

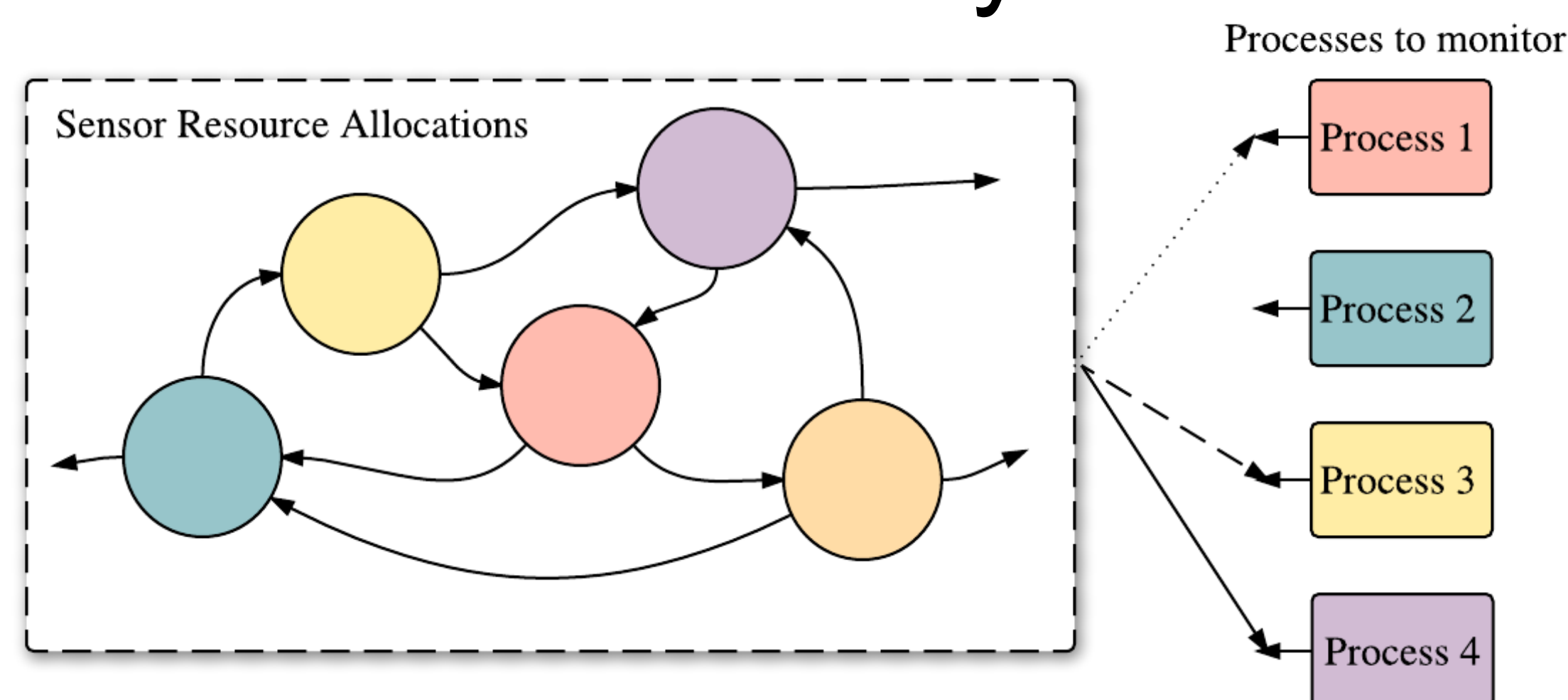
Monitoring multiple systems over channels with usage dependent performance

David Ward and Nuno C. Martins



INSTITUTE FOR SYSTEMS RESEARCH
A. JAMES CLARK SCHOOL OF ENGINEERING

Resource Constrained Dynamic Sensors



Resource constrained sensor systems with dynamics are modeled by a finite state Markov Process. Each Markov chain state represents a different allocation of resources. **Each process is measured over a packet-drop channel with a probability of drop determined by the current resource allocation.** Applications include:

- Battery operated channels with energy harvesting,
- Dynamic workload management of human operators, and
- Monitoring multiple cell cultures.

Designing Sensor Resource Allocation Policies

A sensor resource allocation policy is determined by designing a transition matrix to stabilize the estimation error of all the processes that are being monitored. These processes are modeled by linear systems driven by noise. Also, immediate transitions between different resource allocations may not be possible. Thus, the transition matrix must have a specific zero pattern structure.

Problem: Let $\mathbf{S} \in \{0, 1\}^{m \times m}$ for $i = 1, \dots, t$ be given. Find a stochastic transition matrix \mathbf{Q} with $\mathbf{Q} \leq \mathbf{S}$ such that for $i = 1, \dots, t$,

$$\rho(\mathbf{Q}^T \mathbf{D}_i) < 1,$$

where $\rho(\mathbf{Q}^T \mathbf{D}_i)$ is the spectral radius of $\mathbf{Q}^T \mathbf{D}_i$ and \mathbf{D}_i are diagonal, positive definite, parameter matrices determined by the models of the linear processes that are being monitored.

A Proposed Iterative Algorithm

This sensor resource allocation problem is a difficult, nonconvex problem. We propose an iterative fixed-point like algorithm based on the following technical characterization of the problem. **This algorithm performs well compared to branch and bound methods, scales well with problem size and for specific cases is guaranteed to find a stabilizing transition matrix, if it exists.**

$\exists \mathbf{Q}$ such that $\mathbf{Q} \leq \mathbf{S}$
and for $i = 1, \dots, t$,

$$\rho(\mathbf{Q}^T \mathbf{D}_i) < 1,$$

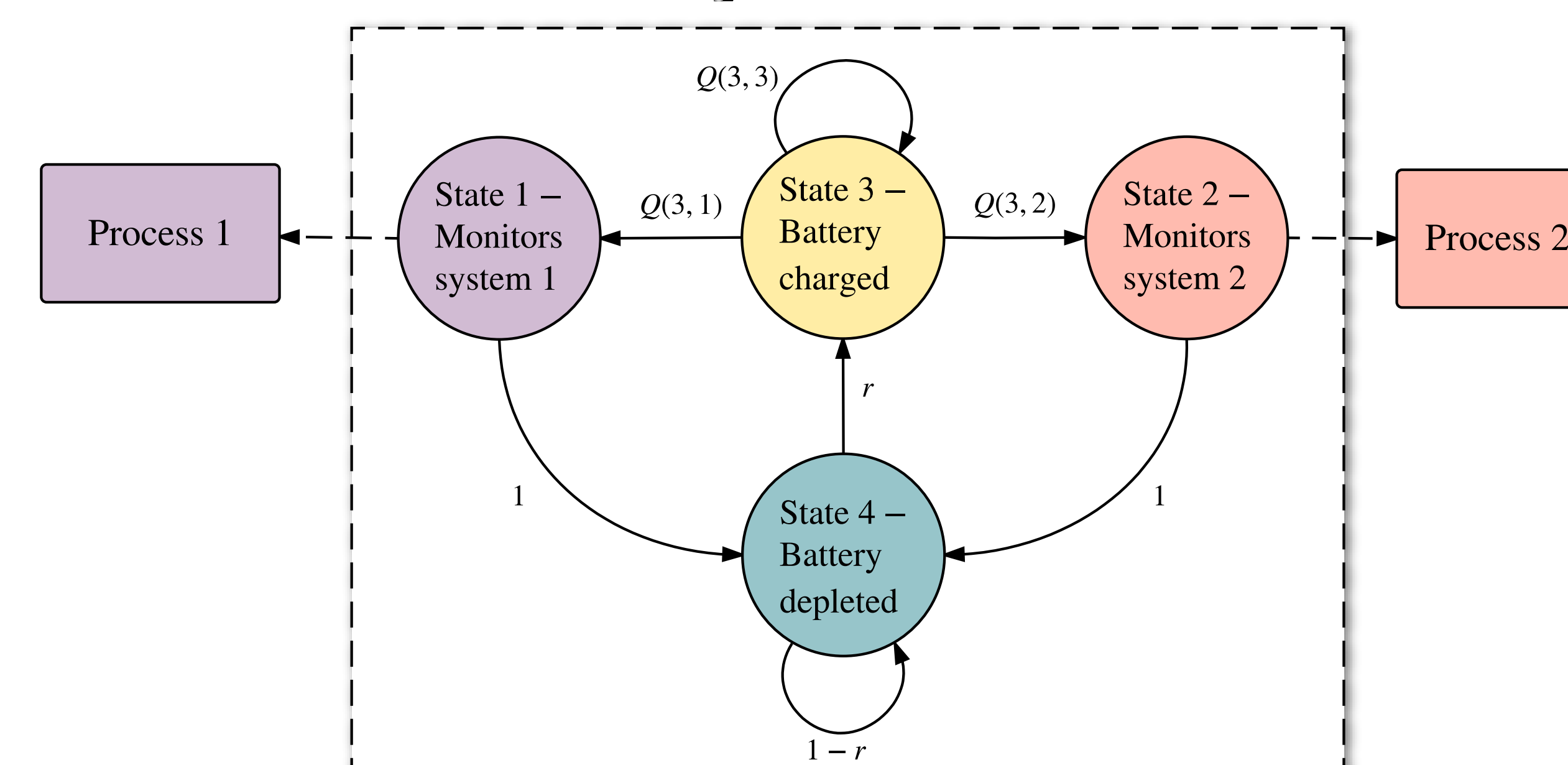


$\exists \mathbf{H} > 0$ such that for all $\mathbf{Z} > 0$
and $k = 1, \dots, m$

$$\mathbf{h}^{(k)} \geq \min_{i \in ON(k)} \mathbf{h}^{(i)} \mathbf{D}^{(k)} \mathbf{z}_k,$$

where \mathbf{z}_k is the k^{th} column of \mathbf{Z}
and $\mathbf{h}^{(k)}$ is the k^{th} row of \mathbf{H}

Example – Monitoring Two Systems with a one-time use battery



In this example, r is the recharge probability of the battery during a single time step. The transition matrix must be designed to balance when to **preserve the battery by not measuring, measure system 1, or measure systems 2.**