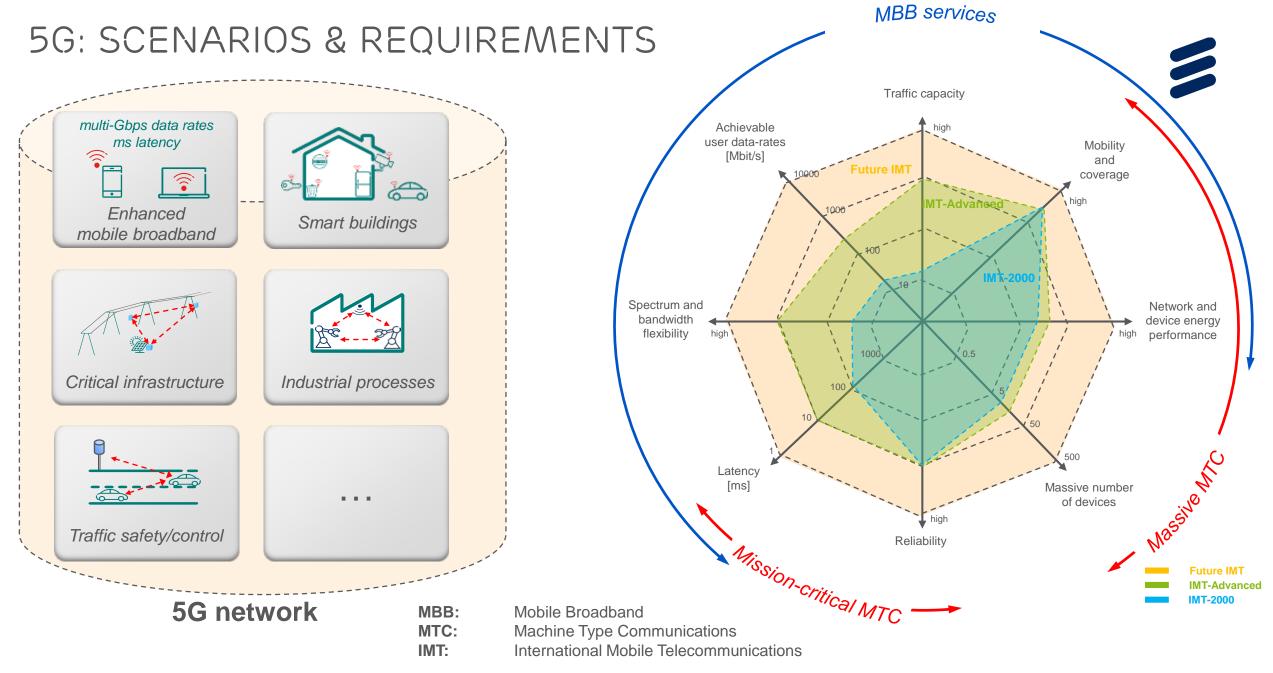


TUNING THE RECEIVER STRUCTURE AND THE PILOT-TO-DATA POWER RATIO IN MULTIPLE INPUT MULTIPLE OUTPUT SYSTEMS

GABOR FODOR ERICSSON RESEARCH ROYAL INSTITUTE OF TECHNOLOGY



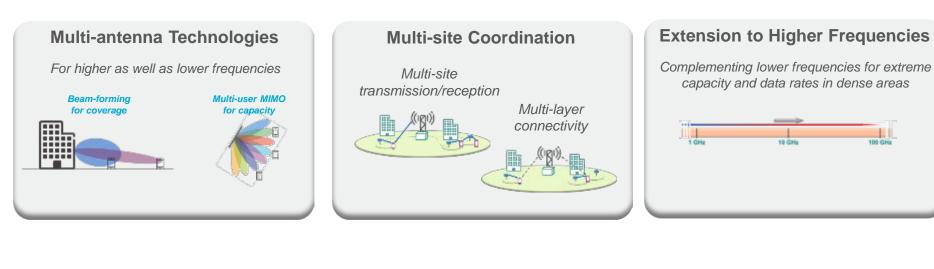
R. Baldemair, E. Dahlman, G. Fodor, G. Mildh, S. Parkvall, Y. Selén, "Evolving Wireless Communications:

Tuning the MU-MIMO Receiver and the PDPR | UMD Seminar |

Addressing the Challenges and Expectations of the Future", IEEE Vehicular Technology Magazine, Vol. 8, No. 1, pp. 24-30, Mar. 2013

5G TECHNOLOGY COMPONENTS





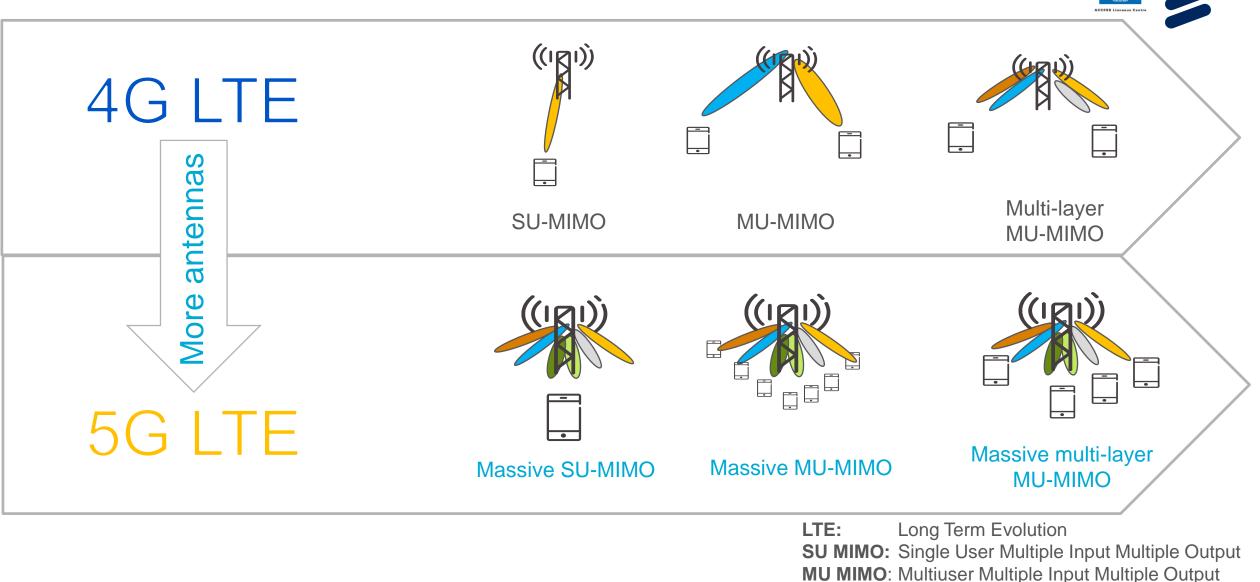
Spectrum	Flexibility
Spectrum sharing Unlicensed Shared licensed Complementing dedicated licensed spectrum	(Full) Duplex Flexibility

Ultra-lean Design Access/backhaul Integration Device-to-Device Communication Direct communication Minimize transmissions not related to user data Same technology for access and backhaul Device-based relaying Separate delivery of user data Cooperative devices Same spectrum for access and backhaul and system information 0----0 0000 Higher data rates and Device-bas Robyin enhanced energy efficiency

H. Shokri-Ghadikolaei, F. Boccardi, C. Fischione, G. Fodor and M. Zorzi, "Spectrum Sharing in mmWave Cellular Networks via Cell Association, Coordination, and Beamforming", IEEE J. on Selected Areas in Communications, Vol. 34, Issue 11, pp. 2902-2917, 2016
 D. Astely, E. Dahlman, G. Fodor, S. Parkvall and J. Sachs, "LTE Release 12 and Beyond", IEEE Comm. Mag., Vol. 51, No. 7, pp. 154-160, July 2013.

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MIMO EVOLUTION



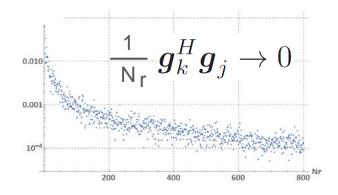
G. Fodor, N. Rajatheva, W. Zirwas, L. Thiele, M. Kurras, K. Guo, A. Tolli, J. H. Sorensen, E. de Carvalho,

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¹ "An Overview of Massive MIMO Technology Components in METIS", IEEE Communications Magazine, Vol. 55, Issue 6, pp. 155-161, June 2017.

WHY FULL DIMENSION MIMO ?

- The vector channel to a desired user becomes orthogonal to the vector channel of a random interfering user;
- Rejecting interference becomes possible simply by aligning the BF vector with the desired channel;
 CSI is important !
- > Ultimate limitation is CSI error





V. Saxena, G. Fodor, E. Karipidis, "Mitigating Pilot Contamination by Pilot Reuse and Power Control Schemes for Massive MIMO Systems", IEEE VTC Spring, Glasgow, Scotland, May 2015.

ACCESS Linnaeus Centre Uniform 250 Linear Array Average sum rate (Mbps/cell) 001 002 > 10 users > Perfect 150 CSI 50 Matched filter Zero-forcing Interference free 0. 100 200 300 400 500 Number of antennas

The capacity performance of conjugate BF and ZF become asymptotically identical. [Yang, Marzetta, *IEEE JSAC* 2013]

BS: Base Station CSI: Channel State Information ZF: Zero Forcing BF: Beam Forming



UL MU MIMO RECEIVER DESIGN QUESTIONS



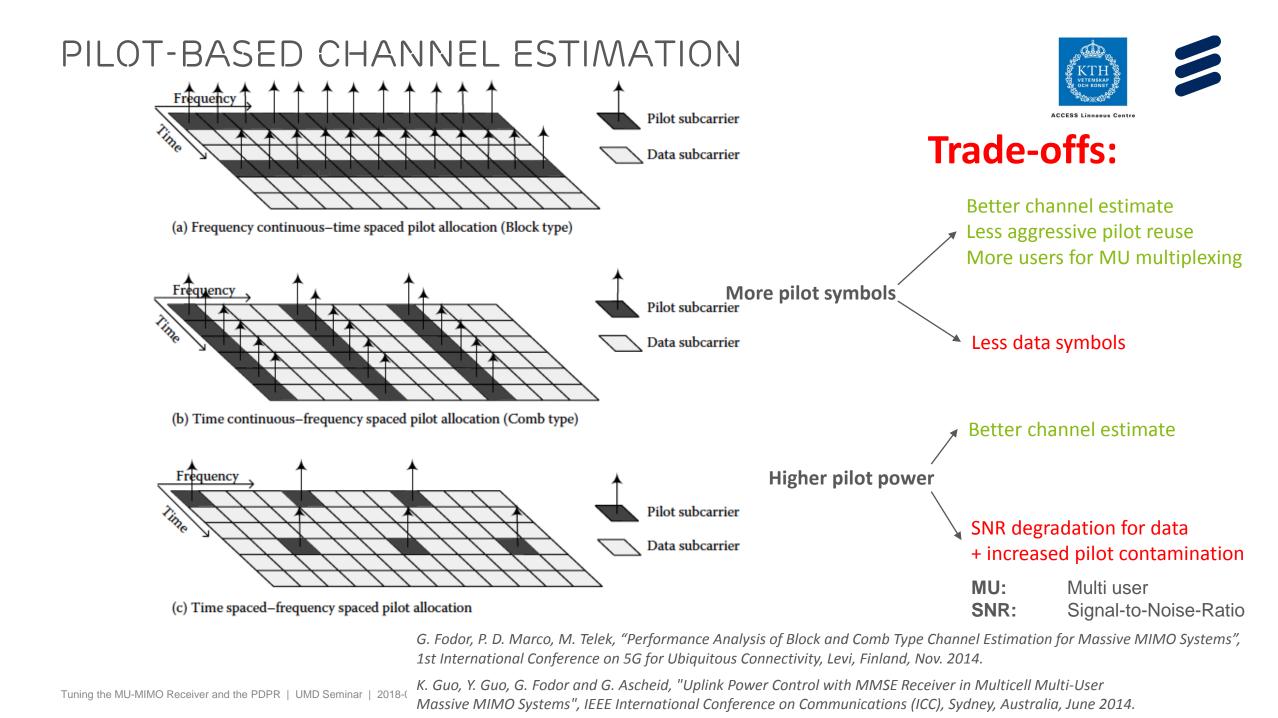
- How can we improve the performance of the MMSE receiver in the presence of CSI errors in terms of:
 - Mean squared error of the received data symbols;
 - Spectral efficiency
- > What are the gains of CSI error aware receivers over naïve receivers ?
- > Do such gains increase/decrease as the number of antennas grows large ?
- > What is the impact of correlated antennas ?

UL:UplinkMU MIMO:Multiuser Multiple Input Multiple OutputMMSE:Minimum Mean Squared ErrorCSI:Channel State Information

N. Rajatheva, S. Suyama, W. Zirwas, L. Thiele, G. Fodor, A.Tölli, E. Carvalho, J. H. Sorensen, "Massive Multiple Input Multiple Output (MIMO) Systems", Chapter 8 in: A. Osseiran, J. F. Monserrat, P. Marsch, "5G Mobile and Wireless Communications Technology", Cambridge University Press, 2016.

Tuning the MU-MIMO Receiver and the PDPR

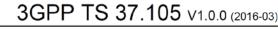
L. S. Muppirisetty, T. Charalambous, J. Karout, G. Fodor, H. Wymeersch, "Location-Aided Pilot Contamination Avoidance for Massive MIMO Systems", IEEE Trans. Wireless Comm, April 2018.



FULL DIMENSION IN 3GPP

> Full Dimension MIMO (FD-MIMO)

- > Greater number of antenna ports
- > Efficient MU MIMO Spatial Multiplexing
- > Robustness against CSI Impairments (e.g. intercell interference)
- 3GPP Technical Report: Study on Elevation Beamforming and FD-MIMO for LTE
 - > See also:
 - 36.873 Study on 3D Channel Model for LTE
 - 37.105 Active Antenna System BS Radio Transmission and Reception



Technical Specification

3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Active Antenna System (AAS) Base Station (BS) radio transmission and reception (Release 13)





Source: Samsund New WID Proposal: Enhancements on Full-Dimension (FD) MIMO for LTE Title: Document for: Approval Agenda Item: 10.1.1 3GPP[™] Work Item Description For guidance, see 3GPP Working Procedures, article 39; and 3GPP TR 21.900. Comprehensive instructions can be found at http://www.3gpp.org/Work-Item Title: Enhancements on Full-Dimension (FD) MIMO for LTE LTE eFDMIMO Acronym Unique identifier NOTE: If this is a RAN WID including Core and Perf. Part, then Title, Acronym and Unique identifier refer to the feature WI. Please tick (X) the applicable box(es) in the table below: This WID includes a Core part x This WID includes a Performance part x 3GPP Work Area Radio Access Core Network Services

3GPP TSG RAN Meeting #71

Göteborg, Sweden, March 7 - 10, 2016

3GPP TR 36.897 V13.0.0 (2015-06)

Technical Report

3rd Generation Partnership Project; Technical Specification Group Radio Access Network; Study on elevation beamforming / Full-Dimension (FD) Multiple Input Multiple Output (MIMO) for LTE (Release 13)





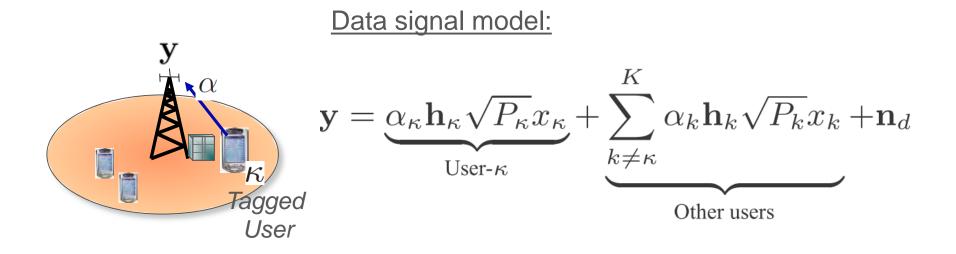


A GLOBAL INITIATIVE



MU MIMO UPLINK SIGNAL MODEL





$$\mathbf{G}^{\text{na\"ive}} = \mathbf{G}^{\text{na\"ive}}(\mathbf{\hat{h}}) = \frac{\alpha \sqrt{P} \mathbf{\hat{h}}^{H}}{\alpha^{2} P ||\mathbf{\hat{h}}||^{2} + \sigma^{2}}$$

The naïve G minimizes the MSE of the received data symbols when perfect channel estimation is available at the receiver.

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G. Fodor, P. Di Marco and M.Telek, "On the Impact of Antenna Correlation on the Pilot-Data Balance in Multiple Antenna Systems" IEEE International Conference on Communications (ICC), London, UK, June 2015.

RELATED WORKS ON MMSE RECEIVERS/ESTIMATORS

M. R. McKay, I. B. Collings, and A. M. Tulino, "Achievable sum rate of MIMO MMSE receivers: A general analytic framework," *IEEE Trans. Inf. Theory*, vol. 56, no. 1, pp. 396 – 410, January 2010.

Y. Jiang, M. K. Varanasi, and J. Li, "Performance analysis of ZF and MMSE equalizers for MIMO systems: An in-depth study of the high SNR regime," *IEEE Trans. on Info. Theory*, vol. 57, no. 4, pp. 2008–2026, March 2011.

A. H. Mehana and A. Nosratinia, "Diversity of MMSE MIMO receivers," *IEEE Transactions on Information Theory*, vol. 58, no. 11, pp. 6788–6805, November 2012.

—, "Diversity of MMSE receivers in MIMO multiple access channels," IEEE W. Comm. Letters, vol. 2, no. 3, pp. 275–278, June 2013.

T.-M. Ma, Y.-S. Shi, and Y.-G. Wang, "A low complexity MMSE for OFDM systems over frequency-selective fading channels," *IEEE Commun. Lett.*, vol. 16, no. 3, Mar. 2012.

E. Eraslan, B. Daneshrad, and C.-Y. Lou, "Performance indicator for MIMO MMSE receivers in the presence of channel estimation error," *IEEE W. Comm. Letters*, vol. 2, no. 2, pp. 211–214, April 2013.

CSI Errors are not considered

Focuses on channel estimation only

Uses the naïve receiver







Pilot signal model:
$$\mathbf{Y}^p = \alpha \sqrt{P_p} \mathbf{h} \mathbf{s}^T + \mathbf{N}$$
 $\mathbf{h} \in \mathbb{C}^{N_r \times 1}$ Estimated channel: $\hat{\mathbf{h}} = \mathbf{h} + \mathbf{w} = \frac{1}{\alpha \sqrt{P_p}} \mathbf{Y}^p \mathbf{s}^* (\mathbf{s}^T \mathbf{s}^*)^{-1} = \mathbf{h} + \frac{1}{\alpha \sqrt{P_p} \tau_p} \mathbf{N} \mathbf{s}^*$ Conditional
channel distribution: $(\mathbf{h} \mid \hat{\mathbf{h}}) \sim \mathbf{D}\hat{\mathbf{h}} + \mathcal{CN}(\mathbf{0}, \mathbf{Q})$ $\mathbf{D} \triangleq \mathbf{C}\mathbf{R}^{-1}$
 $\mathbf{Q} \triangleq \mathbf{C} - \mathbf{C}\mathbf{R}^{-1}$
 $\mathbf{Q} \triangleq \mathbf{C} - \mathbf{C}\mathbf{R}^{-1}$ Covariance of the
estimated channel: $\mathbf{R} \triangleq \mathbb{E}\{\hat{\mathbf{h}}\hat{\mathbf{h}}^H\} = \mathbf{C} + \frac{\sigma_p^2}{\alpha^2 P_p \tau_p} \mathbf{I}_{N_r}$

N. Rajatheva, S. Suyama, W. Zirwas, L. Thiele, G. Fodor, A.Tölli, E. Carvalho, J. H. Sorensen, "Massive Multiple Input Multiple Output (MIMO) Systems", Chapter 8 in: A. ^{Tuning the MU-MIMO Rece} Osseiran, J. F. Monserrat, P. Marsch, "5G Mobile and Wireless Communications Technology", Cambridge University Press, June 2016. ISBN: 9781107130098

 $\mathbf{y} = \underbrace{\alpha_{\kappa} \mathbf{h}_{\kappa} \sqrt{P_{\kappa}} x_{\kappa}}_{\text{User-}\kappa} + \underbrace{\sum_{k \neq \kappa}^{K} \alpha_{k} \mathbf{h}_{k} \sqrt{P_{k}} x_{k} + \mathbf{n}_{d}}_{\text{Other users}}$

 $\mathbf{G}_{\kappa}^{\star} \triangleq \arg\min_{\mathbf{G}} \mathbb{E}\{\mathsf{MSE}\} = \arg\min_{\mathbf{G}} \mathbb{E}\{|\mathbf{Gy} - x_{\kappa}|^{2}\}$

Data signal model:

MU MIMO Receiver

at the BS:

PRELIMINARIES II



HOW TO FIND THE (TRUE) MMSE RECEIVER ? -- APPROACH 1



$$MSE (\mathbf{G}_{\kappa}, \mathbf{h}_{1}, \dots, \mathbf{h}_{K}) = \mathbb{E}_{x, \mathbf{n}_{d}} \{ |\mathbf{G}_{\kappa} \mathbf{y} - x_{\kappa}|^{2} \}$$

$$\blacksquare \mathbb{E}_{\mathbf{h}_{1}, \dots, \mathbf{h}_{\kappa-1}, \mathbf{h}_{\kappa+1}, \dots, \mathbf{h}_{K}}$$

$$MSE (\mathbf{G}_{\kappa}, \mathbf{h}_{\kappa})$$

$$\blacksquare (\mathbf{h} \mid \hat{\mathbf{h}}) \sim \mathbf{D}\hat{\mathbf{h}} + \mathcal{CN}(\mathbf{0}, \mathbf{Q})$$

$$MSE (\mathbf{G}_{\kappa}, \hat{\mathbf{h}}_{\kappa}) = \mathbb{E}_{\mathbf{h}_{\kappa} \mid \hat{\mathbf{h}}_{\kappa}} MSE (\mathbf{G}_{\kappa}, \mathbf{h}_{\kappa})$$

$$\blacksquare$$

$$\mathbf{G}_{\kappa} \triangleq \arg\min \mathbb{E}\{MSE\} = \arg\min \mathbb{E}\{|\mathbf{G}\mathbf{v} - x_{\kappa}|^{2}\}$$

HOW TO FIND THE (TRUE) MMSE RECEIVER ? -- APPROACH 2



> Determine the MSE of a tagged User κ as a function of $\,{\rm G}_{\kappa}^{}\,$ and the estimated channel of all users



- > Closed form expression for the MMSE receiver in the presence of CSI errors
- > Closed form expression for the MSE when using the naïve and the MMSE receiver
- Closed form expressions for the optimum pilot-to-data power ratio when using the MMSE receiver

HOW TO FIND THE (TRUE) MMSE RECEIVER ?

Proposition The optimal $\mathbf{G}_{\kappa}^{\star}$ can be derived as:

$$\mathbf{G}_{\kappa}^{\star} = \alpha_{\kappa} \sqrt{P_{\kappa}} \mathbf{\hat{h}}_{\kappa}^{H} \mathbf{D}_{\kappa}^{H} \left(\alpha_{\kappa}^{2} P_{\kappa} \left(\mathbf{D}_{\kappa} \mathbf{\hat{h}}_{\kappa} \mathbf{\hat{h}}_{\kappa}^{H} \mathbf{D}_{\kappa}^{H} + \mathbf{Q}_{\kappa} \right) + \sum_{\substack{k \neq \kappa}}^{K} \alpha_{k}^{2} P_{k} \mathbf{C}_{k} + \sigma_{d}^{2} \mathbf{I} \right)^{-1}$$
CSI error compensation
by 2nd order statistics (D and Q) MU-MIMO Interference

Elements of proof:

Quadratic Form:

$$MSE\left(\mathbf{G}_{\kappa},\hat{\mathbf{h}}_{\kappa}\right) = -\underbrace{\mathbf{G}_{\kappa}}_{\mathbf{x}}\underbrace{\alpha_{\kappa}\sqrt{P_{\kappa}}\mathbf{D}_{\kappa}\hat{\mathbf{h}}_{\kappa}}_{\mathbf{B}} - \alpha_{\kappa}\sqrt{P_{\kappa}}\hat{\mathbf{h}}_{\kappa}^{H}\mathbf{D}_{\kappa}^{H}\mathbf{G}_{\kappa}^{H} + 1 + \left(\mathbf{X}\mathbf{A}\mathbf{X}^{H} - \mathbf{X}\mathbf{B} - \mathbf{B}^{H}\mathbf{X}^{H} + 1\right) + \mathbf{G}_{\kappa}\underbrace{\left(\alpha_{\kappa}^{2}P_{\kappa}\left(\mathbf{D}_{\kappa}\hat{\mathbf{h}}_{\kappa}\hat{\mathbf{h}}_{\kappa}^{H}\mathbf{D}_{\kappa}^{H} + \mathbf{Q}_{\kappa}\right) + \sum_{k\neq\kappa}^{K}\alpha_{k}^{2}P_{k}\mathbf{C}_{k} + \sigma_{d}^{2}\mathbf{I}}_{\mathbf{A}}\mathbf{G}_{\kappa}^{H}}_{\mathbf{A}} = \mathbf{B}^{H}\mathbf{A}^{-1}$$



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HOW TO FIND THE (TRUE) MMSE RECEIVER ?

Proposition The optimal $\mathbf{G}_{\kappa}^{\star}$ can be derived as:

$$\mathbf{G}_{\kappa}^{\star} = \alpha_{\kappa} \sqrt{P_{\kappa}} \mathbf{\hat{h}}_{\kappa}^{H} \mathbf{D}_{\kappa}^{H} \left(\alpha_{\kappa}^{2} P_{\kappa} \left(\mathbf{D}_{\kappa} \mathbf{\hat{h}}_{\kappa} \mathbf{\hat{h}}_{\kappa}^{H} \mathbf{D}_{\kappa}^{H} + \mathbf{Q}_{\kappa} \right) + \sum_{\substack{k \neq \kappa}}^{K} \alpha_{k}^{2} P_{k} \mathbf{C}_{k} + \sigma_{d}^{2} \mathbf{I} \right)^{-1}$$
CSI error compensation
by 2nd order statistics (D and Q)
$$MU-MIMO \text{ Interference}$$
"Perfect"

Approach 2:

$$\mathbf{G}_{\kappa}^{*} = \alpha_{\kappa} \sqrt{P_{\kappa}} \hat{\mathbf{h}}_{\kappa}^{H} \mathbf{D}_{\kappa}^{H} \left(\sum_{k=1}^{K} \alpha_{k}^{2} P_{k} \left(\mathbf{D}_{k} \hat{\mathbf{h}}_{k}^{*} \hat{\mathbf{h}}_{k}^{H} \mathbf{D}_{k}^{H} + \mathbf{Q}_{k} \right) + \sigma_{d}^{2} \mathbf{I} \right)^{-1}$$

"Estimate"

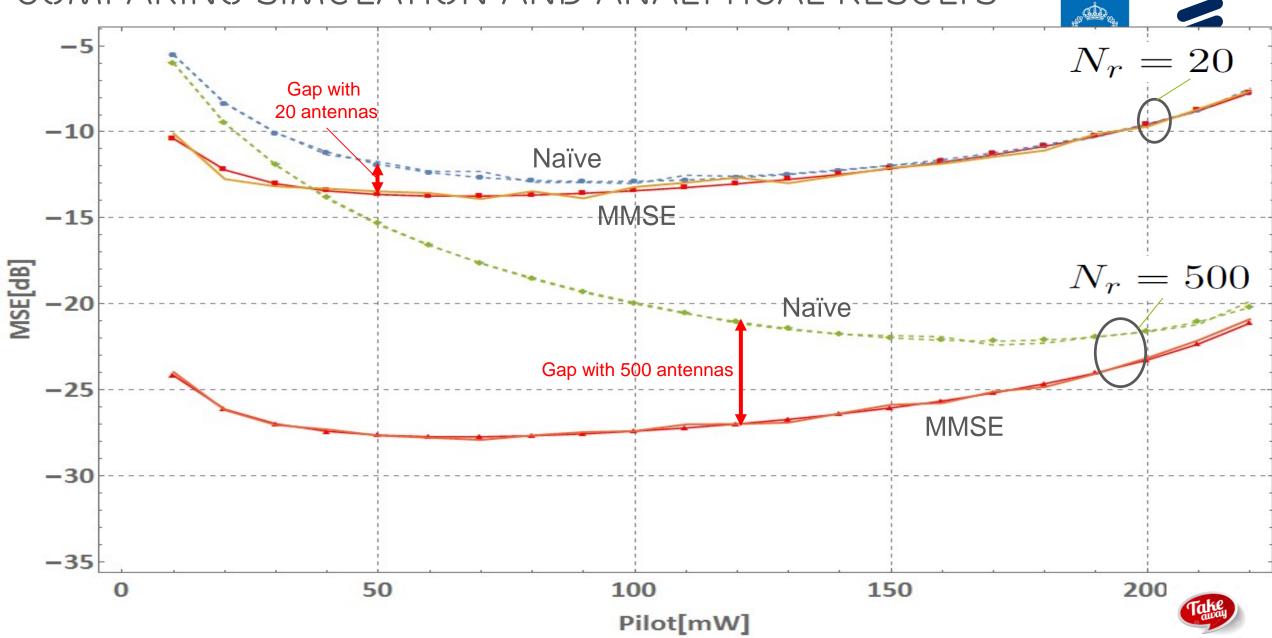




COMPARING ANALYTICAL AND SIMULATION RESULTS

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COMPARING SIMULATION AND ANALYTICAL RESULTS

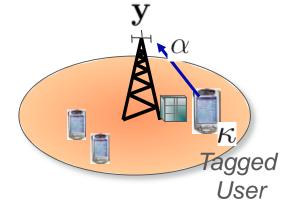


The gain of the (true) MMSE receiver over the naïve receiver increases when the number of antennas increases.

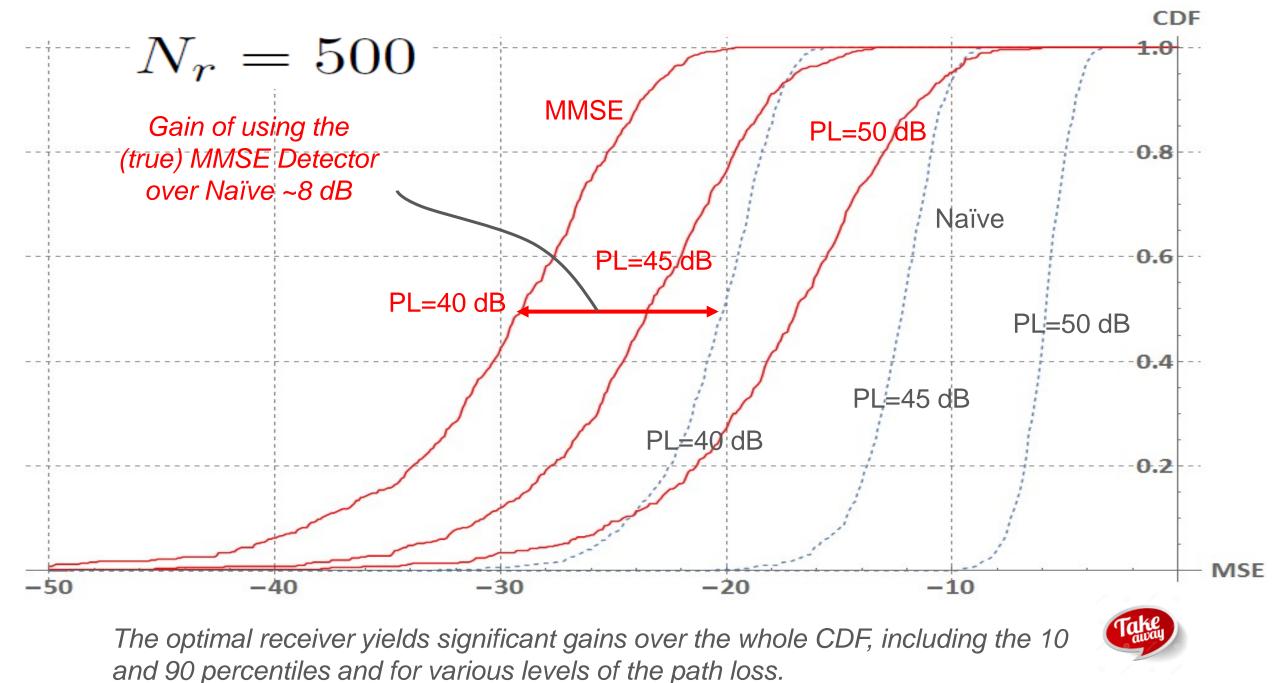
SIMULATION SETUP

• Single user system, that is no MU MIMO interference

Parameter	Value
Number of antennas	$N_r = 2, 4, 8, 10, 20, 50, 100, 500$
Path Loss of tagged User- κ	$\alpha = 40, 45, 50 \text{ dB}$
Number of pilot and data symbols	$\tau_p = 1; \ \tau_d = 11$
Power budget	$ au_p P_p + au_d P = P_{tot}$ =250 mW.





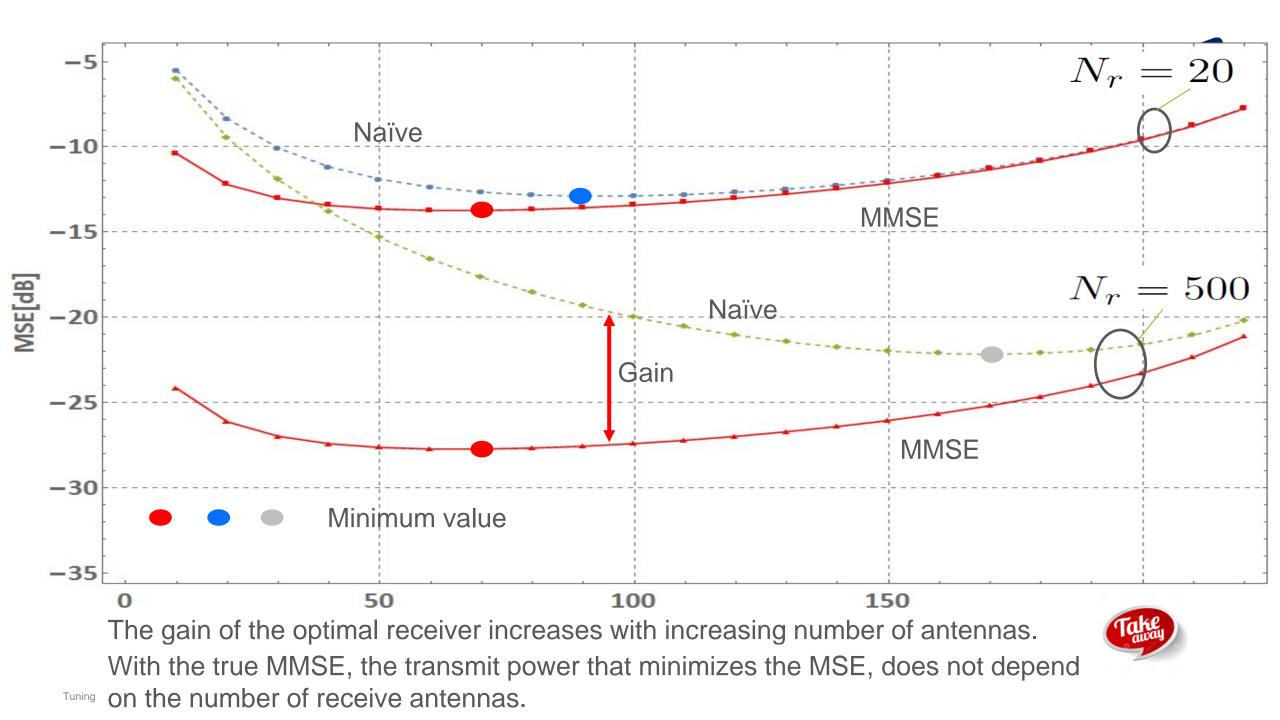


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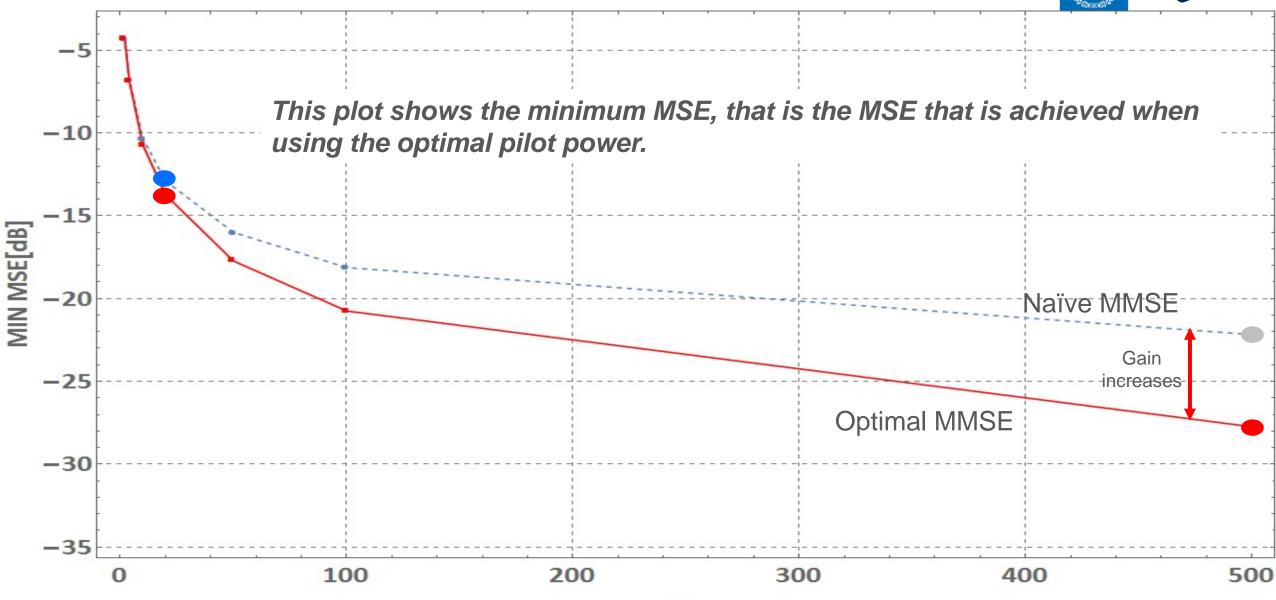


OPTIMUM PILOT POWER SETTING

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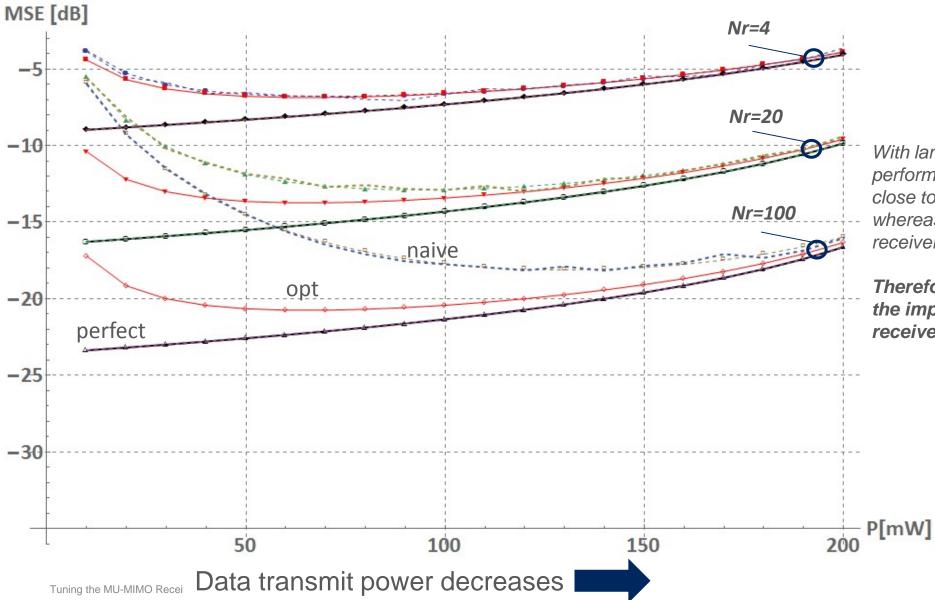


HOW DOES THE GAIN DEPEND ON THE NUMBER OF ANTENNAS ?



2

COMPARISON WITH PERFECT CSI





With large number of antennas, the MSE performance of the optimal receiver remains close to the perfect CSI performance, whereas the performance of the naïve receiver is far from the perfect CSI case.

Therefore, with larger number of antennas, the importance of applying the optimal receiver increases.

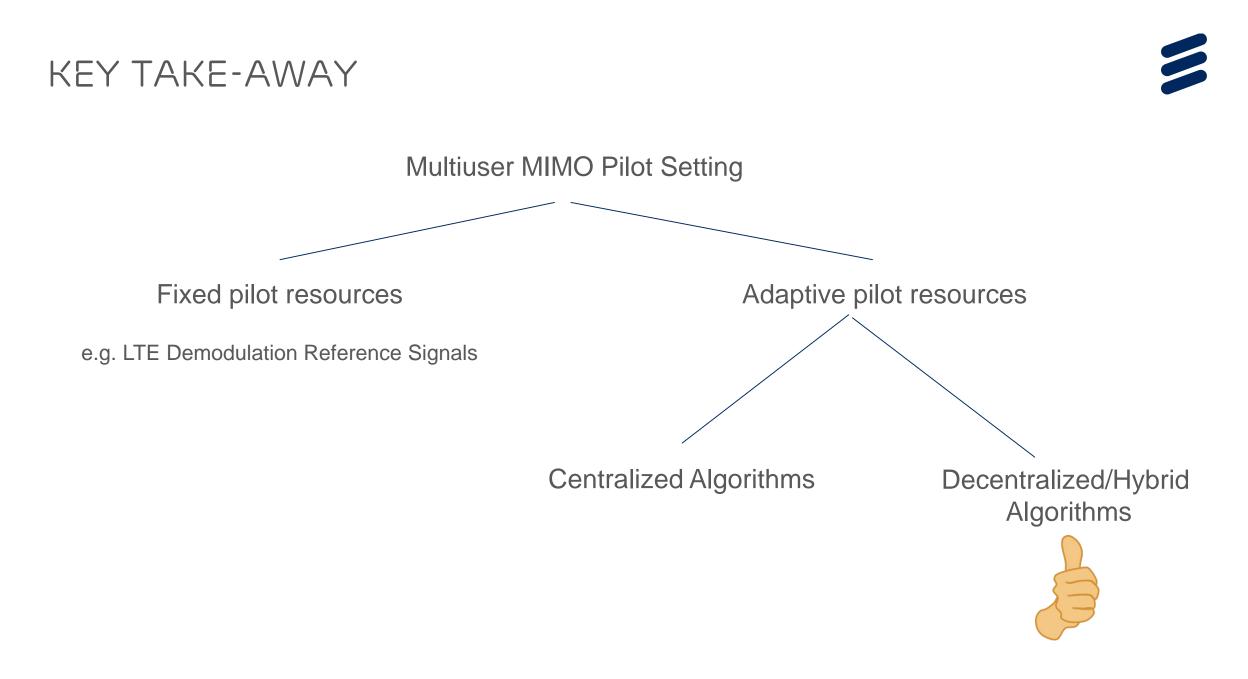


- The gain of the optimal receiver increases with increasing number of antennas. In the massive MIMO domain, this gain can be up to 8-10 dB in terms of MSE;
- The true MMSE receiver well approximates the perfect channel estimation case, independently of the number of antennas (as opposed to the naïve receiver);
- > With the true MMSE, the transmit power that minimizes the MSE, does not depend on the number of receive antennas (as opposed to the naïve receiver);

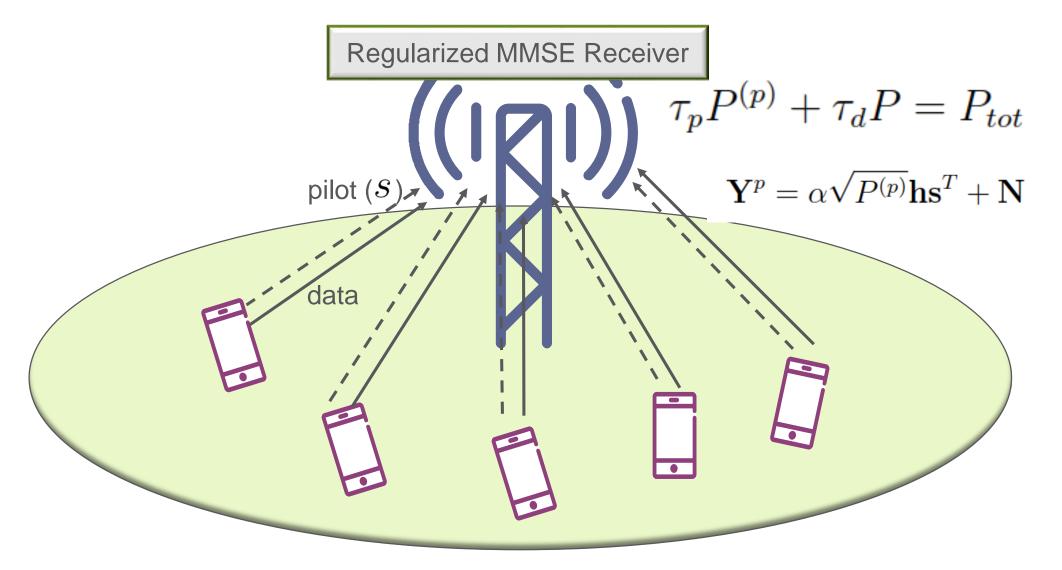


TUNING THE PILOT-TO-DATA POWER RATIO

Tuning the MU-MIMO Receiver and the PDPR | UMD Seminar | 2018-05-23 | Page 27



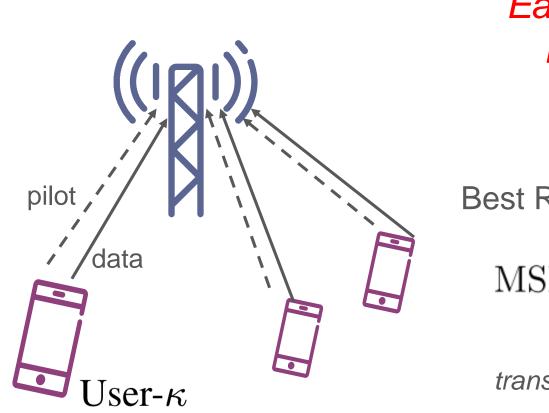
SINGLE CELL MU MIMO MODEL



P. Zhao, G. Fodor, G. Dan, and M. Telek, "A Game Theoretic Approach to Setting the Pilot Power Ratio in Multi-User MIMO Systems", IEEE Transactions on Communications, Vol. 66, Issue 3, March 2018.

MU MIMO GAME





Each user tunes his PPR to minimize the own MSE.

Best Response Power Allocation:

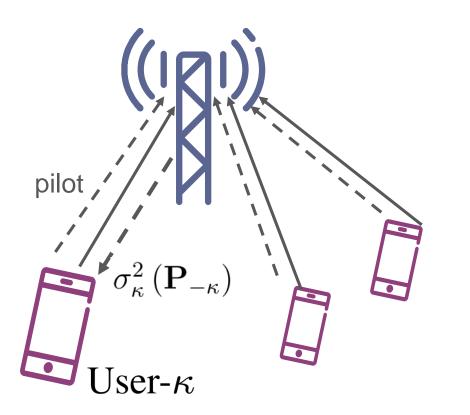
$$\operatorname{MSE}_{\kappa}(P_{\kappa}^{\star}, \mathbf{P}_{-\kappa}) \leq \operatorname{MSE}_{\kappa}(P_{\kappa}, \mathbf{P}_{-\kappa})$$

transmit power of all other players

$$\mathbf{P} \triangleq \{P_1, \dots, P_K\} \in \mathbb{R}^{1 \times K}$$

P. Zhao, G. Fodor, G. Dan, and M. Telek, "A Game Theoretic Approach to Setting the Pilot Power Ratio in Multi-User MIMO Systems", IEEE Transactions on Communications, December 2017.

BEST PILOT-DATA POWER RATIO ALGORITHM





Each user minimizes the own
 MSE by setting the PPR

 BPA converges to a pure strategy Nash equilibrium

BS can help User- κ by signaling $\sigma_{\kappa}^{2}(\mathbf{P}_{-\kappa})$ to User- κ

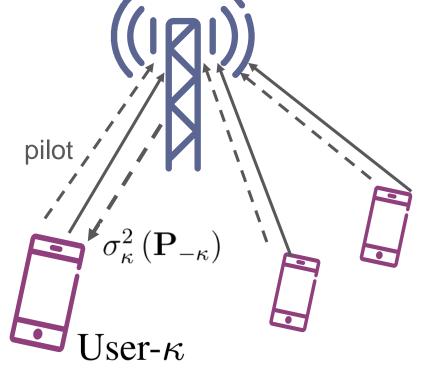
BEST PILOT-DATA POWER RATIO ALGORITHM (BPA)

Non-cooperative Game:

$$\mathcal{G} \triangleq < \mathcal{K}, (\mathcal{P}_d), (MSE_{\kappa}(\mathbf{P})) >$$

 $P_{\kappa}^{\star}(\mathbf{P}_{-\kappa})$: Best response power allocation of the tagged MS, as a function of the currently used transmit power of all other MSs.

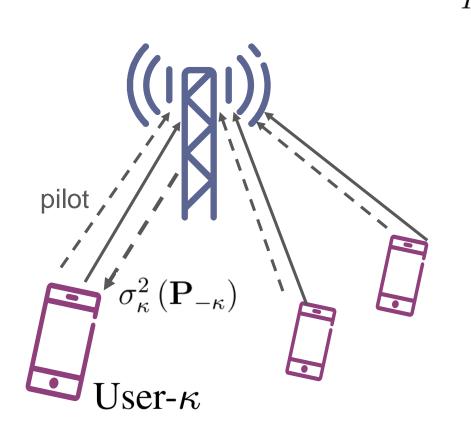
Mapping from \mathbf{P} to \mathbf{P}^{\star} : $\mathbf{f}(\mathbf{P}) \triangleq [P_1^{\star}(P_1, \mathbf{P}_{-1}), \dots, P_K^{\star}(P_K, \mathbf{P}_{-K})]$ $f_j(\mathbf{P}) = P_j^{\star}(\mathbf{P}_{-j})$ $\mathbf{F}(\mathbf{P})_{ij} \triangleq \frac{\partial}{\partial P_i} f_j(\mathbf{P}) = \frac{\partial}{\partial P_i} P_j^{\star}(\mathbf{P})$





BEST PILOT-DATA POWER RATIO ALGORITHM (BPA)





 $P_{\kappa}^{\star}(\mathbf{P}_{-\kappa})$ Best response power allocation of the tagged MS, as a function of the currently used transmit power of all other MSs.

Algorithm 1: Best PPR Algorithm (BPA)	
Input : MSE improvement threshold ϵ ,	
$Mode \in \{\mathrm{MIN}, \mathrm{MAX}\}$	
1 if $Mode ==$ MIN then	
2 Initial data power $P_{\kappa}^{(0)} = P_{\kappa}^{\star}(0), \forall \kappa \in \mathcal{K}$	
3 else	
4 Initial data power $P_{\kappa}^{(0)} = P_{\kappa}^{\star} \left(\frac{P_{tot}}{\tau_d} \mathbf{e} \right), \forall \kappa \in \mathcal{K}$	
5 end	
$6 \ i = 0$	
7 repeat	
8 BS sends σ_p^2 and $\sigma_\kappa^2 \left(\mathbf{P}_{-\kappa}^{(i-1)} \right)$ to MS- $\kappa, \kappa \in \mathcal{K}$	
9 for $\kappa \in \mathcal{K}$ do	
10 if then	
$11 \qquad \qquad P_{\kappa}^{(i)} = P_{\kappa}^{\star} \left(\mathbf{P}_{-\kappa}^{(i-1)} \right)$	
12 else	
$P_{\kappa}^{(i)} = P_{\kappa}^{(i-1)}$	
14 end	
15 end	
16 $i = i + 1$	
17 until $P_{\kappa}^{(i)} == P_{\kappa}^{(i-1)}, \ \forall \kappa \in \mathcal{K};$	
Output : Data power allocation P .	



- > What is the Pilot-to-Data Power Ratio ?
- > MU MIMO Game
- Numerical Results
- Conclusions

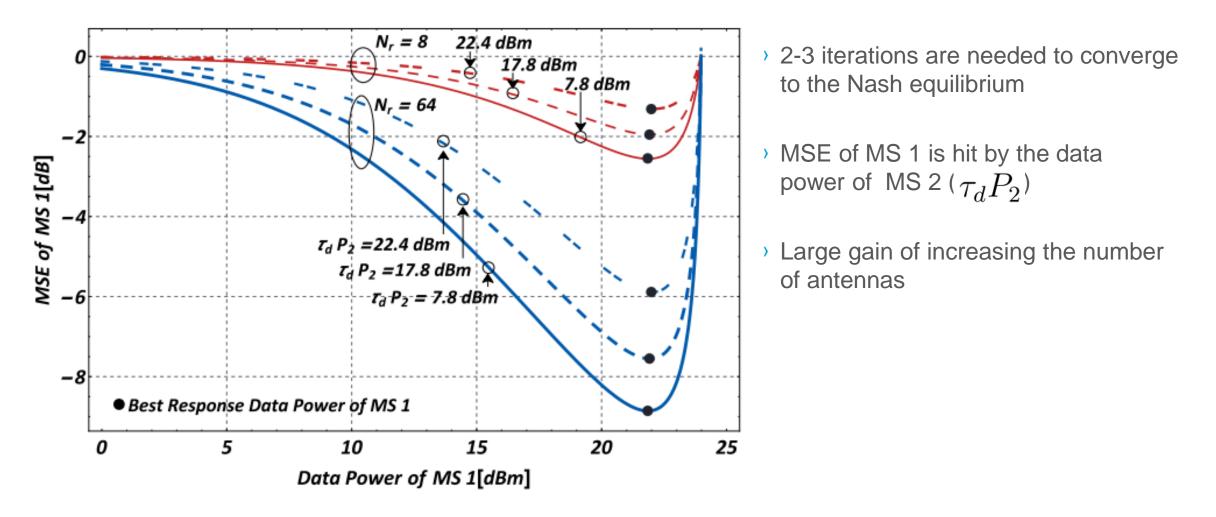
SINGLE CELL PARAMETER SETTING



Parameter	Value
Number of antennas at the BS	$N_r = 8, \dots, 80$
Number of MSs	K = 2, 6
Total number of symbols (per time slot)	F = 12
Number of data symbols (per time slot)	$\tau_d = 6$
Number of pilot symbols (per time slot)	$F - \tau_d$
Power budget	$P_{tot} = 24 \text{ dBm}$
Thermal noise per MHz	-114 dBm

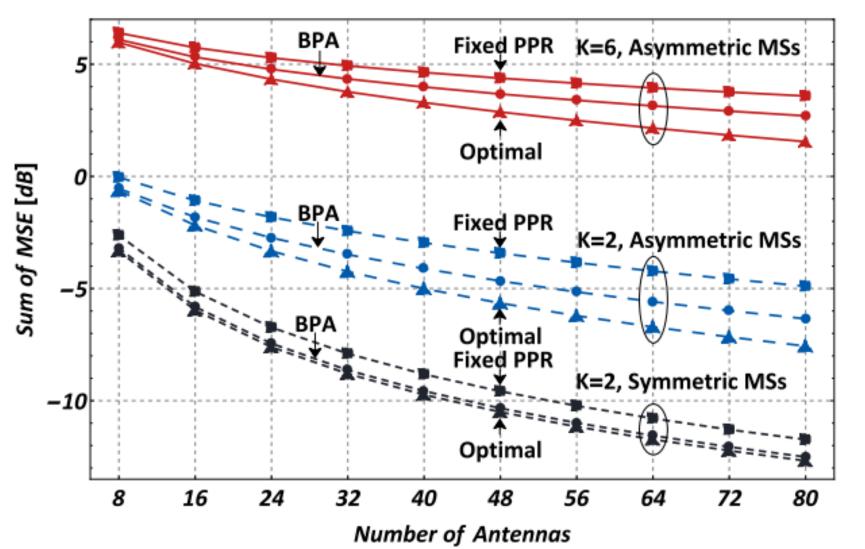
2-PLAYER GAME





2 AND 6-PLAYER GAME





- Adaptive PPR is superior to fixed PPR
- > BPA is close to the optimal PPR





> Adaptive rather than fixed PPR is beneficial for reducing the MSE

 A game theoretic, decentralized PPR setting algorithm quickly converges to a near optimal setting

