Reaching a Target inside a Denied Area: What is the Optimal Control Strategy?

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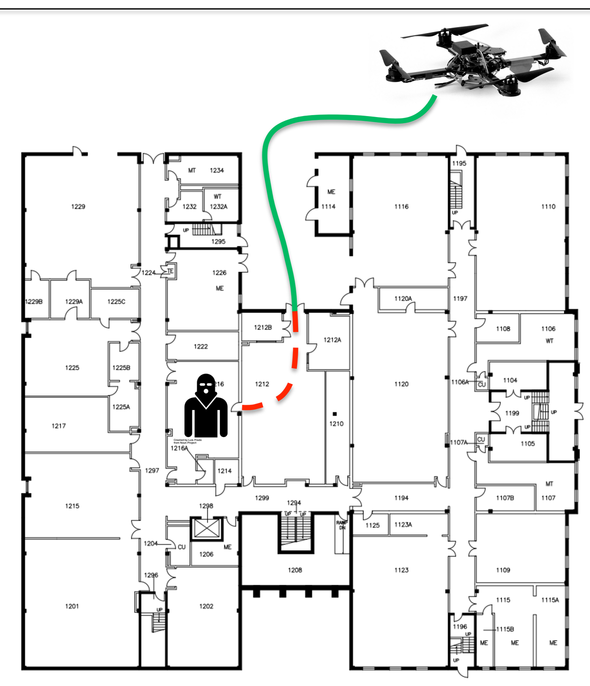
Research Aims

Our goal is to

1. Understand how to control a mobile agent to reach a target enclosed within a denied area with time and energy considerations.
2. Develop a systematic method to find the optimal control strategy to perform such task.

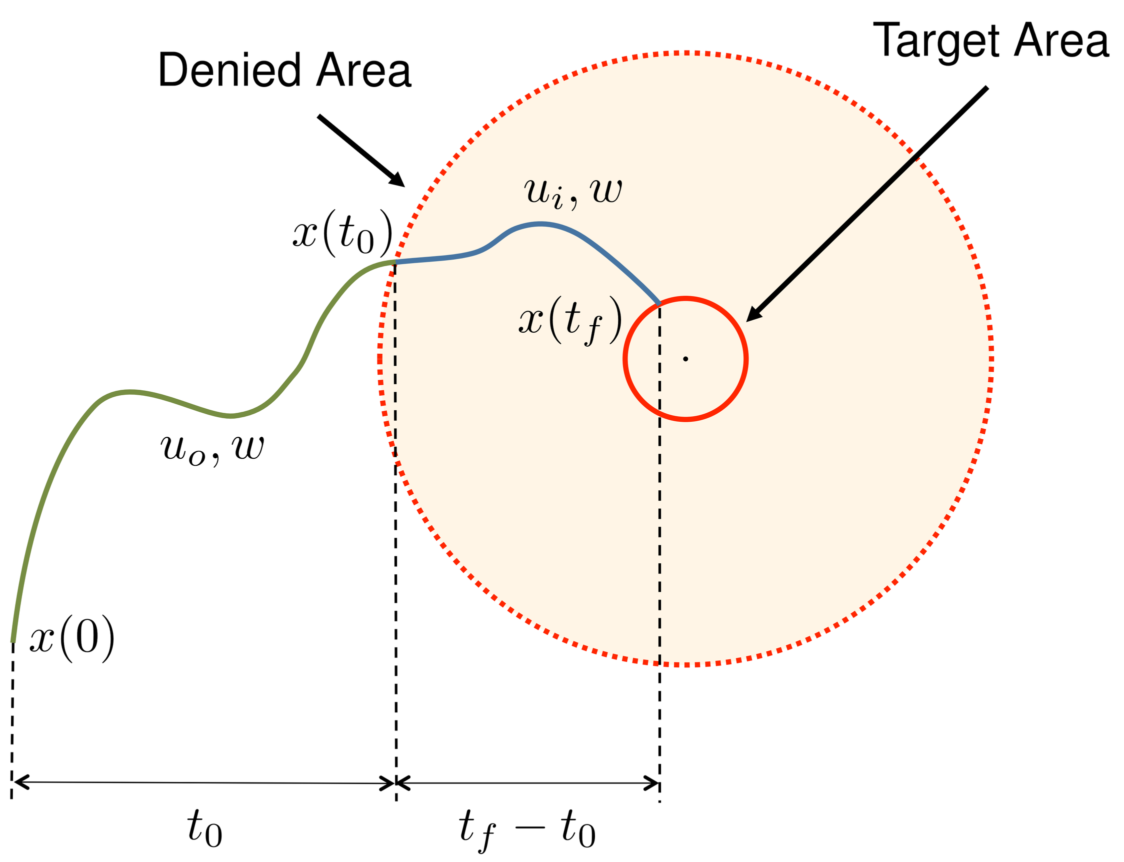
Motivating Scenario

- Hostile target in the building
- Drone knows the map and the target
- No motion measurements inside the building
- Time and energy are limited



Problem Formulation

Consider the motion of a mobile agent in a 2D plane. We formulate an optimization problem to minimize the cost incurred outside and inside the denied area.



$$\underset{t_0, t_f, u(\cdot)}{\text{minimize}} \quad \mathbb{E}_w\left[C_o(x(0), u_o, w[0,t_0], t_0) + C_i(x(t_0), u_i, w[t_0,t_f], t_f - t_0)\right]$$

$$\text{subject to} \quad \dot{x}(t) = Ax(t) + Bu(t) + w(t), x(0)$$

$$u(t) = \begin{cases} u_o(t), & t \in (0, t_0] \\ u_i(t), & t \in (t_0, t_f] \end{cases}$$

$$x(t_0) \in \partial\mathcal{D}_{DA}$$

$$x(t_f) \in \mathcal{D}_{TA}$$

$$0 < t_0 < t_f \le T,$$

where

$$\mathcal{D}_{DA} = \{x \in \mathbb{R}^4 : x^T D x \le \alpha^2\}$$

$$\mathcal{D}_{TA} = \{x \in \mathbb{R}^4 : x^T D x \le \delta^2\}$$

$$D = \begin{bmatrix} D_1 & 0_{2\times 2} \\ 0_{2\times 2} & 0_{2\times 2} \end{bmatrix}, \; D_1 \succ 0$$

$$0 < \delta < \alpha.$$

- Assume the mobile agent has a continuous time, linear time-invariant dynamics.
- Assume no measurement is available to the mobile agent inside the denied area.

Current Focus: What happens in the denied area?

- The cost C_i incurred inside the denied area is composed of control effort and a time cost $\phi(t_f)$.
- ϕ is an increasing function to quantize the probability of missing the target or to penalize the case of taking too much time.
- Shift t_0 to 0 and use u to denote u_i for simplicity.
- Assume no perturbation w inside the denied area.

$$\underset{t_f, u(\cdot)}{\text{minimize}} \quad \int_0^{t_f} u^T(t) R u(t) dt + \phi(t_f)$$

$$\text{subject to} \quad \dot{x}(t) = Ax(t) + Bu(t), x(0)$$

$$x(t_f) \in \mathcal{D}_{TA}$$

$$0 < t_f \le T$$

Results: Optimal terminal time and control

We can find the optimal terminal time t_f^* and control u^* in the following steps:

1. Solve t_f^* such that $J^*(t_f) + \phi(t_f)$ is stationary at t_f^*. $J^*(t_f)$ is the minimum control effort for fixed t_f^*.

$$\underset{t_f}{\text{minimize}} \quad J^*(t_f) + \phi(t_f)$$

$$\text{subject to} \quad 0 < t_f \le T$$

2. Use t_f^* to solve a convex QCQP problem with optimal solution p^* being the optimal landing position in the target area. This problem is the finite dimensional dual problem of the minimum control effort problem.

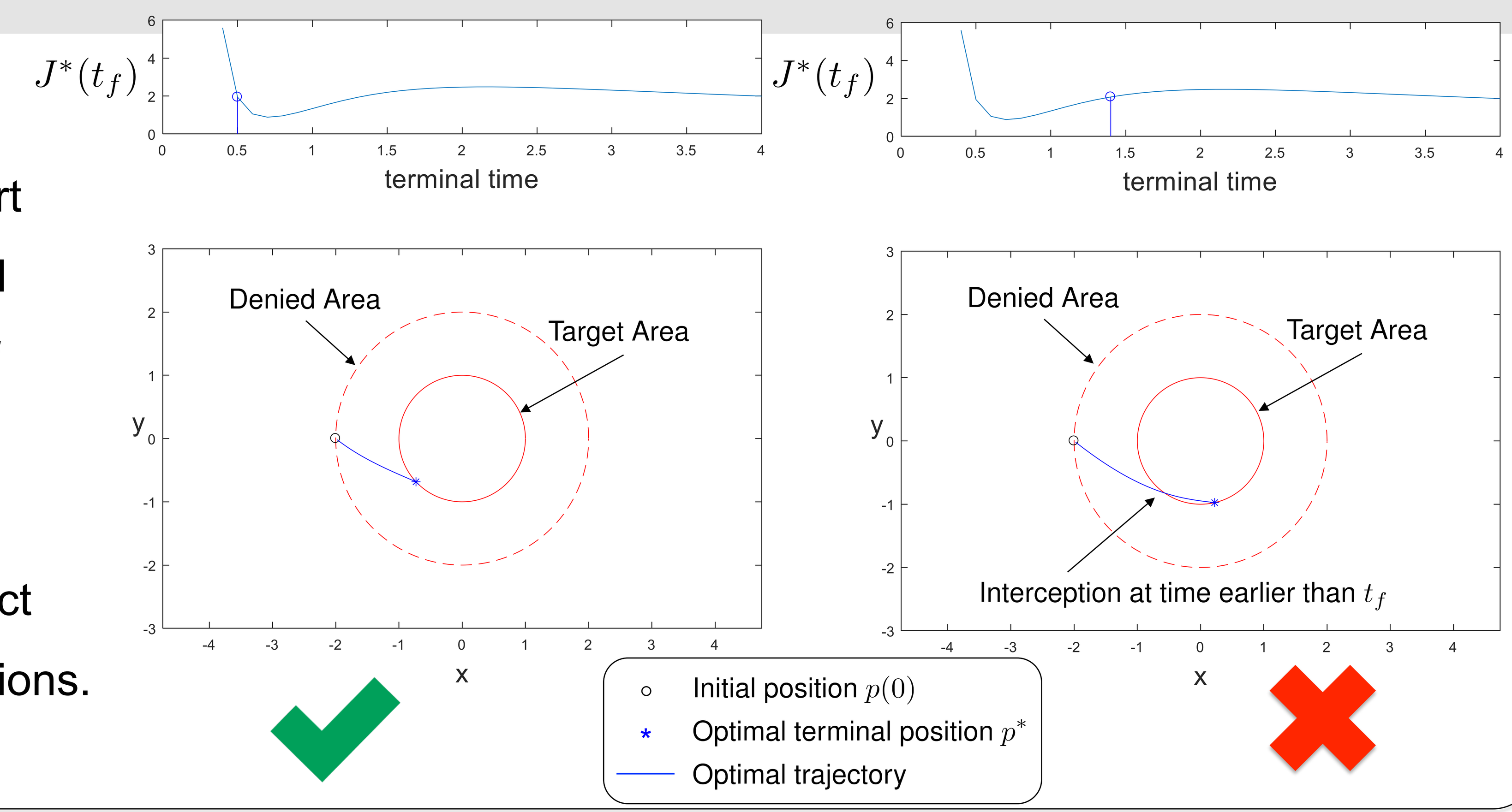
$$J^*(t_f) = \underset{p}{\text{minimize}} \quad (p - p_f)^T \Delta_1^{-1} (p - p_f)$$

$$\text{subject to} \quad p \in \mathcal{D}'_{TA}$$

3. The optimal control u^* is obtained through p^*:

$$u^*(t) = -R^{-1} B^T e^{A^T t_f^*} \Delta^{-1}(t_f^*) \begin{bmatrix} p^* - p_f \\ \Delta_2^T \Delta_1^{-1}(p^* - p_f) \end{bmatrix}$$

Note:
This formulation of the inner part problem guarantees the optimal trajectory is entering, instead of exiting, the target area at the optimal terminal time t_f^*.
Such entering behavior is correct according to empirical observations.



Future Work

1. Propose an iterative algorithm to find the optimal terminal time t_f^* with convergence analysis.
2. Study the cost incurred outside the denied area and complete the whole problem.
3. Conduct tests with quadcopters.

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