What Have We Learned from Reverse-Engineering the Internet's Inter-domain Routing Protocol?

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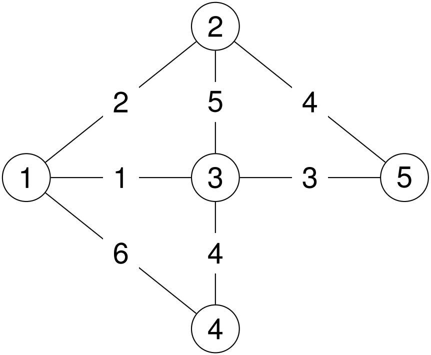
The Answer!

- Hitherto algebraic path problems have focused on global optimality: finding best paths over all possible paths.
- Another notion is local optimality : each node gets the best paths it can obtain given what is available from its neighbors (*routing in equilibrium*).
- The two notions coincide in the classical theory.

We have learned that in some cases ...

- Algebraic path problems admit unique local optima that are distinct from global optima.
- Local optima represent a more meaningful solution.
- We can find local optima in polynomial time.

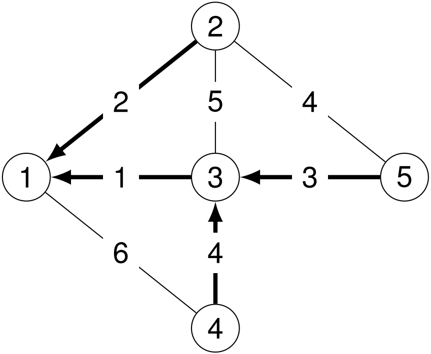
Shortest paths example, $\mathrm{sp} = (\mathbb{N}^{\infty},\ \min,\ +)$



The adjacency matrix

$$\mathbf{A} = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 2 & 1 & 6 & \infty \\ 2 & \infty & 5 & \infty & 4 \\ 1 & 5 & \infty & 4 & 3 \\ 6 & \infty & 4 & \infty & \infty \\ \infty & 4 & 3 & \infty & \infty \end{bmatrix} \end{array}$$

Shortest paths example, continued



Bold arrows indicate the shortest-path tree rooted at 1.

The routing matrix

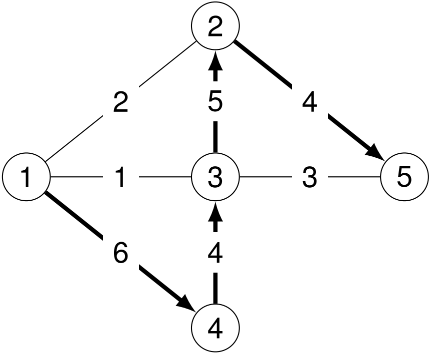
$$\mathbf{A}^* = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 5 & 4 \\ 2 & 0 & 3 & 7 & 4 \\ 1 & 3 & 0 & 4 & 3 \\ 5 & 7 & 4 & 0 & 7 \\ 4 & 4 & 3 & 7 & 0 \end{bmatrix} \end{array}$$

Matrix $\mathbf{A}^*$ solves this global optimality problem:

$$\mathbf{A}^*(i,\ j) = \min_{p \in P(i,\ j)} w(p),$$

where $P(i,\ j)$ is the set of all paths from i to j.

Widest paths example, $(\mathbb{N}^{\infty},\ \max,\ \min)$



Bold arrows indicate the widest-path tree rooted at 1.

The routing matrix

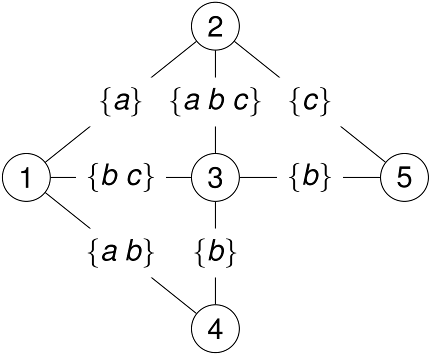
$$
\mathbf{A}^* = \begin{array}{c@{\;}c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\left[\begin{array}{ccccc}
\infty & 4 & 4 & 6 & 4 \\
4 & \infty & 5 & 4 & 4 \\
4 & 5 & \infty & 4 & 4 \\
6 & 4 & 4 & \infty & 4 \\
4 & 4 & 4 & 4 & \infty
\end{array} \right]
\end{array}
$$

Matrix $\mathbf{A}^*$ solves this global optimality problem:

$$\mathbf{A}^*(i,\ j) = \max_{p \in P(i,\ j)} w(p),$$

where $w(p)$ is now the minimal edge weight in p.

Fun example, $(2^{\{a, b, c\}}, \cup, \cap)$



We want a Matrix $\mathbf{A}^*$ to solve this global optimality problem:

$$\mathbf{A}^*(i, j) = \bigcup_{p \in P(i, j)} w(p),$$

where $w(p)$ is now the intersection of all edge weights in p.

For $x \in \{a, b, c\}$, interpret $x \in \mathbf{A}^*(i, j)$ to mean that there is at least one path from i to j with x in every arc weight along the path.

Fun example, $(2^{\{a,\ b,\ c\}},\ \cup,\ \cap)$

The matrix $\mathbf{A}^*$

$$
\begin{array}{c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} &
\begin{bmatrix}
\{a\ b\ c\} & \{a\ b\ c\} & \{a\ b\ c\} & \{a\ b\} & \{b\ c\} \\
\{a\ b\ c\} & \{a\ b\ c\} & \{a\ b\ c\} & \{a\ b\} & \{b\ c\} \\
\{a\ b\ c\} & \{a\ b\ c\} & \{a\ b\ c\} & \{a\ b\} & \{b\ c\} \\
\{a\ b\} & \{a\ b\} & \{a\ b\} & \{a\ b\ c\} & \{b\} \\
\{b\ c\} & \{b\ c\} & \{b\ c\} & \{b\} & \{a\ b\ c\}
\end{bmatrix}
\end{array}
$$

Semirings

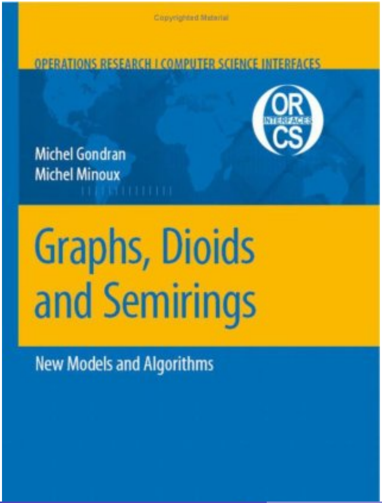
A few examples

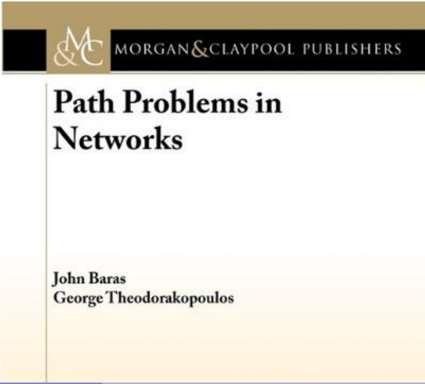
name	S	$\oplus$,	$\otimes$	$\overline{0}$	$\overline{1}$	possible routing use
sp	$\mathbb{N}^{\infty}$	min	$+$	∞	0	minimum-weight routing
bw	$\mathbb{N}^{\infty}$	max	min	0	∞	greatest-capacity routing
rel	$[0, 1]$	max	$\times$	0	1	most-reliable routing
use	$\{0, 1\}$	max	min	0	1	usable-path routing
	2^W	$\cup$	$\cap$	$\{\}$	W	shared link attributes?
	2^W	$\cap$	$\cup$	W	$\{\}$	shared path attributes?

Path problems focus on global optimality

$$\mathbf{A}^*(i, j) = \bigoplus_{p \in P(i, j)} w(p)$$

Recommended Reading





What algebraic properties are needed for efficient computation of global optimality?

Distributivity

$$\begin{array}{lcrcl} \mathrm{L.D} & : & a \otimes (b \oplus c) & = & (a \otimes b) \oplus (a \otimes c), \\ \mathrm{R.D} & : & (a \oplus b) \otimes c & = & (a \otimes c) \oplus (b \otimes c). \end{array}$$

What is this in $\mathrm{sp} = (\mathbb{N}^{\infty},\ \min,\ +)$?

$$\begin{array}{lcrcl} \mathrm{L.DIST} & : & a\ +\ (b\ \min\ c) & = & (a\ +\ b)\ \min\ (a\ +\ c), \\ \mathrm{R.DIST} & : & (a\ \min\ b)\ +\ c & = & (a\ +\ c)\ \min\ (b\ +\ c). \end{array}$$

But some realistic metrics are not distributive! What can we do?

Left-Local Optimality

Say that $\mathbf{L}$ is a left locally-optimal solution when

$$\mathbf{L} = (\mathbf{A} \otimes \mathbf{L}) \oplus \mathbf{I}.$$

That is, for $i \neq j$ we have

$$\mathbf{L}(i,\ j) = \bigoplus_{q \in V} \mathbf{A}(i,\ q) \otimes \mathbf{L}(q,\ j)$$

- $\mathbf{L}(i,\ j)$ is the best possible value given the values $\mathbf{L}(q,\ j)$, for all out-neighbors q of source i.
- Rows $\mathbf{L}(i,\ \_)$ represents **out-trees <u>from</u>** i (think Bellman-Ford).
- Columns $\mathbf{L}(\_,\ i)$ represents **in-trees <u>to</u>** i.
- Works well with hop-by-hop forwarding from i.

Right-Local Optimality

Say that **R** is a right locally-optimal solution when

$$\mathbf{R} = (\mathbf{R} \otimes \mathbf{A}) \oplus \mathbf{I}.$$

That is, for $i \neq j$ we have

$$\mathbf{R}(i,\ j) = \bigoplus_{q \in V} \mathbf{R}(i,\ q) \otimes \mathbf{A}(q,\ j)$$

- $\mathbf{R}(i,\ j)$ is the best possible value given the values $\mathbf{R}(q,\ j)$, for all in-neighbors q of destination j.
- Rows $\mathbf{L}(i,\ \_)$ represents **out-trees <u>from</u>** i (think Dijkstra).
- Columns $\mathbf{L}(\_,\ i)$ represents **in-trees <u>to</u>** i.
- Does not work well with hop-by-hop forwarding from i.

With and Without Distributivity

With

For semirings, the three optimality problems are essentially the same — locally optimal solutions are globally optimal solutions.

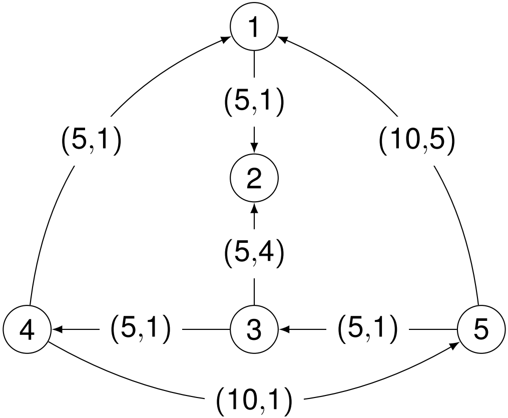
$$\mathbf{A}^* = \mathbf{L} = \mathbf{R}$$

Without

Suppose that we drop distributivity and $\mathbf{A}^*$, $\mathbf{L}$, $\mathbf{R}$ exist. It may be the case they they are all distinct.

Health warning : matrix multiplication over structures lacking distributivity is not associative!

Example



(bandwidth, distance) with lexicographic order (bandwidth first).

Global optima

$$\mathbf{A}^* = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} \quad 1 \quad & \quad 2 \quad & \quad 3 \quad & \quad 4 \quad & \quad 5 \quad \end{array} \\ \left[\begin{array}{ccccc} (\infty, 0) & (5, 1) & (0, \infty) & (0, \infty) & (0, \infty) \\ (0, \infty) & (\infty, 0) & (0, \infty) & (0, \infty) & (0, \infty) \\ (5, 2) & (5, 3) & (\infty, \ 0) & (5, 1) & (5, 2) \\ (10, 6) & (5, 2) & (5, 2) & (\infty, 0) & (10, 1) \\ (10, 5) & (5, 4) & (5, 1) & (5, 2) & (\infty, 0) \end{array} \right] \end{array},$$

Left local optima

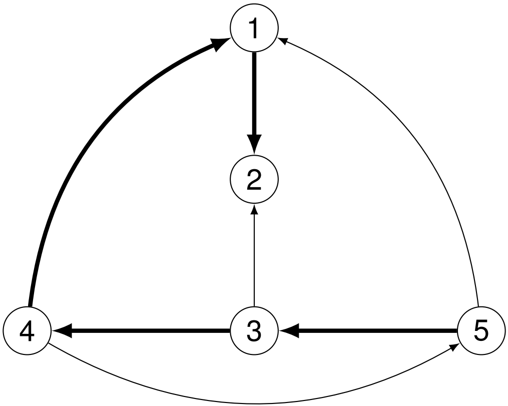
$$\mathbf{L} = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} (\infty, 0) & (5, 1) & (0, \infty) & (0, \infty) & (0, \infty) \\ (0, \infty) & (\infty, 0) & (0, \infty) & (0, \infty) & (0, \infty) \\ (\mathbf{5}, \mathbf{7}) & (5, 3) & (\infty, 0) & (5, 1) & (5, 2) \\ (10, 6) & (5, 2) & (5, 2) & (\infty, 0) & (10, 1) \\ (10, 5) & (5, 4) & (5, 1) & (5, 2) & (\infty, 0) \end{bmatrix} \end{array},$$

Entries marked in **bold** indicate those values which are not globally optimal.

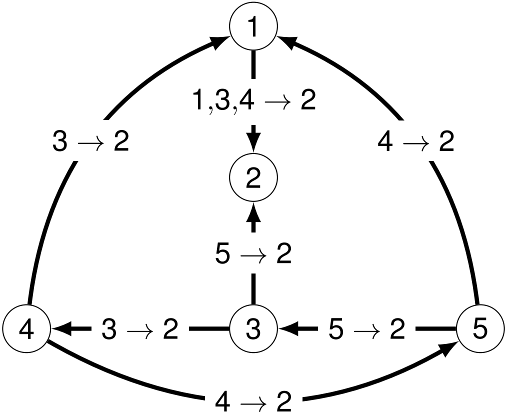
Right local optima

$$\mathbf{R} = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \left[\begin{array}{ccccc} (\infty, 0) & (5, 1) & (0, \infty) & (0, \infty) & (0, \infty) \\ (0, \infty) & (\infty, 0) & (0, \infty) & (0, \infty) & (0, \infty) \\ (5, 2) & (5, 3) & (\infty,\ 0) & (5,\ 1) & (5, 2) \\ (10, 6) & (\mathbf{5}, \mathbf{6}) & (5, 2) & (\infty, 0) & (10, 1) \\ (10, 5) & (\mathbf{5}, \mathbf{5}) & (5, 1) & (5, 2) & (\infty, 0) \end{array} \right] \end{array},$$

Left-locally optimal paths to node 2



Right-locally optimal paths to node 2



Inter-domain routing in the Internet

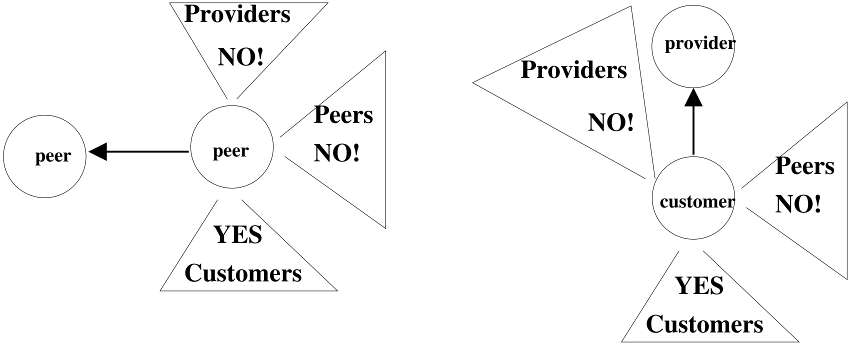
The Border Gateway Protocol (BGP)

- In the distributed Bellman-Ford family.
- Hard-state (not refresh based).
- Complex policy and metrics.
- Primary requirement: connectivity should not violate the economic relationships between autonomous networks.
- At a very high-level, the metric combines economics and traffic engineering.
- This is implemented using a lexicographic product, where economics is most significant.

Simplified model (Gao and Rexford)

- **customer route** : from somebody paying you for transit services.
- **provider route** : from somebody you are paying for transit services.
- **peer route** : from a competitor.
  - If you are at top of food chain you are forced to do this.
  - Smaller networks do this to reduce their provider charges.
- **customer** $<$ **peer** $<$ **provider**

Route visibility restriction



The primary source for violations of distributivity.

Bellman-Ford can compute left-local solutions

$$\begin{array}{rcl} \mathbf{A}^{[0]} & = & \mathbf{I} \\ \mathbf{A}^{[k+1]} & = & (\mathbf{A} \otimes \mathbf{A}^{k}) \oplus \mathbf{I}, \end{array}$$

- Bellman-ford algorithm must be modified to ensure only loop-free paths are inspected.
- $(S, \oplus, \overline{0})$ is a commutative, idempotent, and selective monoid,
- $(S, \otimes, \overline{1})$ is a monoid,
- $\overline{0}$ is the annihilator for $\otimes$,
- $\overline{1}$ is the annihilator for $\oplus$,
- Left strictly inflationarity, L.S.INF : $\forall a, b : a \neq \overline{0} \implies a < a \otimes b$
- Here $a \leq b \equiv a = a \oplus b$.

Convergence to a unique left-local solution is guaranteed. Currently no bound is known on the number of iterations required.

Of course BGP does not satisfy these conditions!

As a result ...

- Protocol will diverge when no solution exists.
- Protocol may diverge even when a solution exists.
- BGP Wedgies, RFC 4264.
  - Multiple stable states may exist.
  - No guarantee that each state implements intended policy.
  - Manual intervention required when system gets stuck in unintended local optima.
  - Debugging nearly impossible when policy is not shared between networks.

Recent observation : Dijkstra's algorithm can work for right-local optima.

Input : adjacency matrix **A** and source vertex $i \in V$,
Output : the i-th row of **R**, $\mathbf{R}(i, \_)$.

begin
  $S \leftarrow \{i\}$
  $\mathbf{R}(i, i) \leftarrow \overline{1}$
  for each $q \in V - \{i\}$: $\mathbf{R}(i, q) \leftarrow \mathbf{A}(i, q)$
  while $S \neq V$
    begin
      find $q \in V - S$ such that $\mathbf{R}(i, q)$ is $\leq_{\oplus}^{L}$ -minimal
      $S \leftarrow S \cup \{q\}$
      for each $j \in V - S$
        $\mathbf{R}(i, j) \leftarrow \mathbf{R}(i, j) \oplus (\mathbf{R}(i, q) \otimes \mathbf{A}(q, j))$
    end
end

Assumptions on $(S, \oplus, \otimes, \overline{0}, \overline{1})$ that guarantee existence of right-local optima

- $(S, \oplus, \overline{0})$ is a commutative, idempotent, and selective monoid,
- $(S, \otimes, \overline{1})$ is a monoid,
- $\overline{0}$ is the annihilator for $\otimes$,
- $\overline{1}$ is the annihilator for $\oplus$,
- Right inflationarity, R.INF : $\forall a, b : a \leq a \otimes b$

Here $a \leq b \equiv a = a \oplus b$.

Using a Link-State approach with hop-by-hop forwarding ...

Need left-local optima!

$$\mathbf{L} = (\mathbf{A} \otimes \mathbf{L}) \oplus \mathbf{I} \quad \Longleftrightarrow \quad \mathbf{L}^T = (\mathbf{L}^T \hat{\otimes}^T \mathbf{A}^T) \oplus \mathbf{I}$$

where $\otimes^T$ is matrix multiplication defined with as

$$a \otimes^T b = b \otimes a$$

and we assume left-inflationarity holds, $\text{L.INF} : \forall a, b : a \leq b \otimes a$.

Each node would have to solve the entire "all pairs" problem.

Functions on arcs

$(S, \oplus, F \subseteq S \to S, \overline{0})$

- $(S, \oplus, \overline{0})$ is a commutative, idempotent, and selective monoid,
- $\forall f \in F : f(\overline{0}) = \overline{0}$
- For local-optima need $\textsc{inf} : \forall a, f : a \leq f(a)$

Simplest model for interdomain routing

- 0 is for *downstream* routes (towards paying customers),
- 1 is for *peer* routes (towards competitor's customers),
- 2 is for *upstream* routes (towards charging providers)

	0	1	2		0	1	2
a	0	1	2	**m**	2	1	2
b	0	1	∞	**n**	2	1	∞
c	0	2	2	**o**	2	2	2
d	0	2	∞	**p**	2	2	∞
e	0	∞	2	**q**	2	∞	2
f	0	∞	∞	**r**	2	∞	∞
g	1	1	2	**s**	∞	1	2
h	1	1	∞	**t**	∞	1	∞
i	1	2	2	**u**	∞	2	2
j	1	2	∞	**v**	∞	2	∞
k	1	∞	2	**w**	∞	∞	2
l	1	∞	∞	**x**	∞	∞	∞

Conclusion

Take away message

If your algebraic model is not distributive, then ask yourself if a left- or right-local solution is reasonable. If so, use Dijkstra's algorithm (with care).

A few open problems

- How many Bellman iterations are needed to find **L**?
- Is there an equational axiomatization of local optimality? (For classical theory we have Kleene Algebras).
- Analytic model of dynamics of hard-state distributed Bellman-Ford.