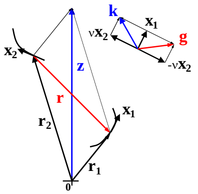
$$u_1 = -\mu \cdot \left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}^{\perp} \right),$$

where μ is the (positive) gain and $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$

• **Mutual Motion Camouflage scenario**: each particle applies the motion camouflage pursuit law, as to pursue the other particle, with a gain inversely proportional to its speed:

$$\begin{aligned} u_1 &= -\tfrac{\mu}{\nu_1} \cdot \left(\tfrac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}^{\perp}\right) = -\mu \cdot \left(\tfrac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}^{\perp}\right) \\ u_2 &= -\tfrac{\mu}{\nu_2} \cdot \left(-\tfrac{\mathbf{r}}{|\mathbf{r}|} \cdot -\dot{\mathbf{r}}^{\perp}\right) = -\tfrac{\mu}{\nu} \cdot \left(\tfrac{\mathbf{r}}{|\mathbf{r}|} \cdot \dot{\mathbf{r}}^{\perp}\right) = \tfrac{u_1}{\nu} \end{aligned}$$

• **Relative motion between the particles, and absolute motion of the center of mass (c.o.m.):**



Relevant vectors:

$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ relative position

$\mathbf{g} = \mathbf{x}_1 - \nu \cdot \mathbf{x}_2$ relative velocity

$\mathbf{h} = \mathbf{y}_1 - \nu \cdot \mathbf{y}_2$ $\mathbf{h} = \mathbf{g}^{\perp} = \dot{\mathbf{r}}^{\perp}$

$\mathbf{z} = \mathbf{r}_1 + \mathbf{r}_2$ c.o.m. position (x2)

$\mathbf{k} = \mathbf{x}_1 + \nu \cdot \mathbf{x}_2$ sum of velocities

$\mathbf{l} = \mathbf{y}_1 + \nu \cdot \mathbf{y}_2$ $\mathbf{k}^{\perp} = \mathbf{l}$

In Mutual Motion Camouflage:

$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{g} & \dot{\mathbf{z}} &= \mathbf{k} \\ \dot{\mathbf{g}} &= -\mu \cdot \left(\tfrac{\mathbf{r}}{|\mathbf{r}|} \cdot \mathbf{h}\right) \cdot \mathbf{h} & \dot{\mathbf{k}} &= -\mu \cdot \left(\tfrac{\mathbf{r}}{|\mathbf{r}|} \cdot \mathbf{h}\right) \cdot \mathbf{l} \\ \dot{\mathbf{h}} &= \mu \cdot \left(\tfrac{\mathbf{r}}{|\mathbf{r}|} \cdot \mathbf{h}\right) \cdot \mathbf{g} & \dot{\mathbf{l}} &= \mu \cdot \left(\tfrac{\mathbf{r}}{|\mathbf{r}|} \cdot \mathbf{h}\right) \cdot \mathbf{k} \end{aligned}$$

<u>First conservation property</u>:

$|\mathbf{g}| = |\mathbf{h}| = \delta$

$|\mathbf{k}| = |\mathbf{l}| = \theta$

are conserved quantities in MMC and in any other scenario with: $u_1 = u_2 \cdot \nu$

Relevant scalar variables:

$\rho = |\mathbf{r}|$ distance between particles

$\gamma = \left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \mathbf{g}\right)$ (scaled) inner product $\mathbf{r}, \dot{\mathbf{r}}$

$\lambda = \left(\frac{\mathbf{r}}{|\mathbf{r}|} \cdot \mathbf{h}\right)$ (scaled) inner product $\mathbf{r}, \dot{\mathbf{r}}^{\perp}$

$\zeta = |\mathbf{z}|$ (2x) distance of c.o.m. from origin

$\xi = \left(\frac{\mathbf{z}}{|\mathbf{z}|} \cdot \mathbf{k}\right)$ (scaled) inner product $\mathbf{z}, \dot{\mathbf{z}}$

$\eta = \left(\frac{\mathbf{z}}{|\mathbf{z}|} \cdot \mathbf{l}\right)$ (scaled) inner product $\mathbf{z}, \dot{\mathbf{z}}^{\perp}$

<u>Second conservation property</u>:

$$\begin{aligned} \delta^2 &= \gamma^2 + \lambda^2 \\ \theta^2 &= \xi^2 + \eta^2 \end{aligned}$$

with the above choice of scalar variables

• **Reduced equations for relative motion and center of mass (c.o.m.) dynamics:**

$$\begin{aligned} \dot{\rho} &= \gamma \\ \dot{\gamma} &= \frac{(\delta^2 - \gamma^2)}{\rho} - \mu \cdot \lambda^2 \\ \dot{\lambda} &= \left(\mu - \frac{1}{\rho}\right) \cdot \lambda \cdot \gamma \\ \dot{\zeta} &= \xi \\ \dot{\xi} &= \frac{(\theta^2 - \xi^2)}{\zeta} - \mu \cdot \lambda \cdot \eta \\ \dot{\eta} &= -\frac{1}{\zeta} \cdot (\xi \cdot \eta) + \mu \cdot \lambda \cdot \xi \end{aligned}$$

Reduced relative motion equations

(RM+COM)

Reduced c.o.m. equations

Relative motion analysis

$$\begin{aligned} \dot{\rho} &= \gamma \\ \dot{\gamma} &= \left(\tfrac{1}{\rho} - \mu\right) \cdot (\delta^2 - \gamma^2) \end{aligned} \qquad \text{(RM)}$$

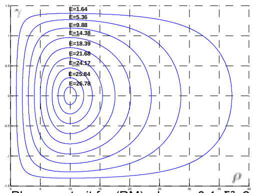
• neutrally stable equilibrium point at: $(\rho, \gamma) = (1/\mu, 0)$

• Motion Camouflage invariant manifold: $\gamma = \pm\delta$

• <u>Every orbit of (RM) starting in</u> $\{(\rho, \gamma) : \rho > 0, -\delta < \gamma < \delta\}$ <u>is a periodic orbit</u>

<u>Third conservation property</u>: for every (periodic) orbit of (RM) the following is a conserved quantity:

$$E(\rho, \gamma) = \rho^2 \cdot (\delta^2 - \gamma^2) \cdot e^{-2\mu\rho}$$



Phase portrait for (RM) when μ=0.1, δ²=2

• $0 < E(\rho, \gamma) \leq E_{MAX}$

• At equilibrium point:

$$E_{MAX} = \frac{\delta^2}{\mu^2} \cdot e^{-2}$$

• For each orbit:

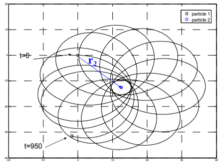
$0 < \rho_{MIN} \leq \rho \leq \rho_{MAX}$

(collision avoidance)

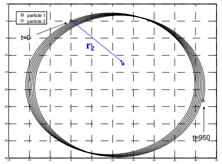
• Infinitely many periodic orbits, with shape dependent on μ (gain) and δ (absolute value of initial relative velocity)

• Initial conditions define which orbit is followed: $E(\rho_0, \gamma_0)$

• In the absolute reference frame this corresponds to the particles following "flower-shaped" orbits around each other, as seen in the "beacon case" (ν = 0):

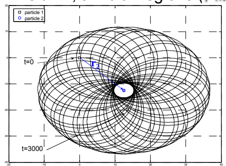


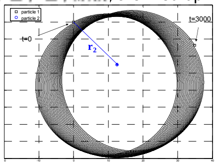
μ=0.1, γ(0)=-0.7,ρ(0)=17, E=4.55



μ=0.05, γ(0)=-0.3,ρ(0)=17, E=48.62

• As t->∞, annular regions ($\rho_{MIN} \leq \rho \leq \rho_{MAX}$) are filled-up:





Center of mass analysis

<u>Theorem</u>: with an appropriate choice for the origin of the absolute reference frame, the reduced dynamics of the center of mass are just a scaled version of the reduced relative motion dynamics:

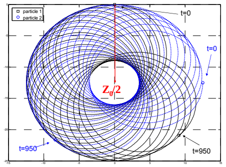
$$\begin{aligned} \zeta(t) &= \sigma \cdot \rho(t) \\ \xi(t) &= \sigma \cdot \gamma(t) \\ \eta(t) &= \sigma \cdot \lambda(t) \end{aligned}$$

where: $\sigma = \frac{\theta}{\delta}$

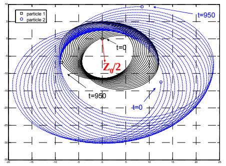
<u>Steps of the proof:</u>

1. The equations (RM+COM) don't change if we substitute $\mathbf{z}$ with its translation $\mathbf{z_t} = \mathbf{z} - \mathbf{z_0}$ in the definition of (ζ, ξ, η)
2. $M = \{(\rho, \gamma, \lambda, \zeta, \xi, \eta) : \zeta = \sigma \cdot \rho, \xi = \sigma \cdot \gamma, \eta = \sigma \cdot \lambda\}$ is an invariant manifold for (RM+COM)
3. For any given initial conditions, there is a (unique) $\mathbf{z_0}$ s.t. $\zeta_0 = \sigma \cdot \rho_0, \quad \xi_0 = \sigma \cdot \gamma_0, \quad \eta_0 = \sigma \cdot \lambda_0$ (e.g. in the "beacon case", take $\mathbf{z_0} = 2 \cdot \mathbf{r_2}$)

The orbits of the center of mass (about the translated origin) and the orbits of each particle w.r.t. the other, determine the overall trajectories of the two-particle system :



μ=0.1, ν=1, θ²=δ²=2, z0=[0;-25]



μ=0.05, ν=2, θ²=3.13, δ²=6.86, z0=[0.6;-13.5]

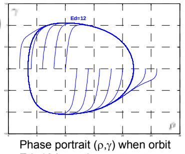
Discussion

• We provided a rigorous analysis of the behavior of the MMC system, which produces rich motion patterns. Pattern flexibility is afforded by simple modifications of gains, speeds and initial positions and orientations.

• Additionally it is possible to make one of the periodic orbits of (RM) asymptotically orbitally stable, with the following modification of the control law:

$$u = \underbrace{-\mu \cdot \lambda}_{\text{MMC}} + \alpha \cdot \lambda \cdot \gamma \cdot (E(\rho, \gamma) - E_d)$$

where $\alpha > 0$ and E_d is the "energy" of the desired orbit.



Phase portrait (ρ,γ) when orbit Ed=12 is made an attractor

• Alternatively, it is also possible to make the equilibrium point of (RM) asymptotically stable, thus achieving circular orbits, with the following control:

$$u = -\mu \cdot \lambda - \beta \cdot \gamma \cdot \lambda^{2k-1}, k \in \mathbb{I} > 0, \beta > 0$$

• In conclusion we note that MMC is a useful building block for the synthesis of sensor network motion patterns, especially to fulfill objectives of spatial coverage.