Modeling problems in air traffic management

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Airport queuing models for delay prediction

David Lovell, Kleoniki Vlachou

- Develop queuing models to quantify benefits of increased precision in future air traffic processes: e.g., reduced interarrival times, reduced variance in interarrival times
- Employ diffusion approximation to model joint probability density function of queue length and time
    - Permits independent specification of mean and variance for service process – in contrast to typical Poisson models
    - Requires assumption of continuous process: valid when many flights considered

Single airport queue model

- Fokker-Planck equation is governing PDE

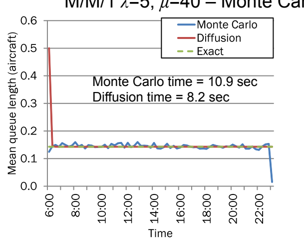
$$\frac{\partial f_i(x;t)}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial x^2} V_i(x;t) f_i(x;t) - \frac{\partial}{\partial x} M_i(x;t) f_i(x;t)$$

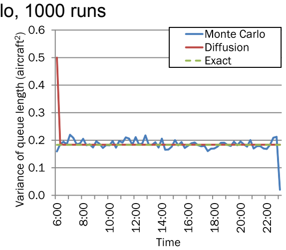
- Boundary conditions enforce realism: queue length must be non-negative and must begin the day empty
- System of coupled partial differential equations solved using Finite Element Method

Model validation and results

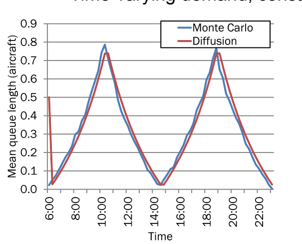
- Validate results using Monte Carlo simulation of system and analytic results, where appropriate
    - Uncongested steady state

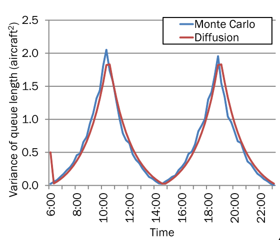
M/M/1 λ=5, μ=40 – Monte Carlo, 1000 runs





- Time-varying demand, constant service rate





Long-term airport congestion management

David Lovell, Michael Ball, Avijit Mukherjee, Andrew Churchill

- Under nominal conditions, some airports experience demand well in excess of their capacity
- Because increasing capacity is often very difficult, approaches for regulating demand are employed

How many operations should be permitted?

- Target number of operations must be set while accounting for:
    - Variations in available capacity: if target equal good weather capacity, then delays will be rampant, but if it equals bad weather capacity, the airport will be underutilized
    - Variations in value: accessing the airport: offering flights during certain hours is clearly more valuable to airlines than offering them during others
- Use stochastic integer program to balance these considerations while hedging against both good and bad capacity outcomes

Objective: Maximize value of available slots

$$\max z = \sum_t V_t Z_t$$

Permit flight cancellations

$$Z_t - X_{t,q} - \sum_i \theta_{t,q}{}^i = 0 \quad \forall t \in \{1,\ldots,T\},\ q \in \{1,\ldots,Q\}$$

Enforce flow balance

$$X_{t,q} + Y_{t-1,q} - Y_{t,q} \le C_{t,q} \quad \forall t \in \{1,\ldots,T+U\},\ q \in \{1,\ldots,Q\}$$

Bound number of available slots

$$D_{\min} \le Z_t \le D_{\max} \qquad \forall t \in \{1,\ldots,T\}$$

Bound cancellations

$$\theta_{t,q}{}^i \le P_i \ \ \forall t \in \{1,\ldots,T\},\ q \in \{1,\ldots,Q\},\ i \in \{1,\ldots,N\}$$

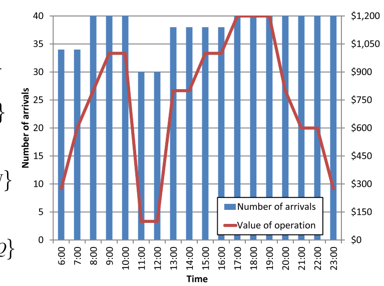
Bound maximum flight delay length

$$Y_{t,q} \le W_{t,q} \quad \forall t \in \{1,\ldots,T+U-1\},\ q \in \{1,\ldots,Q\}$$

Set performance targets

$$\sum_q p_q \sum_i \sum_t \theta^i_{t,q} - \rho \sum_t Z_t \le 0$$

$$\sum_q p_q \sum_t Y_{t,q} - \gamma \sum_t Z_t \le 0$$



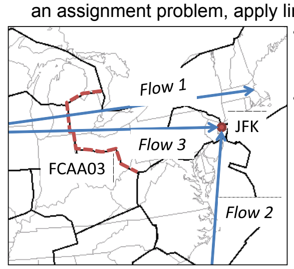
Regional traffic flow management

- Demand-capacity imbalances impact airports and airspace regions, and require intervention to maintain safety and prevent excessive airborne delays
- To regulate aircraft flows, delays are assigned before departures

Coordinate multiple conflicting traffic management initiatives

David Lovell, Michael Ball, Andrew Churchill

- In practice, capacity rationing occurs independently at each congested resource
- Explicit coordination will guarantee feasibility, improve equity between users, and improve efficiency
- Model problem as binary linear program, treating each resource as an assignment problem, apply linking constraints for feasibility



Objective: Minimize assigned arrival delays

$$\min z = \sum_{f \in F} \sum_{i \in V_f} \sum_{s \in Q^i_f} \left(\tau^i_s - \alpha^i_f\right)^{1+\varepsilon} x^i_{fs}$$

Each flight assigned to one slot at each resource

$$\sum_{s \in Q^i_f} x^i_{fs} = 1 \quad \forall f \in F, i \in V_f$$

- More complex formulations may consider random capacity variation
- Objective may induce inequities, requiring alternate formulations to control worst-case deviations

Each slot may receive at most one flight

$$\sum_{f \in F : i \in V_f} x^i_{fs} \le 1 \ \ \forall i \in I, s \in S^i$$

Feasible slot combinations must be assigned

$$x^i_{fs} - \sum_{k \in R^{ij}_{fs}} x^j_{fk} \le 0 \quad \forall f \in F, i \in V_f, j = N^i_f,\ s \in Q^i_f : \left|N^i_f\right| > 0$$

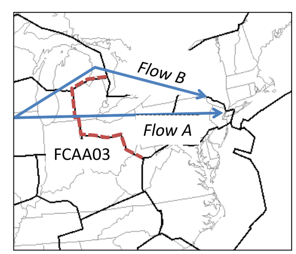
Feasibility range for subsequent resources

$$R^{ij}_{fs} = \left\{ k \in S^j : \max\left(\alpha^j_f, \tau^i_s + \alpha^j_f - \alpha^i_f - \pi_L\right) \le \tau^j_k \le \tau^i_s + \alpha^j_f - \alpha^i_f + \pi_U \right\}$$

Coordinate capacity rationing and dynamic flight rerouting

David Lovell, Michael Ball, Moein Ganji

- Rerouting presents an alternative for offloading flights from congested airspace resources, in place of assigning ground delays
- However, alternative routes are likely longer, increasing travel time and fuel burn
- If disruption clears early, then flights on alternative route may return to nominal route along some hybrid path
- Model problem as binary linear program to make efficient and equitable tradeoffs between ground delay for nominal route and increased cost of using alternative route
- Include random end time and movement for disruption



Objective: Minimize initial cost, less savings for each capacity scenario (early end time)

$$\min z = C(X) - \sum_u P_u S(Y_u)$$

Each flight receives an initial allocation

$$\sum_t x^p_{f,t} + \sum_r x^s_{f,r} = 1 \quad \forall f$$

Resource capacity constraint in initial allocation

$$\sum_f x^p_{f,t} \le C^0_t \quad \forall t$$

Each flight receives an allocation for end times

$$\sum_t y^p_{f,t} \mid u + \sum_r \sum_t y^h_{f,t} \mid u + \sum_r y^s_f \mid u = 1 \quad \forall u, f$$

Resource capacity constraint for each end time

$$\sum_f y^p_{f,t} \mid u + \sum_r \sum_f y^h_{f,t,r} \mid u \le C^u_t \quad \forall u, t$$

Consistent assignment for each end time

$$y^p_{f,t} \mid u = x^p_{f,t} \quad \forall f, u, \quad \forall t \in \{1,\ldots,u+E_f\}$$

Reassignment from primary to alternative route

$$\sum_t y^p_{f,t} \mid u \le 1 - \sum_r x^s_{f,r} \quad \forall u, \forall f \ni D_f < u$$

Prevent increased delay for early end times

$$y^p_{f,t} \mid u \le \sum_{q \ge t} x^p_{f,q} + \sum_r x^s_{f,r} \quad \forall u, f, t$$

Prevent infeasible reallocations for rerouted flights

$$y^h_{f,t,r} \big| u = 0 \ \ \forall f, r, u \quad \forall t \big| t^d_{f,t} \le u$$

Allow rerouting only if flight on alternative route

$$\sum_t y^h_{f,t,r} \mid u + y^s_{f,r} \mid u \le x^s_{f,r} \quad \forall u, f, r$$