Dual-Based Heuristics for the Connected Facility Location and Related Problems

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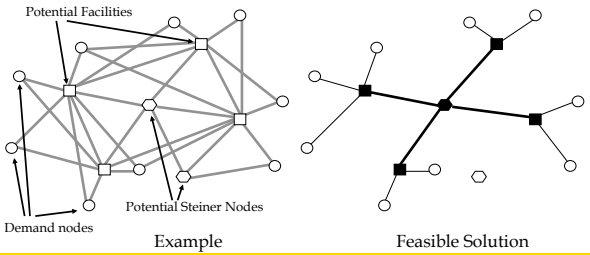
Problem Description

These problems typically arise in the design of telecommunication networks where open facilities need to communicate with each other. For example, in data distribution and management problems.



Objective

- Establish a common framework for a family of related problems: connected facility location (ConFL), rent or buy (ROB), Steiner tree star (STS) and General STS (GSTS).
- Develop an effective heuristic that finds high quality solutions very rapidly.

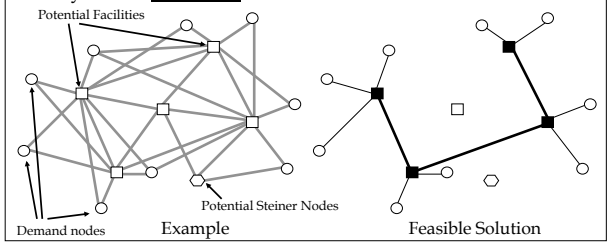
Note that these problems are NP-complete

Heuristic Approach

ConFL, ROB, STS, GSTS → General ConFL → Directed Steiner Tree Problem with unit degree constraint on the root node → Dual-Ascent + Local Search → High Quality Solutions

General Connected Facility Location Problem

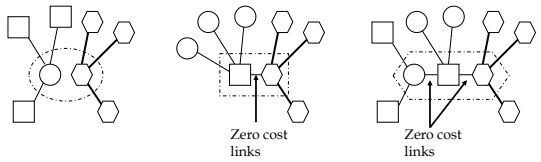
There are three disjoint sets of nodes: demand nodes, potential facility nodes, and potential Steiner nodes.
Facility nodes ALWAYS incur a cost.



Transformation into the General ConFL Problem

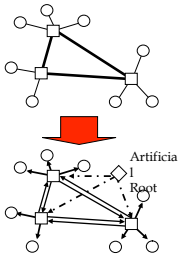
The transformation for each problem is based on node splitting strategy and arcs are added arcs accordingly.

Customer Node | Facility Node | Dual-Role Node



Modeling as a Directed Steiner Tree Problem with Unit Degree Constraint on the Root Node

- Create an artificial node (root node) with unit degree constraint.
- Create a directed arc from the root node and every facility node with cost equal to the fixed cost of the hub.
- Replace each edge by two directed arcs such that the cost of the arc equals the cost of the edge plus the facility opening cost (if any) at the end node.



Dual-Based Heuristic

Dual-Ascent (DA)

- Apply a DA procedure for the Steiner tree problem
- Obtain a dual solution (lower bound) and feasible solution (upper bound) for the Steiner tree problem

Sequential Improvements

- Construct a minimum spanning tree on the set of open facilities and selected Steiner nodes
- Eliminate any Steiner node with degree two or less in the Steiner Tree (when triangle inequality is satisfied)

Local Improvements

- Order open facilities in increasing order of
  - Node degree in the Steiner Tree
  - Number of customers served
- Remove open facilities one at a time in order and compare change in the solution cost
  - If removal results in an improvement, update solution; otherwise, restore open facility into solution
- Repeat until no more improvements are possible

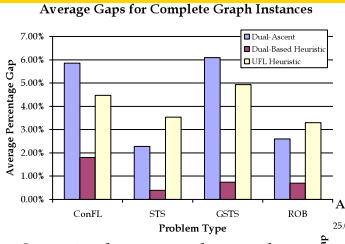
Computational Experiments

We tested our heuristic on randomly generate problems on a 100x100 grid with various characteristics.
In addition, we used large-scale instances and compare results yielded by a variable neighborhood search (VNS) heuristic (Ljubic 2007).
Improvement Steps significantly reduce the upper bound.
Does Dual-Ascent yield "good" starting solutions?

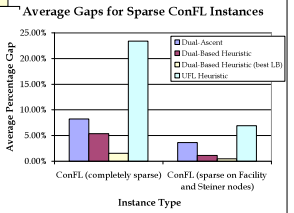
UFL Heuristic

We defined an uncapacitated facility location (UFL) heuristic to evaluate the starting solution yielded by DA.

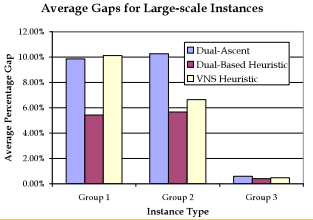
Summary of Results



- Our dual-based solutions are consistently good for all four problems
- ConFL is the hardest problem in the family



- Sparsity between demand nodes and facilities yields harder instances



- Our dual-based heuristic obtained better solutions than the VNS heuristic for most large-scale instances

Conclusions

- For our test problems our heuristic generated solutions that were on average within 6% from optimality, and in the worst case instances within 10% from optimality.
- Results are consistent across a wide range of parameters.
- Our proposed heuristic yields significantly better solutions than the UFL approach and more consistent gaps than the VNS heuristic.
- The procedure is extremely fast on complete graph problems and relative fast on large-scale instances.