Jointly Optimal Quantization, Estimation and Control of Hidden Markov Chains

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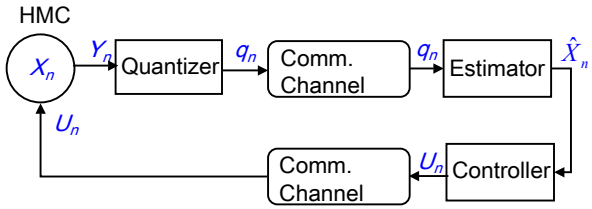
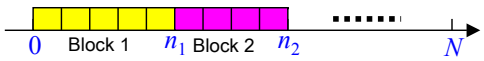
Introduction

- **Motivation**: multiple conflicting objectives in networked control systems
  - System performance
  - Communication bandwidth
  - Delay
  - Hardware/software complexity
- **Question:** how to strike the right balance?
- **Model:** Hidden Markov chains (HMCs)



- X_n: state
- Y_n: output
- q_n: quantized output
- U_n: control

Variable-Length Block-Quantization



- High coding-efficiency
- Inducing delay
- Equi-memory for the encoder and the decoder

Joint Quantization, Estimatin, and Control

- **Joint quantization and estimation**

$$J(\Pi_0, g) = E[\sum_{n=1}^{N} J^c(n) + l_d J^d(n) + l_e J^e(n)],$$

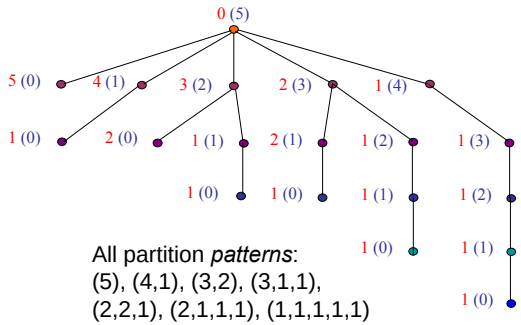
  - Π_0 : *A priori* PMF for X_0
  - γ : Quantization decision
  - $J^c(n)$: Communication cost at time n
  - $J^d(n)$: Delay cost evaluated at time n
  - $J^e(n)$: Estimation error for X_n
- **Joint quantization and control**

$$J(\Pi_0, x) = E[\sum_{n=1}^{N} l_c J^c(n) + J^p(n)],$$

  - ξ: Quantization/control decision
  - $J^c(n)$: Communication cost at time n
  - $J^p(n)$: Running cost at time n
- **Dynamic programming equation**

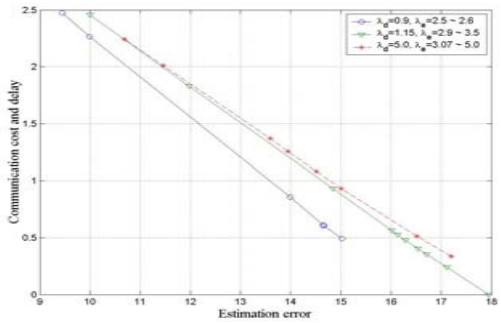
$$V_i(\Pi_{i-1}) = \min_{j \in \{1,\cdots,B\}} \min_{Q_j} \text{Cost for times } \{i,\ldots,i+j-1\} + V_{i+j}(\hat{\Pi}_{i+j-1})$$

  - Challenge: combinatorial problem
  - Tree-structured algorithm for enumeration of partitions

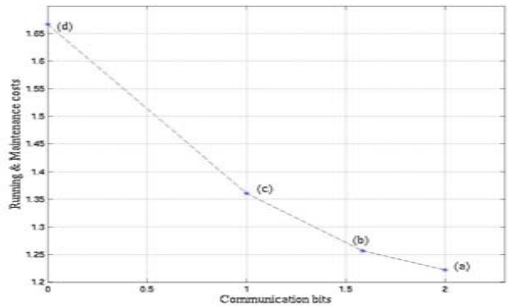


All partition *patterns*:
(5), (4,1), (3,2), (3,1,1),
(2,2,1), (2,1,1,1), (1,1,1,1,1)

Numerical Results



Communication cost & delay *vs.* estimation error.



Running cost *vs. communication bits.*

Conclusion and Future Work

- **Conclusion:**
  - Multi-objective problem in networked control systems
  - DP solutions and Pareto curve
- **Future work:**
  - Efficient approximation method