Space-Time Trellis Code Construction for Fast Fading Channels

Zoltan Safar / K. J. Ray Liu

Motivation and Objective

- Need for high data rates in future wireless communication systems
- The transmitted signal undergoes severe distortion due to fading and interference - system design challenges
- Information theoretic works: the capacity of fading channels is substantially increased when using multiple transmit and/or receive antennas
- Objective: to devise a systematic code design method that provides full spatial diversity for an arbitrary number of transmit antennas and any memoryless constellation

Performance Criteria I

- Notation: K - number of transmit antennas
  L - number of receive antennas
  b - number of source bits per state transition
  $B = 2^b$ - constellation size
- Sent channel symbol by antenna k at time t: c_t^k
- Decoded (erroneous) channel symbol for antenna k at time t: e_t^k
- Channel symbol difference matrix :

$$\mathbf{D} = \begin{bmatrix} c_0^0 - e_0^0 & c_1^0 - e_1^0 & \dots & c_{T-1}^0 - e_{T-1}^0 \\ c_0^1 - e_0^1 & c_1^1 - e_1^1 & \dots & c_{T-1}^1 - e_{T-1}^1 \\ \dots & \dots & \dots & \dots \\ c_0^{K-1} - e_0^{K-1} & c_1^{K-1} - e_1^{K-1} & \dots & c_{T-1}^{K-1} - e_{T-1}^{K-1} \end{bmatrix} = [\mathbf{c}_0 - \mathbf{e}_0, \mathbf{c}_1 - \mathbf{e}_1, \dots, \mathbf{c}_{T-1} - \mathbf{e}_{T-1}]$$

Performance Criteria II

- Results from Tarokh et al., 1998:
  - Assume: for τ discrete time instants $t_0, t_1, \dots, t_{\tau-1}$ the t_ith column of **D** is nonzero, i.e.:

$$\mathbf{c}_{t_i} - \mathbf{e}_{t_i} \neq \mathbf{0} \quad \text{for} \quad i = 0, 1, \dots, \tau - 1$$

  - Probability that the encoder erroneously decodes $\{\mathbf{e}_t\}$ if $\{\mathbf{c}_t\}$ was sent can be upper bounded as:

$$P(\{\mathbf{e}_t\} \mid \{\mathbf{c}_t\}) \leq \left(\frac{E_0}{4KN_0}\right)^{-\tau L} \prod_{i=0}^{\tau-1} \left\| \mathbf{c}_{t_i} - \mathbf{e}_{t_i} \right\|^{-2L}$$

  - Performance criterion:
    - the minimum number of nonzero columns in **D** must be maximized
    - the minimum of the norm products must be maximized

A New Design Criterion

- Assume: the encoder has B^{K-1} states
- Uniqueness criterion: every channel symbol index vector must be unique
  - Viability: - K digit B-ary numbers
    - B branches emanating from B^{K-1} states
- Properties:
  - Diversity advantage:
    - any τ-long error event will achieve a diversity advantage of τL
    - B^{K-1} states: the shortest error event is K state transitions long a diversity advantage of KL is guaranteed
  - Node error probability:
    - the probability of the union of the τ-long error events decreases at least exponentially with τ
    - at high SNR, the shortest error events will dominate

The Design Method I

- Most important objective: to maximize the diversity advantage
  → uniqueness criterion
- Remaining freedom: to increase the value of the minimum norm product (coding advantage)
  - hard problem: nonlinear combinatorial optimization
  - exhaustive computer search: too expensive
  - our approach: - does not guarantee optimality
    - simple (closed form) and flexible
- Basic idea:the method attempts to maximize the number of nonzero entries in the **D** matrix corresponding to the shortest error events
  - indirect maximization of the norm product
  - the shortest error events achieve the minimum diversity advantage

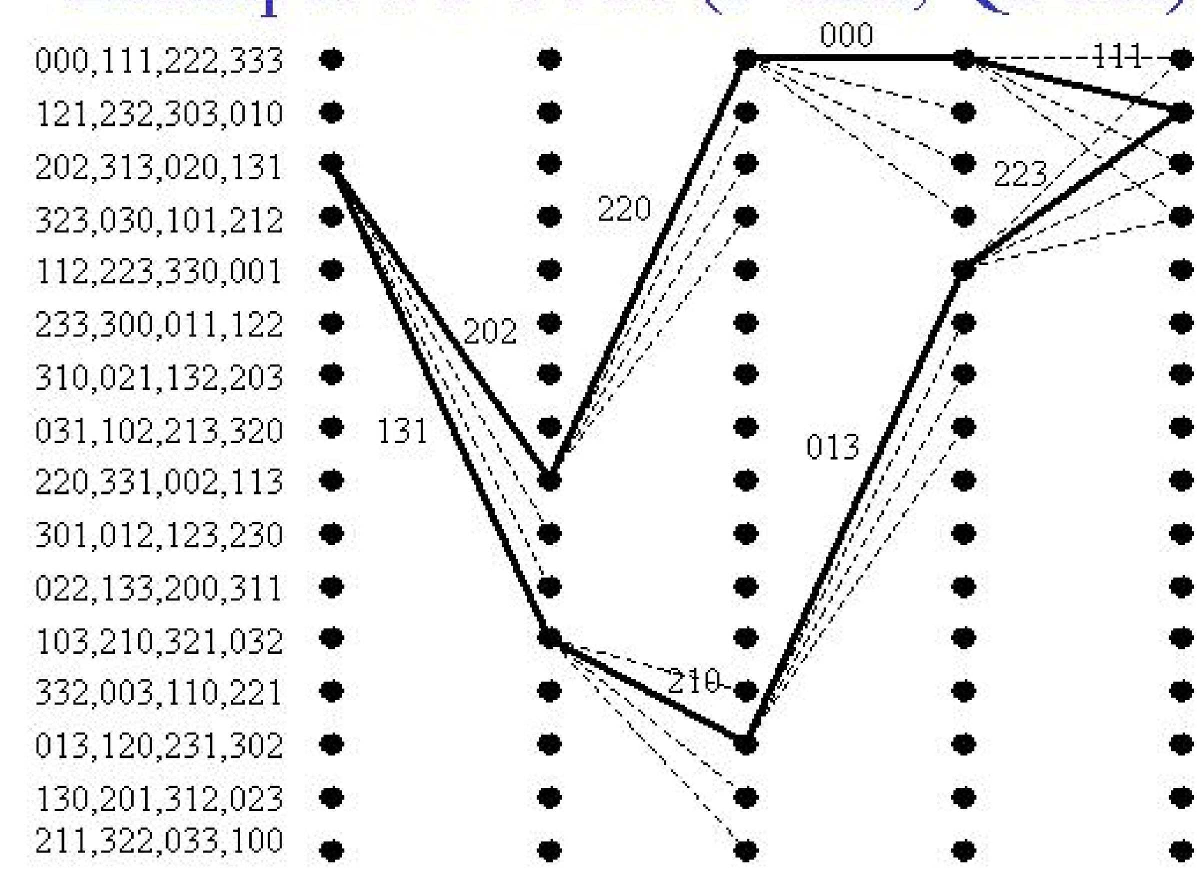
The Design Method II

- ST code example:
  - state $S = 6$, 2nd branch (b=2)
  - S can be expressed as: $S = Bl_2 + l_1$ with $l_1 = 1$ and $l_2 = 2$

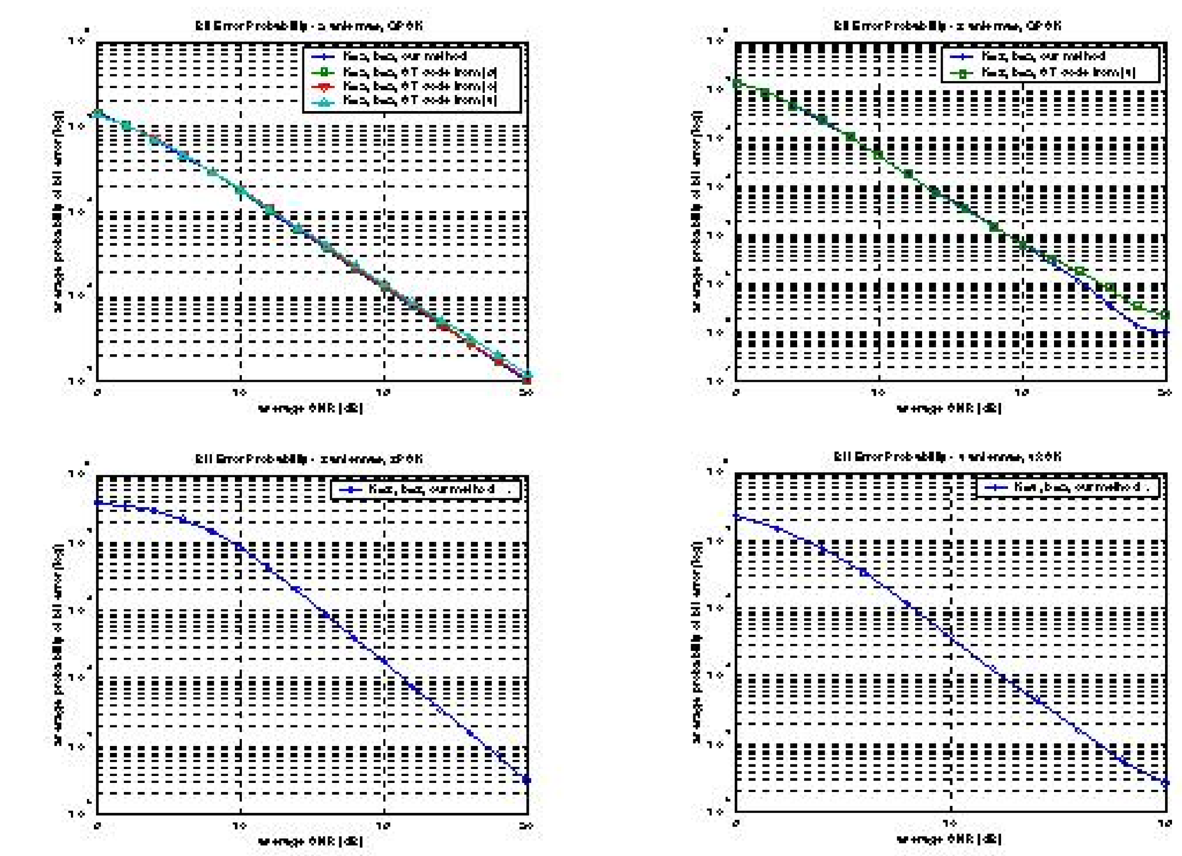
$i^0(S,b) = (b + l_2 + l_1) \bmod B = 1$
$i^1(S,b) = (i^0(S,b) + l_1) \bmod B = 3$ $\quad [1,3,2]^T$
$i^2(S,b) = (i^0(S,b) + l_2) \bmod B = 2$

- It can be shown that the ST codes designed by this method satisfy the uniqueness criterion for any memoryless constellation

Example ST Code (3 ant., QPSK)



Simulation Results



Future Directions

- Extension to correlated Rayleigh fading channels (space-time correlation function must be obtained)
- Variable Rate ST Trellis Codes
  - MTCM construction: changing the multiplicity = changing the rate
  - Possibility of unequal error protection
- Extension to broadband (frequency selective) channel models
  - Equalization: joint decoding/channel estimation/equalization
  - ST coded OFDM: existing work: quasi-static fading