Photon Pulse-shape Engineering

Matt James
ARC Centre for Quantum Computation
and Communication Technology
Research School of Engineering
Australian National University

Joint work with

- Hendra Nurdin
- Naoki Yamamoto
- Guofeng Zhang
- John Gough
- Michael Hush
- Andre Carvalho
- Ruvi Lecamwasam

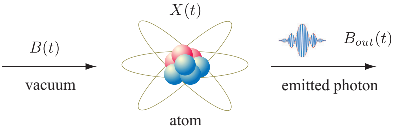
Outline

Introduction

Single-photon light fields have found important applications in quantum communication, quantum computation, quantum cryptography, and quantum metrology.

Photons are the fundamental units in quantum descriptions of light.

Photons are emitted, for example, from atoms.



A theory for spontaneous and stimulated emission goes back to Einstein.

Mathematically, photon states $|1_\xi\rangle$ may be 'created' from the vacuum $|0\rangle$:

$$|1_\xi\rangle = B^*(\xi)|0\rangle = \int_{-\infty}^{\infty} \xi(r) b^*(r) dr |0\rangle$$

The function ξ describes the shape of the photon wavepacket.

Fields $b(t)$ in a single photon state $|1_\xi\rangle$ have zero mean

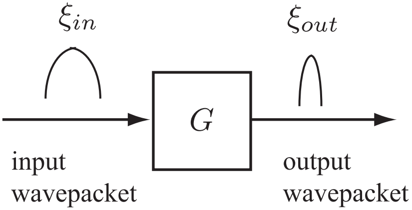
$$\langle 1_\xi | b(t) | 1_\xi \rangle = 0,$$

and intensity

$$\langle 1_\xi | b^*(t) b(t) | 1_\xi \rangle = |\xi(t)|^2$$

giving the probability of detection per unit time.

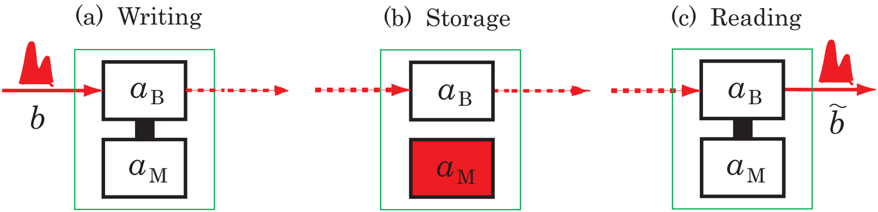
We are interested in how photons can be transformed (scattered)



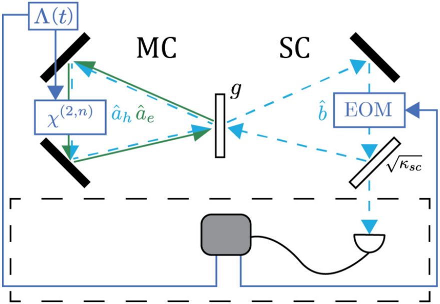
For example, a photon encountering a beamsplitter may be either transmitted or reflected (multichannel).

The determination of the state of the output field is a key problem.

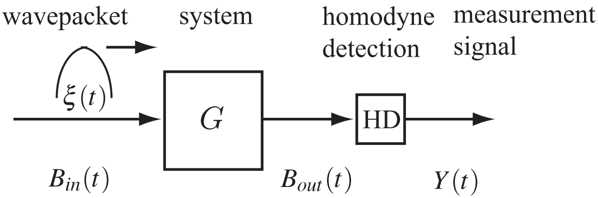
Wavepacket shapes are important for perfect absorption. This leads to a zero dynamics principle, which together with the concept of decoherence free subspaces may be applied to quantum memories.



An important experimental problem is to create photons on demand with prescribed wavepacket shapes, high efficiency, and high fidelity.



I also discuss the problem of finding the quantum filter for a system driven by a single photon state $|1_\xi\rangle$.



Some Quantum Mechanics

A little history

- Black body radiation (Plank)
- Photoelectric effect (Einstein)
- Atomic quantization (Bohr)
- Quantum probability (Born)
- Spontaneous and stimulated emission of light (Einstein)
- Matter waves (De Broglie)
- Matrix mechanics, uncertainty relation (Heisenberg)
- Wave functions (Schrodinger)
- Entanglement (EPR)
- Axiomatization, quantum probability (von Neumann)

Non-commuting observables

$$[Q, P] = QP - PQ = i\hbar\, I$$

Expectation

$$\langle Q \rangle = \int q|\psi(q,t)|^2 dq$$

Heisenberg uncertainty

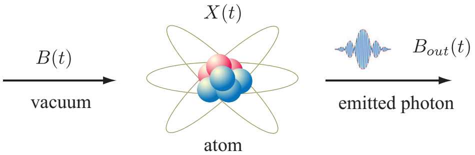
$$\Delta Q \Delta P \geq \frac{1}{2}|\langle i[Q,P]\rangle| = \frac{\hbar}{2}$$

Schrodinger equation

$$i\hbar \frac{\partial \psi(q,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(q,t)}{\partial q^2} + V(q)\psi(q,t)$$

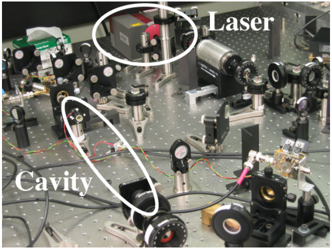
Quantum Stochastic Models

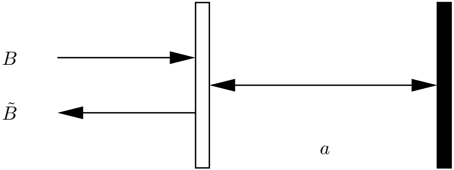
Recall that an *open quantum system* is a system interacting with an external environment. A basic example is an atom in an electromagnetic field.



We now describe dynamical models for open quantum systems in terms of quantum stochastic models in continuous time. Upon integration and expectation, these models yield quantum operation descriptions.

Quantum stochastic models describe open systems with inputs and outputs.





external free field with
input and output components

cavity mode

Quantum fields (boson)

Infinitely many quantum oscillators $b(t)$ (or $b(x)$ or $b(\omega)$)

Singular commutation relations

$$[b(t), b^*(t')] = \delta(t - t')$$

Quantum stochastic representation

$$B(t) = \int_0^t b(s)ds$$

Ito product rule

$$dB(t)dB^*(t) = dt$$

An open quantum system is specified by the triple

$$(S, L, H)$$

Schrodinger equation

$$dU(t) = \{LdB^*(t) - L^*dB(t) - (\frac{1}{2}L^*L + iH(u))dt\}U(t)$$

where $B(t)$ is a *quantum Wiener process*.

[Hudson-Parthasarathy (1984), Gardiner-Collett (1985)]

System operators X and output field $\tilde{B}(t)$ evolve in the Heisenberg picture:

$$\begin{aligned} X(t) = j_t(X) &= U^*(t)(X \otimes I)U(t) \\ \tilde{B}(t) &= U^*(t)(I \otimes B(t))U(t) \end{aligned}$$

Dynamics for $X(t) = j_t(X)$—a *quantum Markov process* (given u)—and output measurement signal $Y(t)$ (homodyne detection, for example):

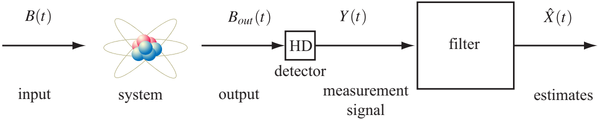
$$\begin{aligned} dj_t(X) &= j_t(\mathscr{L}^{u(t)}(X))dt + dB^*(t)j_t([X, L]) + j_t([L^*, X])dB(t) \\ dY(t) &= j_t(L + L^*)dt + dB(t) + dB^*(t) \end{aligned}$$

where

$$\mathscr{L}^u(X) = -i[X, H] + \frac{1}{2}L^*[X, L] + \frac{1}{2}[L^*, X]L$$

Measurement of the output field (e.g. amplitude quadrature observables)

$$Y(t) = \tilde{B}(t) + \tilde{B}^*(t)$$

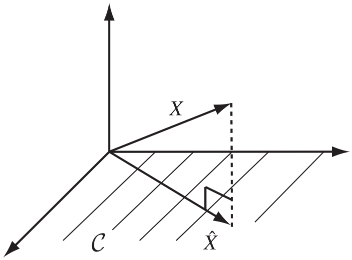


Conditional expectation

Let X commute with a commutative subspace $\mathscr{C}$. The conditional expectation

$$\hat{X} = \pi(X) = \mathbb{E}[X|\mathscr{C}]$$

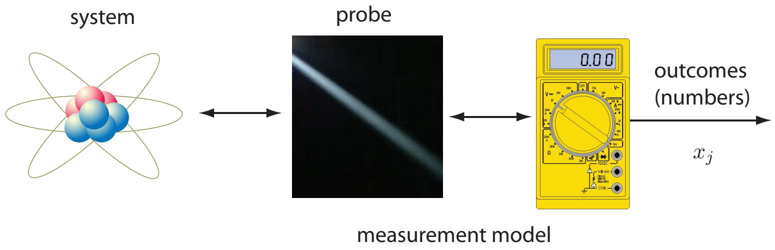
is the orthogonal projection of $X \in \mathscr{A}$ onto $\mathscr{C}$.



$\hat{X}$ is the minimum mean square estimate of X given $\mathscr{C}$.

By the spectral theorem, $\hat{X}$ is equivalent to a classical random variable.

Probe model for quantum measurement



Information about the system is transferred to the probe.

Quantum conditional expectation is well defined.

The von Neumann "projection postulate" is a special case.

In continuous time, this leads to *quantum filtering*.

Quantum conditional expectation

$$\pi_t(X) = \mathbb{E}[j_t(X) | Y(s), 0 \leq s \leq t]$$

Quantum filter [stochastic Schrodinger equation]

$$\begin{aligned} d\pi_t(X) &= \pi_t(\mathscr{L}^{u(t)}(X))dt \\ &\quad + (\pi_t(XL + L^* X) - \pi_t(X)\pi_t(L + L^*))(dY(t) - \pi_t(L + L^*)dt) \end{aligned}$$

[Belavkin (1993), Carmichael (1993)]

Open quantum harmonic oscillator

Single oscillator a interacting with field $b(t)$ - energy exchange:

$$H_{int} = i\sqrt{\gamma}(b^*(t)a - a^*b(t))$$

Dynamics (Ito form) [more to come on this]

$$\begin{aligned} dU(t) = \{&\sqrt{\gamma} a dB^*(t) - \sqrt{\gamma} a^* dB(t) \\ &-\frac{\gamma}{2} a^* a dt - i\omega a^* a dt\} U(t), \end{aligned}$$

Motion of oscillator mode $a(t) = U^*(t) a U(t)$

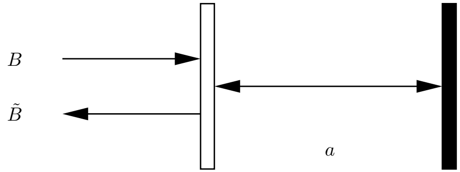
$$da(t) = -(\frac{\gamma}{2} + i\omega) a(t) dt - \sqrt{\gamma}\, dB(t)$$

The commutation relations are preserved

$$[a(t), a^*(t)] = [a, a^*] = 1$$

The output field $B_{out}(t) = U^*(t)B(t)U(t)$ is given by

$$dB_{out}(t) = \sqrt{\gamma}\, a(t) + dB(t)$$



external free field with
input and output components

cavity mode

The amplitude quadrature

$$Q(t) = B(t) + B^*(t)$$

is self-adjoint, and commutes with itself at different times ($[Q(t), Q(s)] = 0$), and so by the spectral theorem it turns out that $Q(t)$ is equivalent to a classical Wiener process (with respect to the vacuum state).

The phase quadrature

$$P(t) = -i(B(t) - B^*(t))$$

which is also equivalent to a classical Wiener process, but note that $[Q(t), P(t)] \neq 0$.

Quantum linear system

$$\dot{\breve{a}}(t) = A\breve{a}(t) + BS\breve{b}(t), \quad \breve{a}(t_0) = \breve{a},$$
$$\breve{b}_{out}(t) = C\breve{a}(t)dt + S\breve{b}(t)$$

where

$$\breve{a} = \begin{bmatrix} a \\ a^\sharp \end{bmatrix}, \quad \breve{b}(t) = \begin{bmatrix} b(t) \\ b(t)^\sharp \end{bmatrix}$$

is a vectors of system (mode) and field annihilation/creation operators, and A, B and C depend on physical parameters (Hamiltonian, field couplings, channel scattering):

$$S = \Delta(S_-, 0), C = \Delta(C_-, C_+), B = -C^\flat, A = -\frac{1}{2}C^\flat C - iJ_n H.$$

Notation:

$$\Delta\left(U,V\right)=\left[\begin{array}{cc} U & V \\ V^{\#} & U^{\#} \end{array}\right], \quad J=\left[\begin{array}{cc} I & 0 \\ 0 & -I \end{array}\right],$$
$$X^{\flat}=JX^{\dagger}J$$

Transfer function G has the form

$$G=\Delta(G^{-},G^{+})$$

and satisfies

$$G(\omega)^{\flat}G(\omega)=G(\omega)G(\omega)^{\flat}=I$$

This characterizes physical realizability.

Gaussian Mode States

Annihilation operator a, commutation relations $[a, a^*] = 1$.
Characteristic function for zero mean Gaussian:

$$\mathbb{E}[\exp(iz^* a + iza^*)] = \exp(\frac{1}{2} n(n+1)|z|^2 + m^* z^2 + m(z^*)^2)$$

where

$$n = n^* \geq 0, \quad |m|^2 \leq n(n+1)$$

Second moments

$$\begin{aligned} \mathbb{E}[aa^*] &= n+1 & \qquad \mathbb{E}[aa] &= m \\ \mathbb{E}[a^* a^*] &= m^* & \qquad \mathbb{E}[a^* a] &= n \end{aligned}$$

Gaussian Field States

Annihilation operator $b(t)$, commutation relations $[b(t), b^*(s)] = \delta(t-s)$.

$$B(f) = \int f^*(t) b(t) dt = \int f^*(t) dB(t)$$

Characteristic function

$$\mathbb{E}[\exp(iB(f) + iB^*(f))]$$
$$= \exp(-\tfrac{1}{2}\langle f|(2N+1)f\rangle - \tfrac{1}{2}\langle Mf|f^*\rangle - \tfrac{1}{2}\langle f^*|Mf\rangle)$$

N and M are operators on $\mathfrak{H} = L^2$ such that $N = N^* \geq 0$, $|M|^2 \leq N(N+1)$, and $[N, M] = 0$.

Second moments

$$\mathbb{E}[B(f)B^*(g)] = \langle f|Ng\rangle + \langle f|g\rangle, \quad \mathbb{E}[B(f)B(g)] = \langle Mf|g^*\rangle$$
$$\mathbb{E}[B^*(f)B^*(g)] = \langle f^*|Mg\rangle, \quad \mathbb{E}[B^*(g)B(f)] = \langle f|Ng\rangle$$

Absorption and Emission of Photons

Consider a cavity driven by a field $b(t)$ in a single photon state $|1_\xi\rangle$

$$\begin{aligned} \dot{a}(t) &= -\frac{\gamma}{2} a(t) - \sqrt{\gamma}\, b(t) \\ b_{out}(t) &= \sqrt{\gamma}\, a(t) + b(t) \end{aligned}$$

Solving for the cavity mode we have

$$a(t_1) = e^{-\frac{\gamma}{2} t_1} a_0 - \sqrt{\gamma} \int_{t_0}^{t_1} e^{-\frac{\gamma}{2}(t_1 - s)} b(s) ds$$

The cavity number operator is

$$n(t_1) = a^*(t_1) a(t_1)$$

Mean occupation in steady state, resulting from a pulse on $(-\infty, 0]$:

$$E[n(0)] = \langle n(0) \rangle = \langle 01_\xi | n(0) | 01_\xi \rangle$$

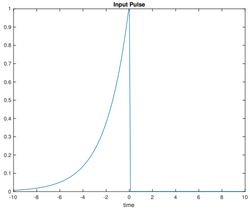
This may be computed as follows

$$\begin{aligned}
a(t_1)|0\rangle|1_\xi\rangle &= e^{-\frac{\gamma}{2}t_1} a_0|0\rangle|1_\xi\rangle - \sqrt{\gamma}\int_{t_0}^{t_1} e^{-\frac{\gamma}{2}(t_1-s)} b(s)ds|0\rangle|1_\xi\rangle \\
&= 0 - \sqrt{\gamma}\int_{t_0}^{t_1} e^{-\frac{\gamma}{2}(t_1-s)} b(s)ds|0\rangle_S \int_{-\infty}^{\infty} \xi(r)b^*(r)dr|0\rangle_F \\
&= -|0\rangle_S\sqrt{\gamma}\int_{t_0}^{t_1}\int_{-\infty}^{\infty} e^{-\frac{\gamma}{2}(t_1-s)}\xi(r)b(s)b^*(r)dsdr|0\rangle_F \\
&= -|0\rangle_S\sqrt{\gamma}\int_{t_0}^{t_1}\int_{-\infty}^{\infty} e^{-\frac{\gamma}{2}(t_1-s)}\xi(r)(b^*(s)b(r) + \delta(s-r))dsdr \\
&= 0 - |0\rangle_S\sqrt{\gamma}\int_{t_0}^{t_1} e^{-\frac{\gamma}{2}(t_1-r)}\xi(r)ds|0\rangle_F
\end{aligned}$$

Note the convolution.

Now suppose that the pulse is a rising exponential tuned to the cavity dynamics:

$$\xi(t) = \begin{cases} -\sqrt{\gamma}\, e^{\frac{\gamma}{2}t} & t \leq 0 \\ 0 & t > 0 \end{cases}$$



We then have for $t_0 \to -\infty$, $t_1 = 0$,

$$E[n(0)] = 1$$

The corresponds to perfect absorption: the cavity contains exactly one photon.

The transfer function from the input to the output field is

$$\Xi(s) = \frac{s - \frac{\gamma}{2}}{s + \frac{\gamma}{2}}$$

Stable pole $s = -\frac{\gamma}{2}$
Unstable zero: $s = \frac{\gamma}{2}$

$$\Xi(\frac{\gamma}{2}) = 0$$

The inverse Laplace transform of

$$\xi(s) = \frac{1}{s - \frac{\gamma}{2}}$$

is the rising exponential

$$\xi(t) = e^{\frac{\gamma}{2}t}, \quad -\infty < t \leq 0.$$

The output response is

$$\xi_{out}(s) = \frac{s - \frac{\gamma}{2}}{s + \frac{\gamma}{2}} \frac{1}{s - \frac{\gamma}{2}} = \frac{1}{s + \frac{\gamma}{2}}$$

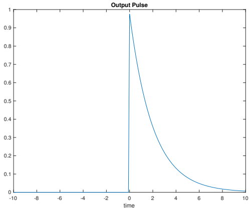
or

$$\xi_{out}(t) = 0, \quad -\infty < t \leq 0,$$

and

$$\xi_{out}(t) = e^{-\frac{\gamma}{2}t} \quad 0 \leq t < +\infty$$

Decaying exponential (for $t > 0$).



Cavity has emitted photon to ambient environment - emission.

Zero Dynamics Principle

Energy balance identity:

$$\int_{t_0}^{t} b_{out}^{\dagger}(s) b_{out}(s) ds + a^{\dagger}(t) a(t) = \int_{t_0}^{t} b_{in}^{\dagger}(s) b_{in}(s) ds + a^{\dagger}(t_0) a(t_0)$$

In terms of the envelope equations and coherent or photon input states,

$$\int_{t_0}^{t} |\beta_{out}(s)|^2 ds + |\alpha(t)|^2 = \int_{t_0}^{t} |\beta_{in}(s)|^2 ds + |\alpha(0)|^2$$

where

$$\begin{aligned} \dot{\alpha}(t) &= A\alpha(t) - C^{\dagger}\beta_{in}(t) \\ \beta_{out}(t) &= C\alpha(t) + \beta_{in}(t) \end{aligned}$$

If all input energy is stored internally, then we must have

$$\beta_{out}(\cdot) = 0$$

The output pulse is zero (though the output field will be vacuum).

Now $\beta_{out}(t) = 0$ implies

$$\beta_{in}(t) = -C\alpha(t) = -\alpha(t)^T C^T$$

and so the internal zero dynamics is

$$\dot{\alpha}(t) = (A + C^\dagger C)\alpha(t) = -A^\dagger \alpha(t)$$

On the time interval $(-\infty, t_1]$ we have

$$\beta_{in}(t) = -\alpha_1^T e^{-A^\sharp (t-t_1)} C^T \Theta(t_1 - t)$$

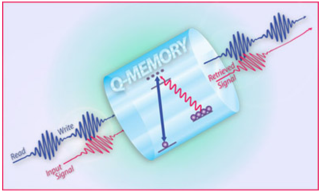
where $\Theta(\cdot)$ is the Heavyside step function, and

$$\beta_{out}(t) = 0$$

The input $\beta_{in}(\cdot)$ is a is a rising exponential.

Quantum Memory

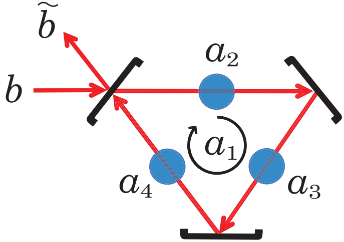
- Stores quantum states
- For example, optical states temporarily mapped onto atomic states.
- Applications include quantum repeaters and other devices in quantum information systems.
- Excellent experimental progress.



[http://archive.nrc-cnrc.gc.ca/eng/news/sims/2010/03/07/qmemory.html]

Perfect Quantum Memory using Atomic Ensembles

Networks of atomic ensembles may be engineered to have *tunable* decoherence-free subsystems.



Combined with input matched pulse shapes designed using the zero dynamics principle, perfect quantum memories can be realized.

[Yamamoto and James, 2014]

In suitable coordinates this ensemble network is described by a finite dimensional linear quantum system of the form:

$$\frac{d}{dt}\begin{bmatrix} a_B \\ a_M \end{bmatrix} = \begin{bmatrix} A_B & \Delta A_{BM} \\ \Delta A_{MB} & \Delta A_M \end{bmatrix}\begin{bmatrix} a_B \\ a_M \end{bmatrix} - \begin{bmatrix} C_B^\dagger \\ 0 \end{bmatrix} b_{in}$$

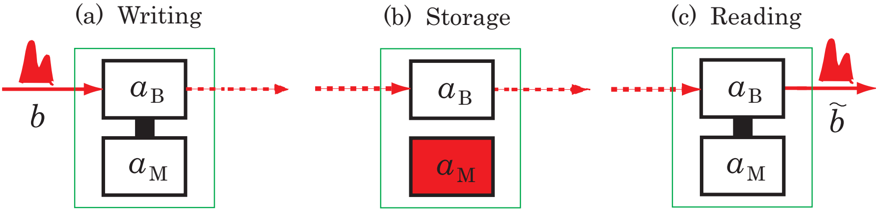
$$b_{out} = C_B a_B + b_{in}$$

The mode a_M does not appear in the output.
When $\Delta = 0$ mode a_M is decoherence free:

$$\frac{d}{dt}\begin{bmatrix} a_B \\ a_M \end{bmatrix} = \begin{bmatrix} A_B & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a_B \\ a_M \end{bmatrix} - \begin{bmatrix} C_B^\dagger \\ 0 \end{bmatrix} b_{in}$$

[Yamamoto 2012]

During the write and read stages, $\Delta \neq 0$ and all modes interact with the input.

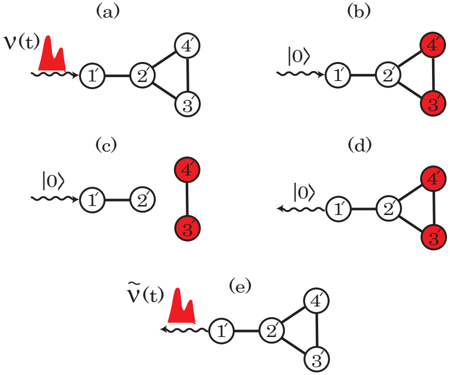


For storage, $\Delta = 0$ isolating the mode a_M from the input.

By suitably shaping the input pulse

$$\nu(t) = \sum_k s_k \nu_k(t)$$

input field states may be perfectly stored and retrieved from specified modes of the decoherence free subsystem.



Define the vector of rising exponentials

$$\beta(t) = e^{-A^{\sharp}(t-t_1)} C^T \Theta(t_1 - t)$$

and $\alpha_1 = (s_1, \ldots, s_n)^T$.

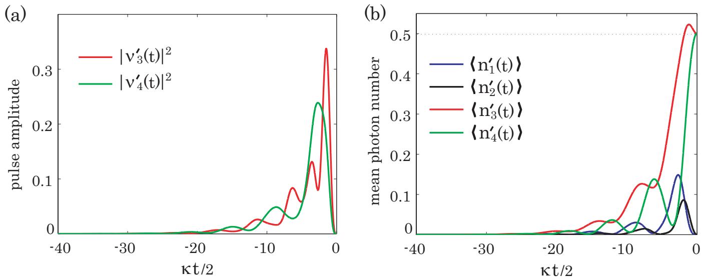
Then the input pulse

$$\beta_{in}(t) = -\alpha_1^T \beta(t) = \sum_k s_k \beta_k(t)$$

is perfectly transferred into the memory on the time interval $(-\infty, t_1]$.

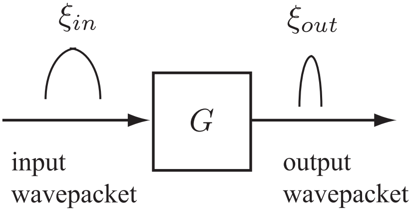
The data may be stored internally on a time interval $[t_1, t_2]$, and subsequently perfectly retrieved on $[t_2, \infty)$.

Pulse amplitude and mean internal photon number during the write stage.

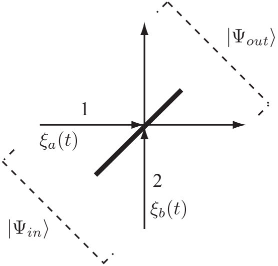


Wavepacket Transformation

Linear optical devices (for example) may be used to shape photon wavepackets.



Example: beamsplitter



$$\xi_{in}^{-}(t) = \begin{bmatrix} \xi_a(t) & 0 \\ 0 & \xi_b(t) \end{bmatrix} \rightarrow \xi_{out}^{-}(t) = \frac{1}{\sqrt{2}} \begin{bmatrix} \xi_a(t) & \xi_b(t) \\ -\xi_a(t) & \xi_b(t) \end{bmatrix}$$

$$\begin{aligned} |\Psi_{in}\rangle &= (B_1^*(\xi_a)|0\rangle) \otimes |0\rangle + |0\rangle \otimes (B_2^*(\xi_b)|0\rangle) \qquad \rightarrow \\ |\Psi_{out}\rangle &= \frac{1}{2}(B_1^*(\xi_a)B_1^*(\xi_b)|0\rangle) \otimes |0\rangle + \frac{1}{\sqrt{2}}(B_1^*(\xi_a)|0\rangle) \otimes (B_2^*(\xi_b)|0\rangle) \\ &\quad -(1-\frac{1}{\sqrt{2}})(B_1^*(\xi_b)|0\rangle) \otimes (B_2^*(\xi_a)|0\rangle) - \frac{1}{2}|0\rangle \otimes (B_2^*(\xi_a)B_2^*(\xi_b)|0\rangle) \end{aligned}$$

Multichannel passive case with $G^+ = 0$:

$$G = \Delta(G^-, 0)$$

Matrix of pulse shapes:

$$\xi_{in} = \Delta(\xi_{in}^-, 0)$$

where

$$\xi_{in}^- = [\xi_{in,jk}^-]$$

describes pulse shapes in each channel, and cross-channel superpositions.

Input state

$$|\Psi_{in}\rangle = \Pi_k \Sigma_j B_j^*(\xi_{in,jk}^-)|0\rangle$$

where $\xi_{in,jk}^-$ satisfy a normalization condition.

The output state (stationary) produced by a passive linear quantum system is given by

$$|\Psi_{out}\rangle = \Pi_k \Sigma_j B_j^*(\xi^-_{out,jk})|0\rangle$$

where

$$\xi^-_{out}(\omega) = G^-(\omega)\xi^-_{in}(\omega)$$

The output state is again normalized.

The proof involves careful use of *stable* inverses of linear systems.

[Zhang-James, April 2011]

In general $G^+ \neq 0$ since the linear quantum system may contain *active* elements.

Degenerate parametric amplifiers (active devices) produce Gaussian states $|\Phi_R\rangle$ from the vacuum, characterized by a correlation function $R(\tau)$:

$$|0\rangle \quad \to \quad |\Phi_R\rangle$$

The states produced from a single photon state are non-trivial:

$$|1_\xi\rangle \quad \to \quad (B^*(\xi^-_{out}) - B(\xi^+_{out}))|\Phi_R\rangle$$

So we introduce a class $\mathscr{F}$ of *pulsed-Gaussian* states $|\Psi\rangle$ of the form

$$|\Psi\rangle = \Pi_k \Sigma_j (B_j^*(\xi_{jk}^-) - B_j(\xi_{jk}^+))|\Phi_R\rangle$$

where we write

$$\xi = \Delta(\xi^-, \xi^+)$$

States are characterized by a pair

$$|\Psi\rangle \equiv (\xi, R)$$

that satisfy a normalization condition.

The class $\mathscr{F}$ of pulsed-Gaussian states is invariant under the steady state action of a quantum linear system.

The state transfer is given by

$$|\Psi_{in}\rangle \equiv (\xi_{in}, R_{in}) \quad \rightarrow \quad |\Psi_{out}\rangle \equiv (\xi_{out}, R_{out})$$

where

$$\begin{aligned} \xi_{out}(\omega) &= G(\omega)\xi_{in}(\omega) \\ R_{out}(\omega) &= G(\omega)R_{in}(\omega)G(\omega)^{\dagger} \end{aligned}$$

Expected values of quadratic forms, field intensities, etc may be explicitly evaluated.

[Zhang-James, 2011]

Wavepacket Shaping

Given a desired wavepacket shape $\xi(\cdot)$, how do we create a photon with this shape?

One approach is to modulate the coupling of the system to the field.

Consider the two-level (qubit) system

$$(S, L, H) = (I, \lambda(t)\sigma_-, 0)$$

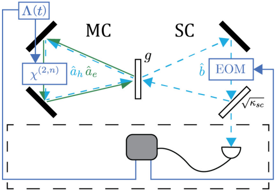
initially prepared in its excited state $|\uparrow\rangle$, where

$$\lambda(t) = \frac{1}{\sqrt{\int_t^\infty |\xi(s)|^2 \, ds}} \xi(t).$$

Then the desired photon is emitted:

$$|\psi_\infty\rangle = |\downarrow\rangle \otimes B^\dagger(\xi)|0\rangle \equiv |\downarrow\rangle \otimes |1_\xi\rangle.$$

In practice, a basic experimental challenge is to create photons on demand with high efficiency and fidelity. One way to reduce the randomness inherent in the photon creation process is to use feedback. [Furusawa et al, 2013]



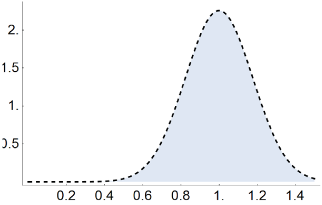
Successful charging occurs at at random herald time τ_{herald}. If the desired release time is T_d, the user should store the photon energy for a time

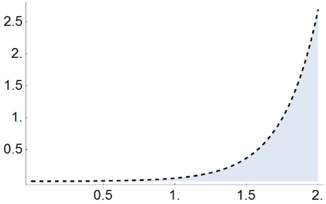
$$\tau_{store} = T_d - \tau_{herald}$$

This is a simple but important example of feedback control.

This experimental setup may be modulated to create photons with desired wavepacket shapes on demand.

[Lecamwasam, Hush, James, Carvalho 2017]

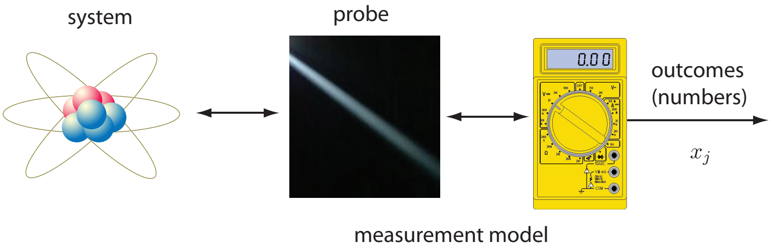




Single Photon Filtering

Quantum filtering

[Belavkin, 1980's]



Information about the system is transferred to the probe.

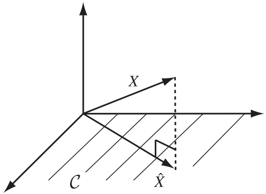
The filtering problem is to use the measurement data $Y(s)$, $0 \leq s \leq t$ to estimate system variables X at time $t \geq 0$.

Quantum conditional expectation

$X(t)$ commutes with $Y(s), 0 \leq s \leq t$. The conditional expectation

$$\hat{X}(t) = \pi_t(X) = \mathbb{E}[X(t) | Y(s), 0 \leq s \leq t]$$

is well defined.

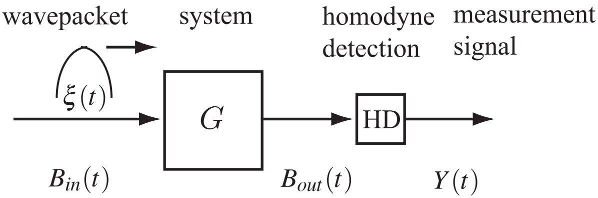


$\hat{X}(t)$ is the minimum mean square estimate of $X(t)$ given $Y(s), 0 \leq s \leq t$.

Quantum filter due to V.P. Belavkin - vacuum input $|0\rangle$.

Single Photon Input

We now consider the problem of finding the quantum filter if the vacuum field state $|0\rangle$ is replaced by a single photon state $|1_\xi\rangle$.



Single Photon Master Equation and Filter

[Gough, James, Nurdin]

Single photon field state:

$$|1_\xi\rangle = B^\dagger(\xi)|0\rangle$$

The basic action of the annihilation operator is

$$b(t)|1_\xi\rangle = \xi(t)|0\rangle$$

Cross expectations

$$\varpi_t^{jk}(X) = \mathbb{E}_{jk}[j_t(X)] = \langle \eta\phi_j | A | \eta\phi_k \rangle$$

$$\phi_j = \left| \begin{array}{ll} |0\rangle, & j=0; \\ |1_\xi\rangle, & j=1. \end{array} \right\}$$

Using the quantum stochastic calculus, we can derive the master equation

$$
\begin{aligned}
\dot{\varpi}_t^{11}(X) &= \varpi_t^{11}(\mathscr{L}X) + \varpi_t^{01}(S^\dagger[X,L])\xi^*(t) \\
&\quad + \varpi_t^{10}([L^\dagger,X]S)\xi(t) + \varpi_t^{00}(S^\dagger XS - X)|\xi(t)|^2 \\
\dot{\varpi}_t^{10}(X) &= \varpi_t^{10}(\mathscr{L}X) + \varpi_t^{00}(S^\dagger[X,L])\xi^*(t) \\
\dot{\varpi}_t^{01}(X) &= \varpi_t^{01}(\mathscr{L}X) + \varpi_t^{00}([L^\dagger,X]S)\xi(t) \\
\dot{\varpi}_t^{00}(X) &= \varpi_t^{00}(\mathscr{L}X)
\end{aligned}
$$

In contrast to the vacuum case, this is a *system of coupled equations*.

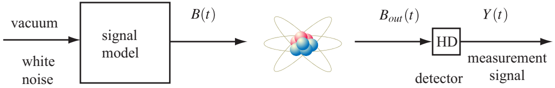
[Gheri, et al, 1998]

We wish to determine the *single photon conditional expectation*

$$\pi_t^{11}(X) = \mathbb{E}_{\eta\xi}[X(t)|Y(s), 0 \le s \le t]$$

Signal model: a two-level system initial state $\rho_a = |\uparrow\rangle\langle\uparrow|$ (excited state)

$$(S_M, L_M, H_M) = (I, \lambda(t)\,\sigma_-, 0)$$



The consistency requirement

$$\mathbb{E}_{\eta\xi}[X(t)] = \mathbb{E}_{\uparrow\eta 0}[\tilde{U}^\dagger(t)(I \otimes X \otimes I)\tilde{U}(t)]$$

is satisfied for a suitable choice of $\lambda(t)$ (as discussed earlier).

Extended system conditional expectation

$$\tilde{\pi}_t(A \otimes X) = \mathbb{E}_{\uparrow\eta 0}[\tilde{U}^\dagger(t)(A \otimes X)\tilde{U}(t) | I \otimes Y(s), 0 \leq s \leq t]$$

Filtering equation is standard, but using parameters for extended system:

$$\begin{aligned} d\tilde{\pi}_t(A \otimes X) &= \tilde{\pi}_t(\mathscr{L}_{G_T}(A \otimes X))dt \\ &\quad +(\tilde{\pi}_t(A \otimes XL_T + L_T^\dagger A \otimes X) \\ &\quad -\tilde{\pi}_t(L_T + L_T^\dagger)\tilde{\pi}_t(A \otimes X))dW(t), \end{aligned}$$

where $dW(t) = dY(t) - \tilde{\pi}_t(L_T + L_T^\dagger)dt$.

The single photon conditional expectation is given by

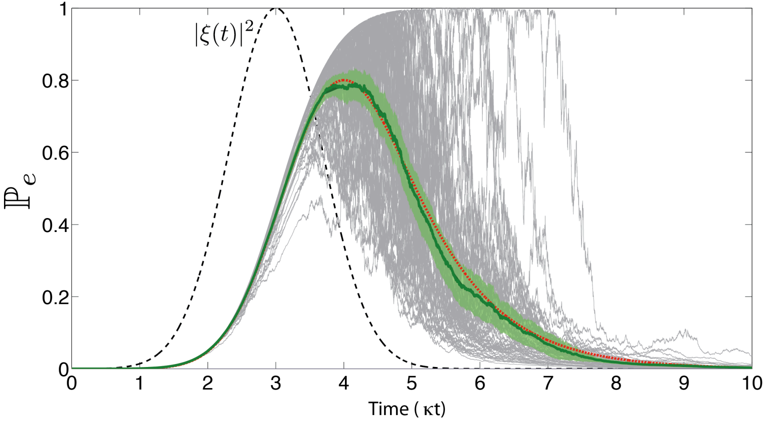
$$\pi_t^{11}(X) = \tilde{\pi}_t(I \otimes X)$$

and may be computed from the *single photon quantum filter*:

$$\begin{aligned}
d\pi_t^{11}(X) =& \big\{\pi_t^{11}(\mathcal{L}X) + \pi_t^{01}(S^\dagger[X,L])\xi^*(t) + \pi_t^{10}([L^\dagger,X]S)\xi(t) + \pi_t^{00}(S^\dagger XS - X)|\xi(t)|^2\big\}dt \\
&+ \big\{\pi_t^{11}(XL + L^\dagger X) + \pi_t^{01}(S^\dagger X)\xi^*(t) + \pi_t^{10}(XS)\xi(t) - \pi_t^{11}(X)K_t\big\}dW(t), \\
d\pi_t^{10}(X) =& \big\{\pi_t^{10}(\mathcal{L}X) + \pi_t^{00}(S^\dagger[X,L])\xi^*(t)\big\}dt \\
&+ \big\{(\pi_t^{10}(XL + L^\dagger X) + \pi_t^{00}(S^\dagger X)\xi^*(t) - \pi_t^{10}(X)K_t\big\}dW(t), \\
d\pi_t^{01}(X) =& \big\{\pi_t^{01}(\mathcal{L}X) + \pi_t^{00}(S^\dagger[X,L])\xi^*(t)\big\}dt \\
&+ \big\{(\pi_t^{01}(XL + L^\dagger X) + \pi_t^{00}(S^\dagger X)\xi^*(t) - \pi_t^{01}(X)K_t\big\}dW(t), \\
d\pi_t^{00}(X) =& \pi_t^{00}(\mathcal{L}X)dt + \big\{\pi_t^{00}(XL + L^\dagger X) - \pi_t^{00}(X)K_t\big\}dW(t).
\end{aligned}$$

$$dW(t) = dY(t) - (\pi_t^{11}(L + L^*) + \pi_t^{01}(S)\xi(t) + \pi_t^{10}(S^*)\xi^*(t))dt$$

Unconditional and conditional evolution of system number operator $n = \sigma_+ \sigma_-$.



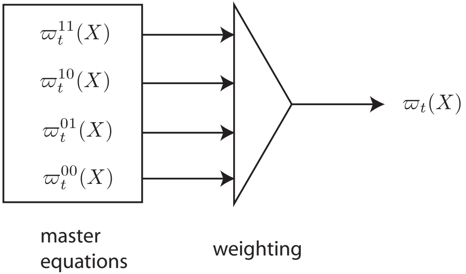
[Gough, James, Nurdin, 2012]

For combinations of single photon and vacuum field states, we use the field density operator

$$\rho_{\text{field}} = \sum_{jk} \gamma_{kj} |\phi_j\rangle\langle\phi_k|$$

The unconditional expectation can be computed from a sum:

$$\mathbb{E}_{\eta\rho_{\text{field}}}[X(t)] = \sum_{jk} \gamma_{jk} \varpi_t^{jk}(X)$$



The corresponding conditional expectation is a form of Bayes' rule:

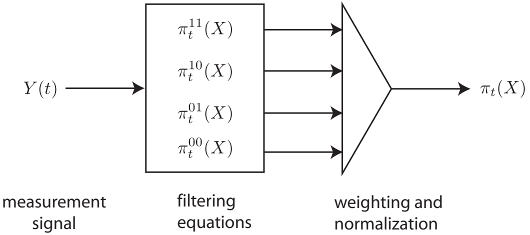
$$\pi_t(X) = \frac{\tilde{\pi}_t(R(t) \otimes X)}{\tilde{\pi}_t(R(t) \otimes I)}$$

where

$$R(t) = \sum_{jk} \frac{\gamma_{jk}}{w_{jk}(t)} Q_{jk}$$

for certain $w_{jk}(t)$, Q_{jk}. Then

$$\pi_t(X) = \frac{\sum_{j,k} \gamma_{jk} \pi_t^{jk}(X)}{\sum_{j,k} \gamma_{jk} \pi_t^{jk}(I)}.$$



Absorption, Multichannel Transfer, Amplification

Other related projects:

- Nurdin, James, Yamamoto, Perfect single device absorber of arbitrary traveling single photon fields with a tunable coupling parameter: A QSDE approach, CDC 2016.
- Yamamoto, Nurdin, James, Quantum state transfer for multi-input linear quantum systems, CDC 2016.
- Li, Carvalho, James, Continuous-mode operation of a noiseless linear amplifier, PRA 2017.

Thank you for your attention!