



Control of Cyber-Physical Systems: Fundamental Challenges and Applications to Energy and Transportation Networks

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Joint work with

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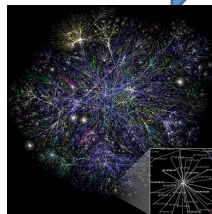


ISR Distinguished Lecture Series, University of Maryland, Dec 5, 2013

Cyber-physical systems are engineered systems
whose operations are monitored and controlled
by a computing and communication core
embedded in objects and structures in the
physical environment.

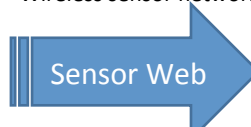
Towards Cyber-Physical Systems

- Internet
- WWW
- Ubiquitous computing



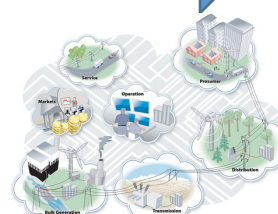
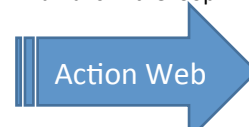
The Internet

- Remote sensing
- Monitoring environments
- Wireless sensor networks



Monitoring storm petrels at Great Duck Island

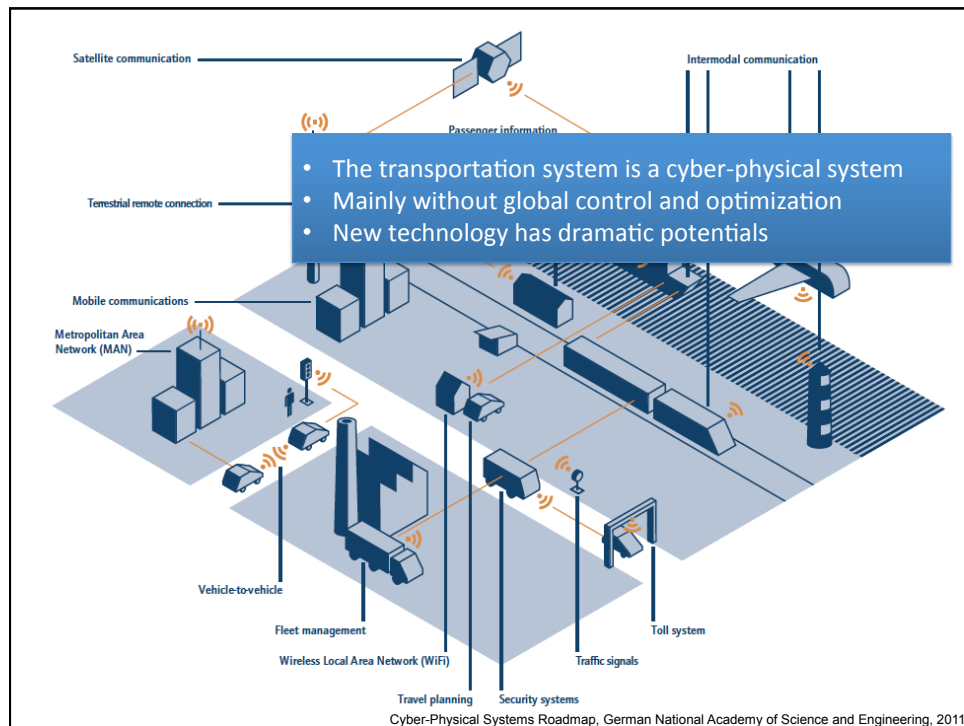
- Closing the loop
- Critical infrastructures
- Humans in the loop



The smart energy grid

Outline

- Introduction
- Case study I: Goods transportation
- Case study II: Building management
- Cross-cutting scientific challenges
- Conclusions

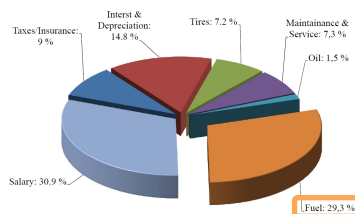


Demands from Goods Road Transportation

- Goods transportation accounts for 30% of CO₂ emissions
- 15% of greenhouse gas emissions of the global fossil fuel combustion
- Expected to increase by 50% for 2000-2020

International Transport Forum (2010), EC (2006)

Life cycle cost for European heavy-duty vehicles



Total fuel cost 80 k€/year/vehicle

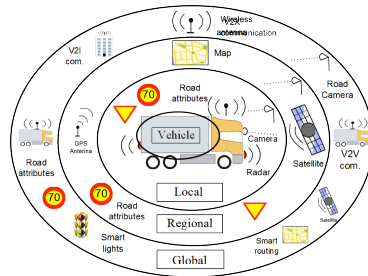
Schittler, 2003

- 24% of long haulage trucks run empty
- 57% average load capacity

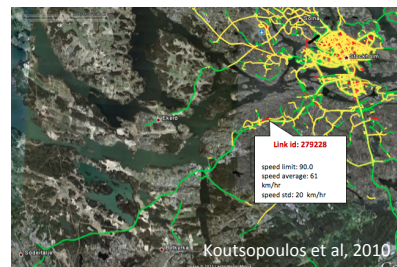
Dr. H. Ludanek, CTO, Scania

Technology Push

Sensor and communication technology



Real-time traffic information



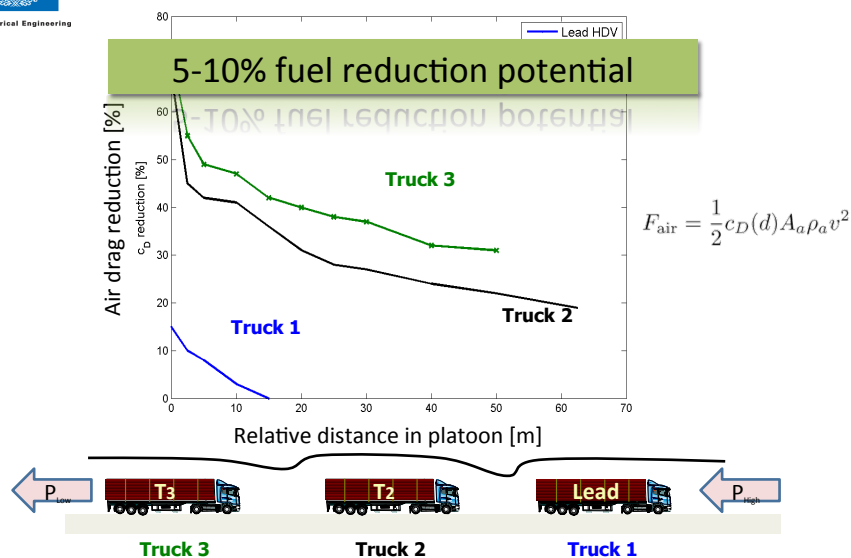
Vehicle platooning and semi-autonomous driving



KTH Electrical Engineering

Air Drag Reduction in Truck Platooning

5-10% fuel reduction potential

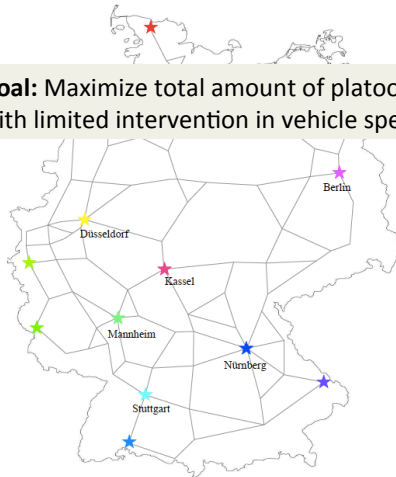


Wolf-Heinrich & Ahmed (1998), Bonnet & Fritz (2000), Scania CV AB (2011)

Fuel-Optimal Goods Transportation

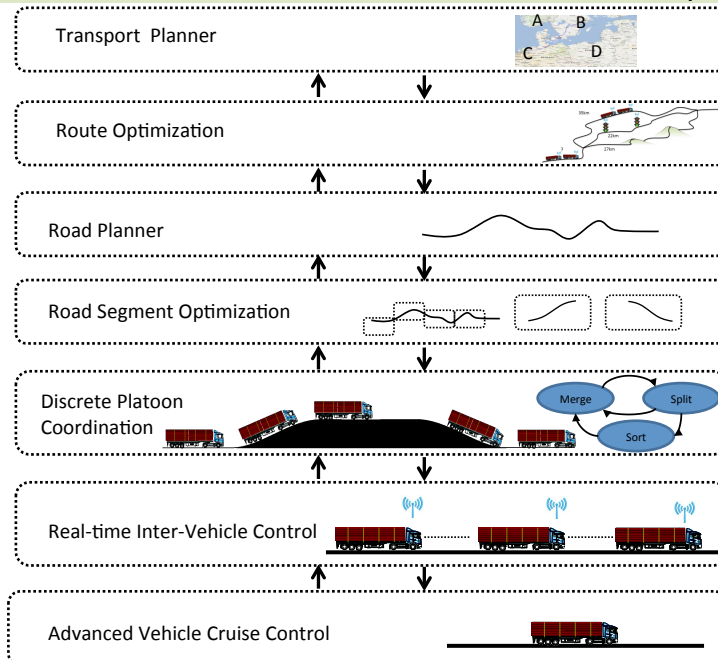
- Goods transported between cities over European highway network
- 2 000 000 long haulage trucks in European Union (400 000 in Germany)
- Large distributed control systems with no real-time coordination today

Goal: Maximize total amount of platooning with limited intervention in vehicle speed and route



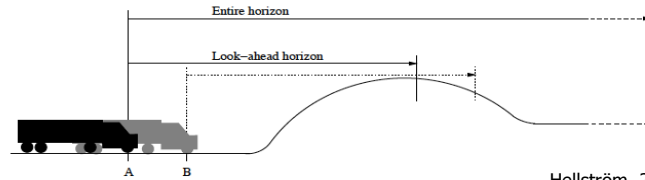
Larson et al., 2013

Architecture for Future Coordinated Goods Transportation



Alam et al., 2012

Receding Horizon Cruise Control for Single Vehicle



Hellström, 2007

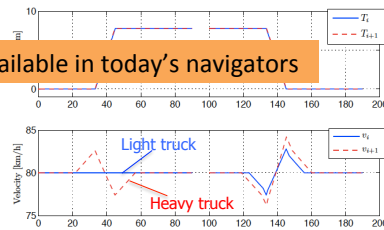
Adjust driving force to **minimize fuel consumption based on road topology** info:

The total fuel consumption over time T is:

$$f_t = \int_0^T \delta(t) \left(\frac{1}{2} \rho_a A_a C_D v^2(t) + mg c_r \cos \alpha + mg \sin \alpha \right) dt \quad (3)$$

Require knowledge of road grade α , not available in today's navigators

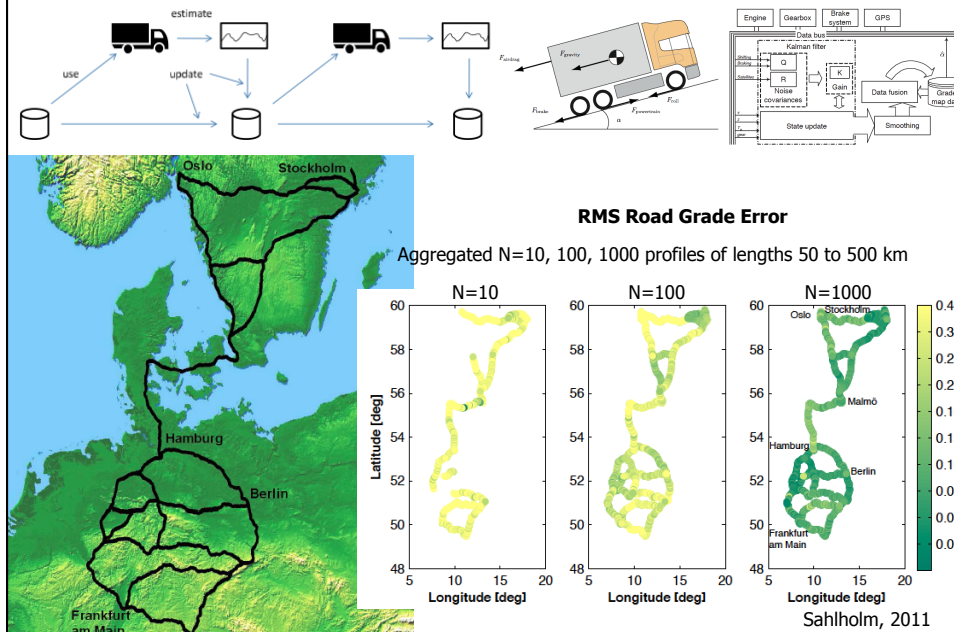
$$\begin{aligned} m_t \frac{dv}{dt} &= F_{eng} - F_b - F_{ad}(v, d) - F_r(\alpha) - F_g(\alpha) \\ &= F_{eng} - F_b - \frac{1}{2} \rho_a A_a C_D v^2 \phi(d) \\ &\quad - mg c_r \cos \alpha - mg \sin \alpha \end{aligned}$$



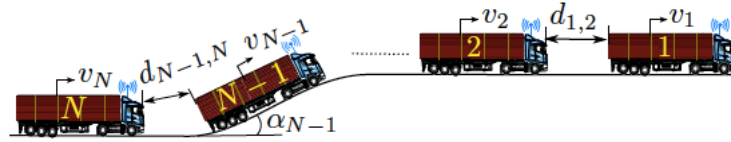
Implemented as velocity reference change in advance cruise controller

Alam et al., 2011

Distributed Road Grade Estimation



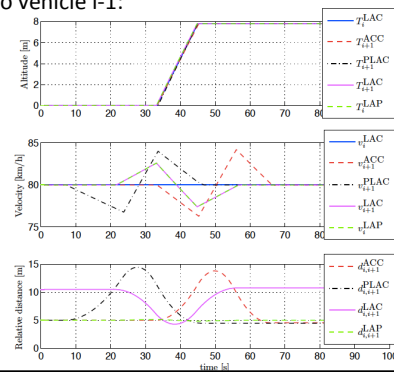
Receding Horizon Cruise Control for Platoon



- How to jointly minimize fuel consumption for a platoon of vehicles?
 - Uphill and downhill segments; heavy and light vehicles

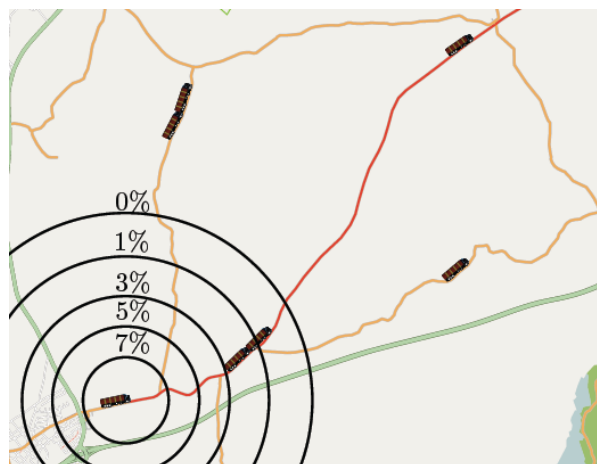
Dynamics of vehicle i depend on distance $d_{i-1,i}$ to vehicle $i-1$:

$$\begin{aligned} \frac{dd_{i-1,i}}{dt} &= v_{i-1} - v_i \\ m_{t_i} \frac{dv_i}{dt} &= F_{\text{engine}}(\delta_i, \omega_{e_i}) - F_{\text{brake}} - F_{\text{air drag}}(v_i, d_{i-1,i}) \\ &\quad - F_{\text{roll}}(\alpha_i) - F_{\text{gravity}}(\alpha_i) \\ &= k_i^e T_e(\delta_i, \omega_{e_i}) - F_{\text{brake}} - k_i^d v_i^2 f_i(d_{i-1,i}) \\ &\quad - k_i^{\text{fr}} \cos \alpha_i - k_i^g \sin \alpha_i \end{aligned}$$

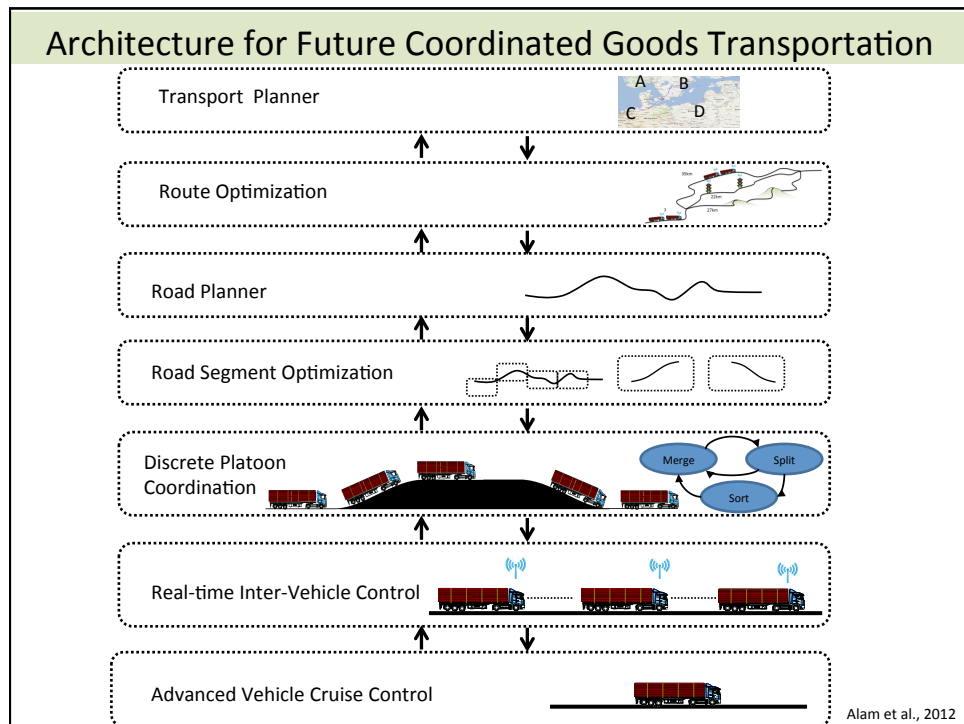


Alam et al., 2013

When is it Fuel Efficient for a Heavy-Duty Vehicle to Catch Up with a Platoon?

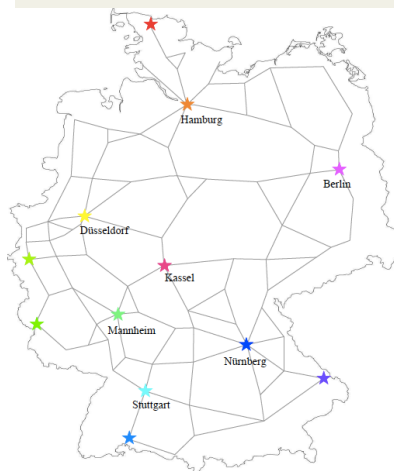


Liang et al., 2013



When and where to create platoons?

Goal: Maximize total amount of platooning with limited intervention in vehicle speed and route



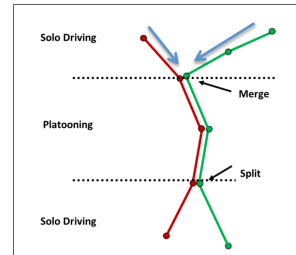
Larson et al., 2013

Platoon merge and split

Heavy-duty vehicle traffic without platooning



Merge and split platoons at highway intersections

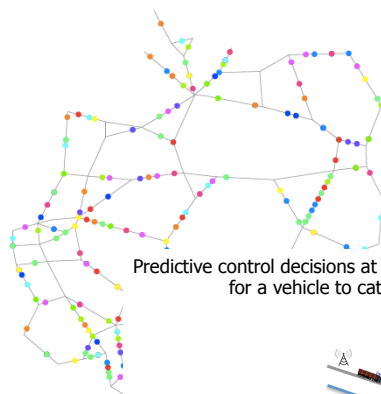


Only vehicles that are relatively close in space and time platoon

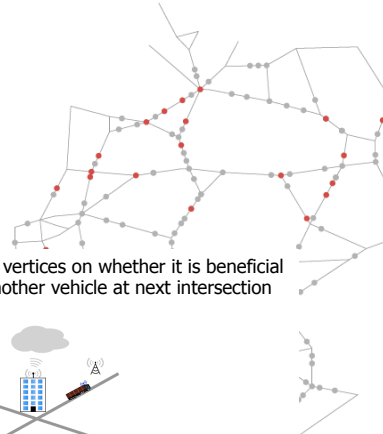
Larson et al., 2013

Distributed optimization of platooning

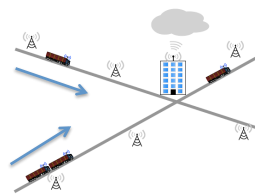
Heavy-duty vehicle traffic without platooning



With platooning

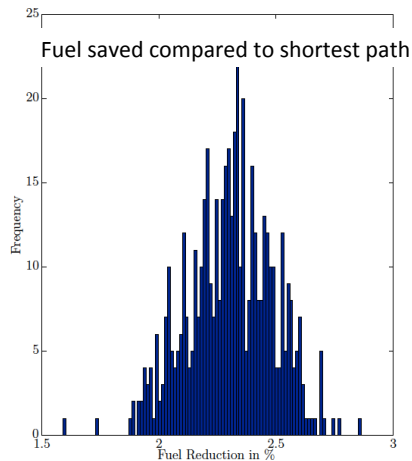


Predictive control decisions at network vertices on whether it is beneficial for a vehicle to catch up another vehicle at next intersection

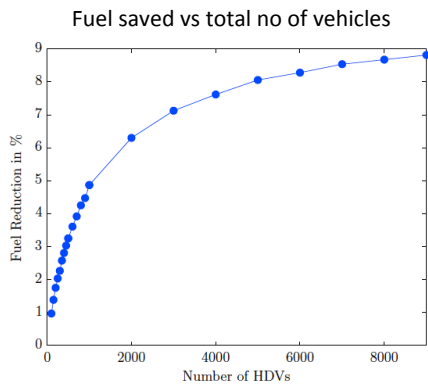


Larson et al., 2013

Numerical evaluations

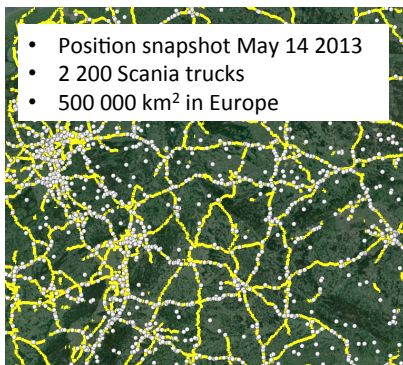


- German road network with 300 trucks
- Random starting points and destinations
- 500 experiments

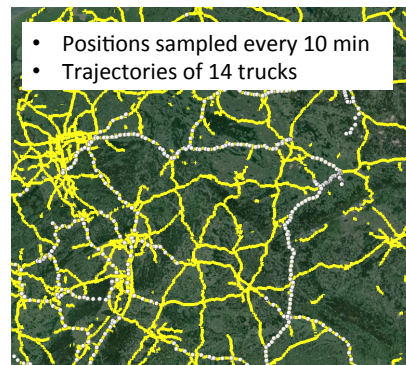


2-5% deployment enough for substantial benefit

Feasibility Study Based on Real Truck Data

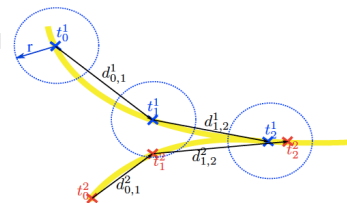


- Position snapshot May 14 2013
- 2 200 Scania trucks
- 500 000 km² in Europe



- Positions sampled every 10 min
- Trajectories of 14 trucks

- 875 long-haulage trucks over European region
- Trucks close in time and space ($< r$ m) could adjust speed to platoon and then save 10% fuel during platooning
- Benefits:
 - $r = 0.2$ km: 78 trucks platooned, 0.16% savings
 - $r = 1$ km: 241 trucks platooned, 0.38% savings
 - $r = 5$ km: 778 trucks platooned, 1.2% savings

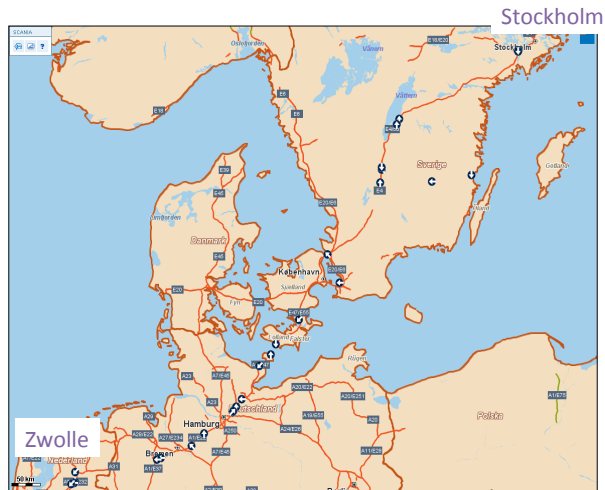


Larson et al., 2013

Stockholm-Zwolle 24/7 Testing

- Real-time fleet management
- Platooning in real traffic
- Fuel reductions and safety
- Driver acceptance
- Public acceptance

Scania Transport Lab
Internal haulage company
20 trucks, 360.000 km/year
75 trailers, 92% loaded
65 drivers, 40 h work/week



Demonstrations

Rapport on vehicle platooning developed by KTH and Scania (Oct, 2011)



*PhD student Assad Alam on
Discovery Channel (Jan, 2012)*



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Stockholm Royal Seaport

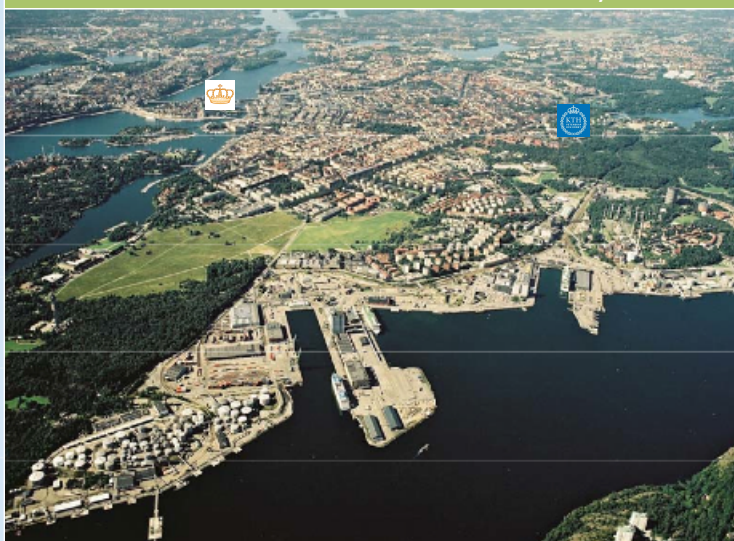
2010

- Oil depot
- Container terminal
- Ports
- Gas plant

2030

- 10,000 new homes
- 30,000 new work spaces
- 600,000 m² commercial space
- Modern port and cruise terminal
- 236 hectares sustainable urban district
- Walking distance to city centre

From a brown field area to a sustainable city district



Stockholm Royal Seaport

2010

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From a brown field area to a sustainable city district

Project Goals

- CO₂ emissions <1.5 tons per person by 2020 (today 4.5)
- Fossil fuel-free by 2030



Energy Consumption and Enabling Technologies

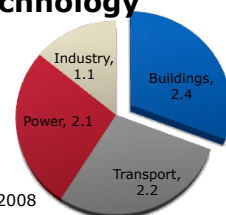


Energy consumption in Europe

- 40% of total energy use is in buildings
- 76% of building energy is for comfort

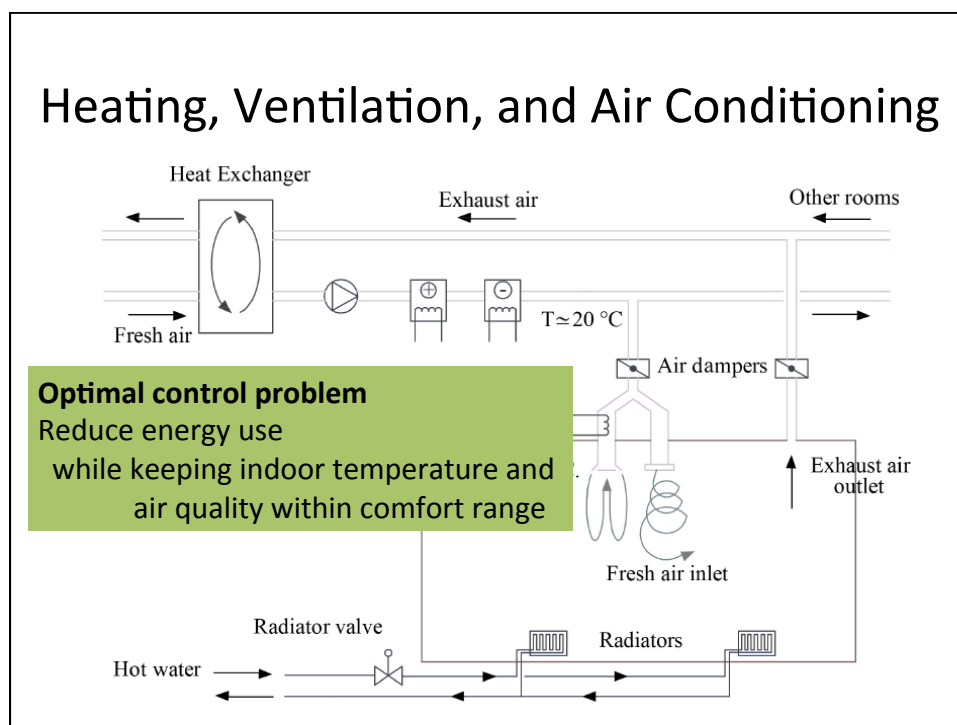
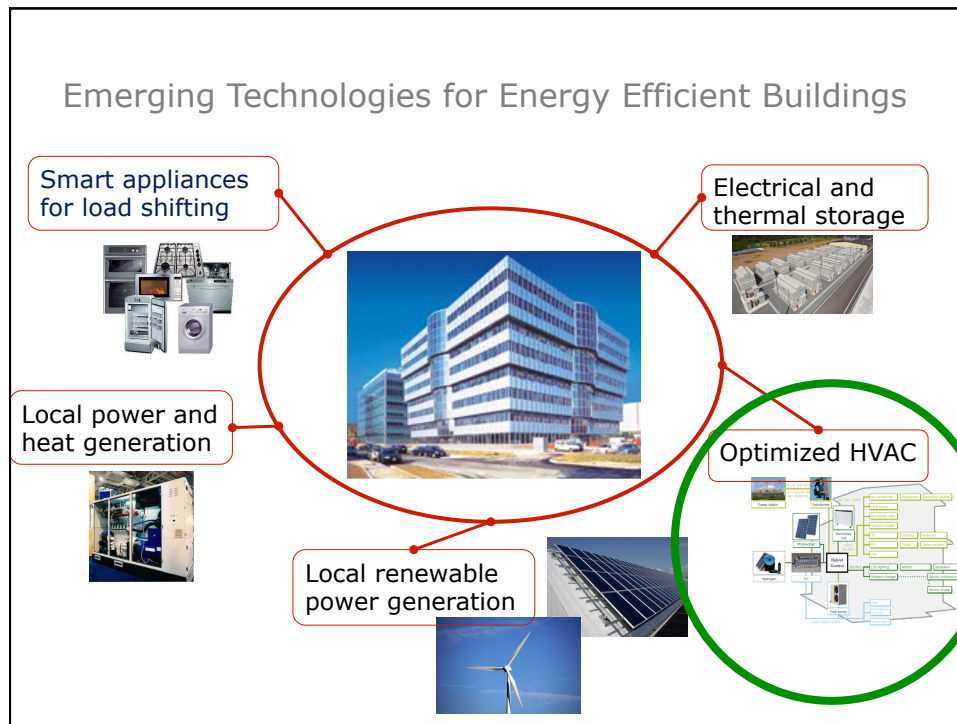
Enabling Information and Communication Technology

- Total energy savings of up to 15% by 2020
- Buildings can save 2.4 GtCO₂e
- Enormous CPS potentials

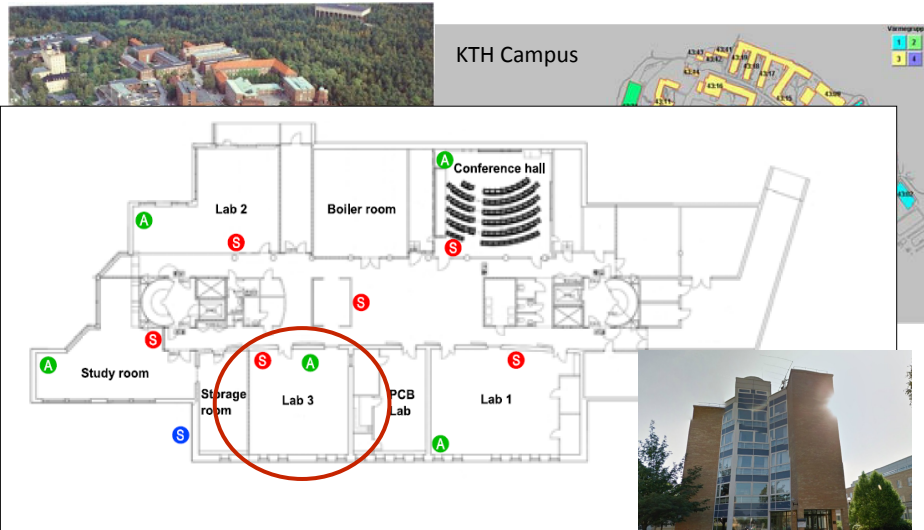


Energy efficiency requirements in building codes, International Energy Agency, Report, 2008

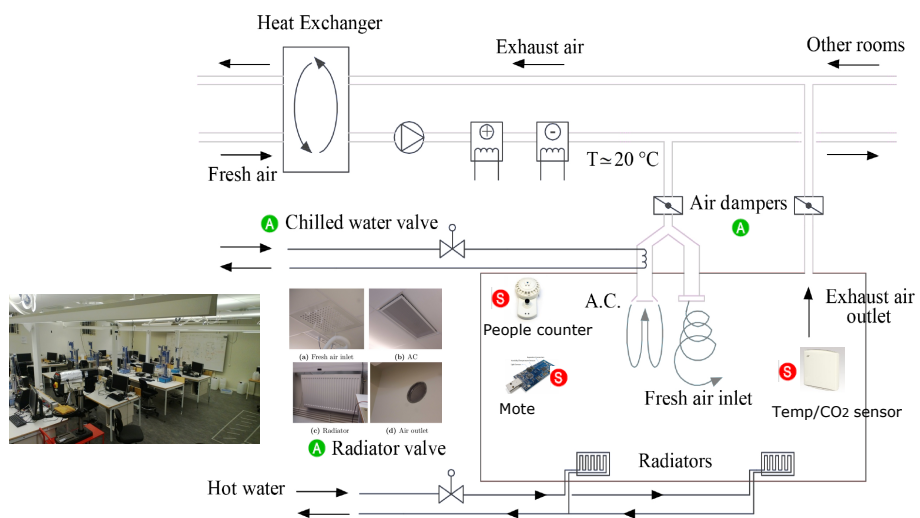
SMART 2020: Enabling the low carbon economy in the information age, The Climate Group, Report, 2008



KTH HVAC Testbed



KTH HVAC Testbed



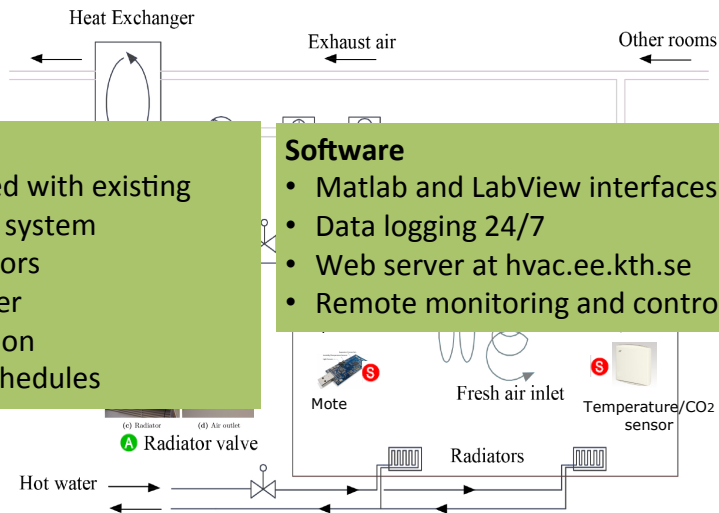
KTH HVAC Testbed

Hardware

- PLC integrated with existing HVAC SCADA system
- Wireless sensors
- People counter
- Weather station
- Occupancy schedules

Software

- Matlab and LabView interfaces
- Data logging 24/7
- Web server at hvac.ee.kth.se
- Remote monitoring and control

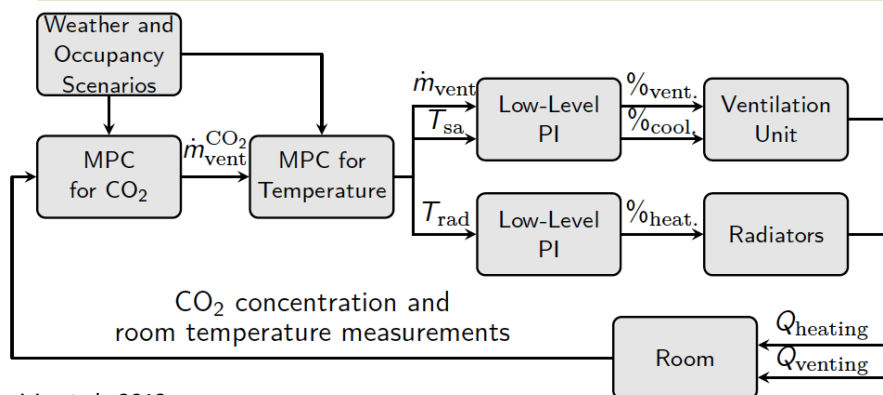


HVAC Control Architecture

Goal: Minimize energy use while satisfying comfort constraints

Approach: Scenario-based Model Predictive Control

- CO₂ MPC generates constraints for temperature MPC
- Probabilistic models of occupancy and weather forecasts errors
- Learn statistics from building operation to generate scenarios
- Air flow and temperature control from scenario-based optimization



Parisio et al., 2013

CO₂ model

$$x_{\text{CO}_2}(k+1) = ax_{\text{CO}_2}(k) + bu_{\text{CO}_2}(k) + ew_{\text{CO}_2}(k)$$

$$y_{\text{CO}_2}(k) = x_{\text{CO}_2}(k)$$

$$w_{\text{CO}_2}(k) = \text{occupancy at } k, \quad u_{\text{CO}_2}(k) = \dot{m}_{\text{vent}}(k)x_{\text{CO}_2}(k)$$

Temperature model

$$x_T(k+1) = A_T x_T(k) + B_T u_T(k) + E_T w_T(k)$$

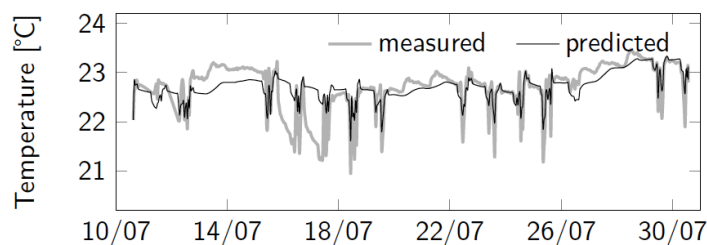
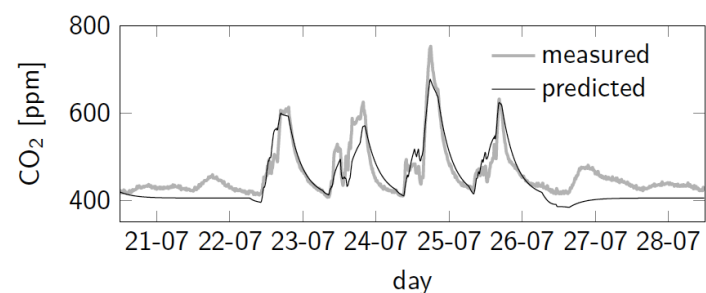
$$y_T(k) = C_T x_T(k)$$

$$w_T(k) = (\text{outside temperature, solar radiation, internal heat gain})$$

$$u_T(k) \rightarrow |Q_{\text{venting}}|, Q_{\text{heating}} \rightarrow (\dot{m}_{\text{vent}}(k), T_{\text{sa}}(k), T_{\text{rad}}(k))$$

Parisio et al., 2013

Model Validation



Parisio et al., 2013

Scenario-based **CO₂** MPC

Chance Constraints

$$\mathbb{P} \left[\dot{m}_{\text{vent}}^{\min} x_{\text{CO}_2}(k) \leq u_{\text{CO}_2}(k) \leq \dot{m}_{\text{vent}}^{\max} x_{\text{CO}_2}(k) \right] \geq 1 - \alpha \quad (\text{flow rate})$$

$$\mathbb{P} \left[y_{\min} \leq y_{\text{CO}_2}(k) \leq y_{\max} \right] \geq 1 - \alpha \quad (\text{air quality})$$

Inputs Constraints

$$u_{\min} \leq u_{\text{CO}_2}(k) \leq u_{\max}$$

Cost Function

$$\sum_{k=0}^{N-1} c'(u(k)\Delta k) \quad (\text{minimize energy use})$$

Compute Control Inputs

$$\dot{m}_{\text{vent}}^{\text{CO}_2}(k) = \frac{u_{\text{CO}_2}(k)}{x_{\text{CO}_2}(k)}$$

Parisio et al., 2013

Scenario-based **Temp** MPC

Chance Constraints

$$\mathbb{P} \left[y_{\min} \leq y_T(k) \leq y_{\max} \right] \geq 1 - \alpha_T \quad (\text{thermal comfort})$$

Inputs Constraints

$$u_{\min} \leq u_T(k) \leq u_{\max}$$

Cost Function

$$\sum_{k=0}^{N-1} c'_T(u_T(k)\Delta k) \quad (\text{minimize energy use})$$

Compute Setpoints for the Low-level Controllers

$$\left(\dot{m}_{\text{vent}}(k), T_{\text{sa}}(k), T_{\text{rad}}(k) \right) = f \left(\dot{m}_{\text{vent}}^{\text{CO}_2}(k), u_T(k) \right)$$

Parisio et al., 2013

How to Handle Chance Constraints

ω := random variable (weather, occupancy, ...)

Uncertainty Modeling

$$\omega(k) = \bar{\omega}(k) + \tilde{\omega}(k)$$

- $\bar{\omega}(k)$:= forecast
- $\tilde{\omega}(k)$:= forecast error

How to Handle Chance Constraints

ω := random variable (weather, occupancy, ...)

Uncertainty Modeling

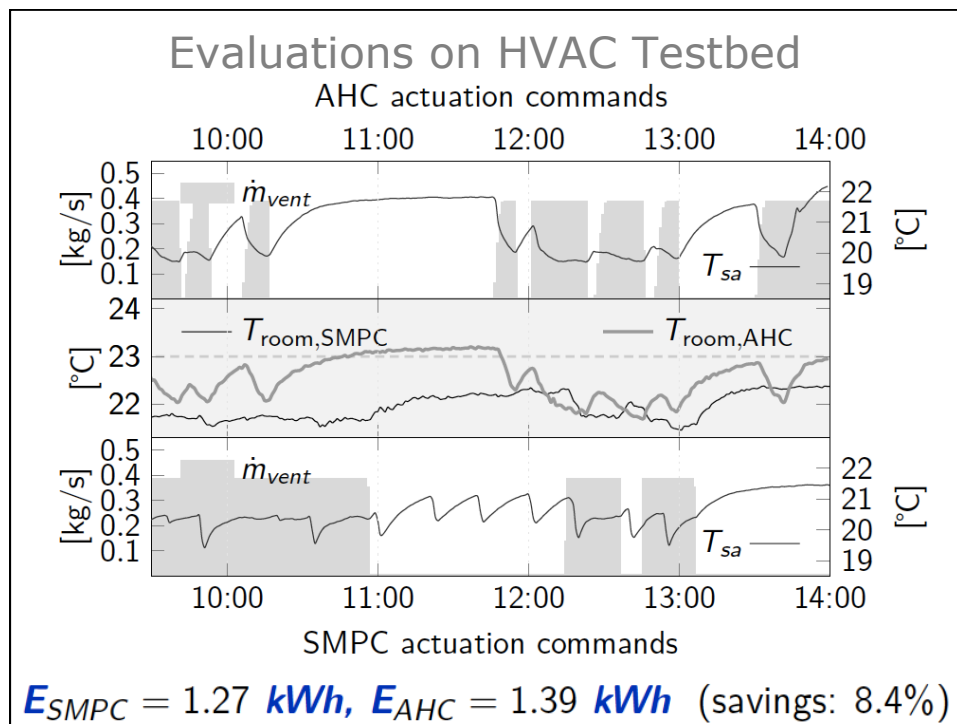
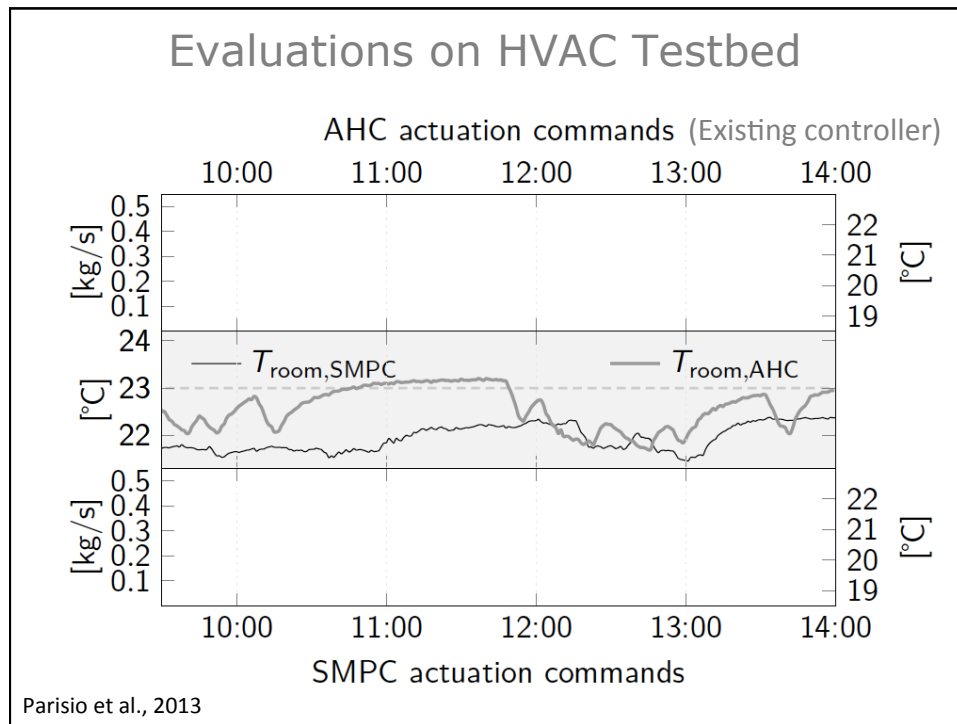
$$\omega(k) = \bar{\omega}(k) + \tilde{\omega}(k)$$

- $\bar{\omega}(k)$:= forecast
- $\tilde{\omega}(k)$:= forecast error

Approximating Chance Constraints

[Calafiore, 2010]

- extract a limited number $S = \frac{2}{\alpha} \left(\ln \left(\frac{1}{\beta} \right) + N \cdot n_u \right)$ of i.i.d. outcomes (called **scenarios**)
- approximate $\mathbb{P}[y_{\min} \leq y(k) \leq y_{\max}] \geq 1 - \alpha$ with $y_{\min} \leq y(\hat{\omega}^j(k)) \leq y_{\max}, \quad \forall j = 1, \dots, S$
- remove redundant constraints: $\max_j \{y(\hat{\omega}^j(k))\} \leq y_{\max}$



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Cyber-Physical Systems Challenges

Societal Scale

- Global and dense instrumentation of physical phenomena
- Interacting with a computational environment: closing the loop
- Security, privacy, usability

Distributed Services

- Self-configuring, self-optimization
- Reliable performance despite uncertain components, resilient aggregation

Programming the Ensemble

- Local rules with guaranteed global behavior
- Distributing control with limited information

Network Architectures

- Heterogeneous systems: local sensor/actuator networks and wide-area networks
- Self-organizing multi-hop, resilient, energy-efficient routing
- Limited storage, noisy channels

Real-Time Operating Systems

- Extensive resource-constrained concurrency
- Modularity and data-driven physics-based modeling

1000 Radios per Person

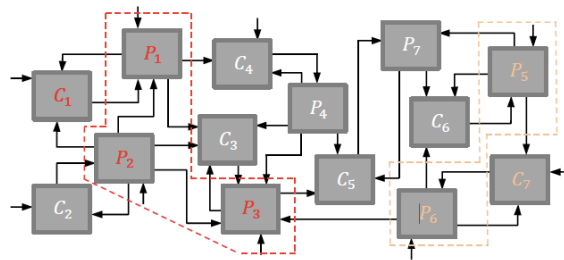
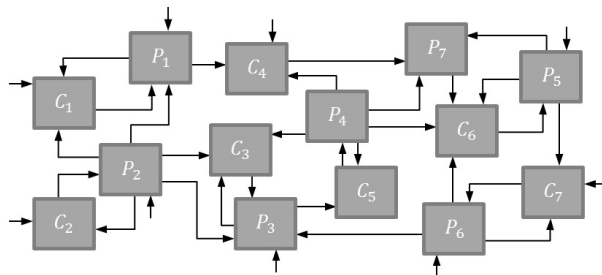
- Low-power processors, radio communication, encryption
- Coordinated resource management, spectrum efficiency

Sastry & J, 2010



How to analyze, design, and implement networked control with

- Guaranteed **global objective** from local interactions
- **Physical dynamics** coupled with information interactions
- Tradeoff **computation-communication-control** complexities
- **Robustness to** external disturbances other **uncertainties**



- Decentralized control extensively studied:
Witsenhausen; Ho & Chu; Sandell & Athans; Anderson & Moore; Siljak; Davison & Chang; Rotkowitz & Lall; etc
- Typically assumes full model information (knowledge of all P_i)
- What if at the design of C_1 only surrounding P_j 's are known?

The role of plant model information



Inter-vehicle distances d_{12} and d_{23} are locally controlled through vehicle torques u_i

$$\begin{bmatrix} \dot{v}_1(t) \\ \dot{d}_{12}(t) \\ \dot{v}_2(t) \\ \dot{d}_{23}(t) \\ \dot{v}_3(t) \end{bmatrix} = \begin{bmatrix} -\rho_1/m_1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -\rho_2/m_2 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -\rho_3/m_3 \end{bmatrix} \begin{bmatrix} v_1(t) \\ d_{12}(t) \\ v_2(t) \\ d_{23}(t) \\ v_3(t) \end{bmatrix} + \begin{bmatrix} b_1/m_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_2/m_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_3/m_3 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} + \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \\ w_4(t) \\ w_5(t) \end{bmatrix}$$

How does knowledge of the vehicle mass m_i influence performance?

Example

$$\begin{aligned} x_1(k+1) &= a_{11}x_1(k) + a_{12}x_2(k) + u_1(k) \\ x_2(k+1) &= a_{21}x_1(k) + a_{22}x_2(k) + u_2(k) \end{aligned} \quad J = \sum_{k=1}^{\infty} \|x(k)\|^2 + \|u(k)\|^2$$

Keep J small, when

Controller 1 knows only a_{11} and a_{12}

Controller 2 knows only a_{21} and a_{22}

$$\begin{aligned} u_1(k) &= -a_{11}x_1(k) - a_{12}x_2(k) \\ u_2(k) &= -a_{21}x_1(k) - a_{22}x_2(k) \end{aligned} \quad \text{achieves } J \leq 2J^*$$

No limited plant model information strategy can do better.

Why Limited Plant Model Information?

Complexity

Controllers are easier to implement and maintain if they mainly depend on local model information



Availability

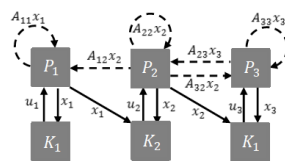
The model of other subsystems is not available at the time of design



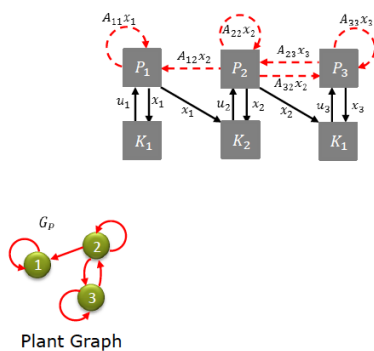
Privacy

Competitive advantages not to share private model information

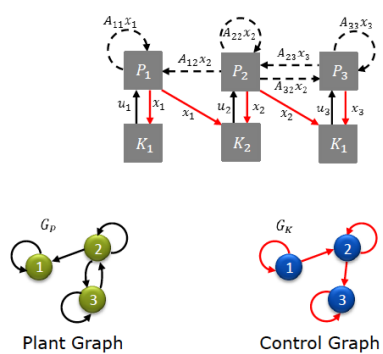
Networked Control System



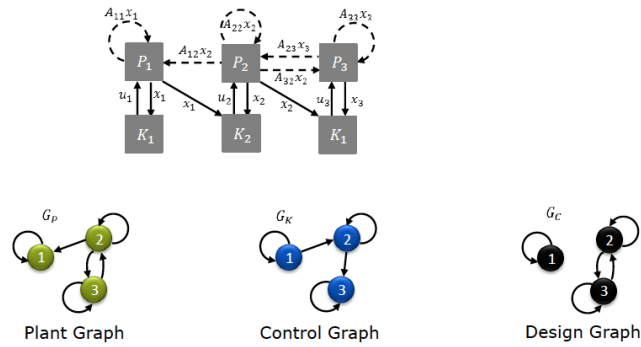
Networked Control System



Networked Control System



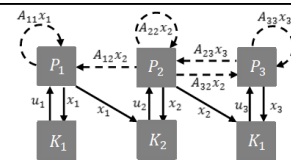
Networked Control System



Physical Constraints

Model Information Limitations

Plant Graph

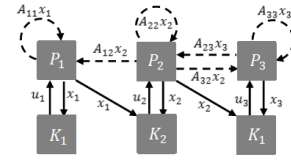


$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j \neq i} A_{ij}x_j(k) + B_{ii}u_i(k)$$

Plant: $P = (A, B, x_0) \in \mathcal{A} \times \mathcal{B} \times \mathbb{R}^n$

$x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{n_i}$

Plant Graph

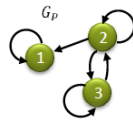


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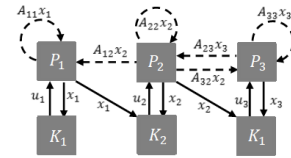
$$\mathcal{A} = \{ A \in \mathbb{R}^{n \times n} \mid A_{ij} = 0 \in \mathbb{R}^{n_i \times n_j} \text{ for all } 1 \leq i, j \leq q \text{ such that } (s_p)_{ij} = 0 \}$$



$$S_P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0_{n_1 \times n_3} \\ 0_{n_2 \times n_1} & A_{22} & A_{23} \\ 0_{n_3 \times n_1} & A_{32} & A_{33} \end{bmatrix}$$

Plant Graph

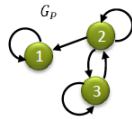


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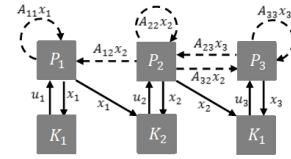
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$$\mathcal{B} = \{ B \in \mathbb{R}^{n \times n} \mid \underline{\sigma}(B) \geq \epsilon, B_{ij} = 0 \in \mathbb{R}^{n_i \times n_j} \text{ for all } 1 \leq i \neq j \leq q \}$$

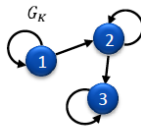
$$B = \begin{bmatrix} B_{11} & 0_{n_1 \times n_2} & 0_{n_1 \times n_3} \\ 0_{n_2 \times n_1} & B_{22} & 0_{n_2 \times n_3} \\ 0_{n_3 \times n_1} & 0_{n_3 \times n_2} & B_{33} \end{bmatrix}$$

Control Graph



$$u(k) = Kx(k)$$

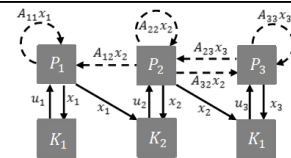
$$\mathcal{K} = \{ K \in \mathbb{R}^{n \times n} \mid K_{ij} = 0 \in \mathbb{R}^{n_i \times n_j} \text{ for all } 1 \leq i, j \leq q \text{ such that } (s_K)_{ij} = 0 \}$$



$$S_K = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

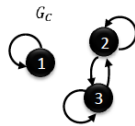
$$K = \begin{bmatrix} K_{11} & 0_{n_1 \times n_2} & 0_{n_1 \times n_3} \\ K_{21} & K_{22} & 0_{n_2 \times n_3} \\ 0_{n_3 \times n_1} & K_{32} & K_{33} \end{bmatrix}$$

Design Graph



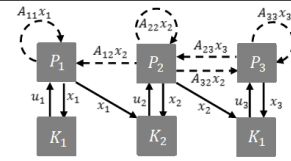
$$K = \Gamma(P) = \Gamma(A, B)$$

The map $[\Gamma_{i1} \ \cdots \ \Gamma_{iq}]$ is only a function of $\{[A_{j1} \ \cdots \ A_{jq}], B_{jj} \mid (s_C)_{ij} \neq 0\}$.



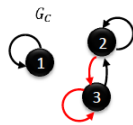
$$S_C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Design Graph



$$K = \Gamma(P) = \Gamma(A, B)$$

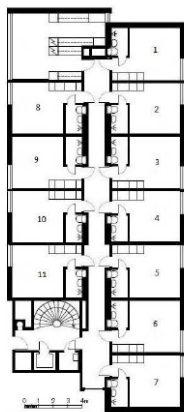
The map $[\Gamma_{i1} \ \cdots \ \Gamma_{iq}]$ is only a function of $\{[A_{j1} \ \cdots \ A_{jq}], B_{jj} \mid (s_C)_{ij} \neq 0\}$.



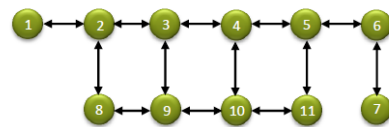
$$S_C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$[\Gamma_{31} \ \Gamma_{32} \ \Gamma_{33}]$ is a function of $\{[A_{21} \ A_{22} \ A_{23}], B_{22}, [A_{31} \ A_{32} \ A_{33}], B_{33}\}$

HVAC Control Example



Plant Graph:



Design Graph:



Performance Metric

The **competitive ratio** of a control design method Γ is defined as

$$r_P(\Gamma) = \sup_{P \in \mathcal{P}} \frac{J_P(\Gamma(A, B))}{J_P(K^*(P))}$$

Performance Metric

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$$r_P(\Gamma) = \sup_{P \in \mathcal{P}} \frac{J_P(\Gamma(A, B))}{J_P(K^*(P))}$$

A control design method Γ' is said to **dominate** another control design method Γ if

$$J_P(\Gamma'(A, B)) \leq J_P(\Gamma(A, B)), \quad \text{for all } P = (A, B, x_0) \in \mathcal{P}$$

with strict inequality holding for at least one plant.

When no such Γ' exists, we say that Γ is **undominated**.

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$$J_P(K) = \sum_{k=1}^{\infty} x(k)^T Q x(k) + \sum_{k=0}^{\infty} u(k)^T R u(k)$$

Q and R are block-diagonal positive definite matrices.

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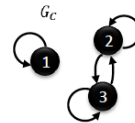
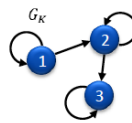
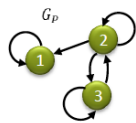
Remark: When G_K is a complete graph

$$K^*(P) = -(R + B^T X B)^{-1} B^T X A \\ A^T X A - A^T X B (R + B^T X B)^{-1} B^T X A - X + Q = 0$$

Problem Formulation

Find the best control design strategy with limited model information:

$$\min_{\Gamma \in \mathcal{C}} r_P(\Gamma)$$



Characterize the influence from

- Plant structure (G_P)
- Controller communication (G_K)
- Model limitation (G_C)

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Assumptions

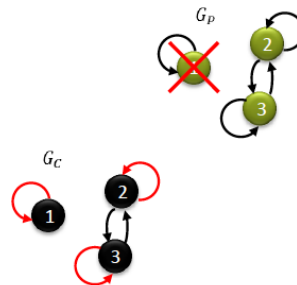
- All subsystems are fully actuated:

$$B \in \mathbb{R}^{n \times n} \text{ and } \underline{\sigma}(B) \geq \epsilon > 0.$$

- G_P contains no isolated node.

- G_C contains all self-loops.

- To simplify the presentation, fix $\epsilon = 1$ and $Q = R = I$.

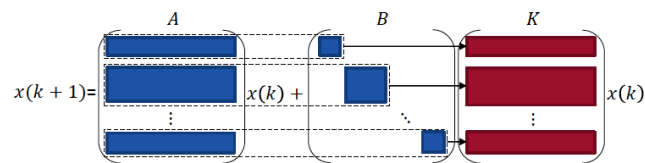


Deadbeat Control Design

$$\Gamma^\Delta(A, B) = -B^{-1}A$$

Subcontroller i depends only on subsystem i 's model:

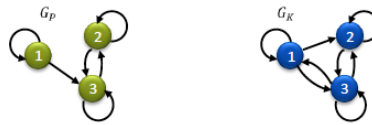
$$[\Gamma_{i1}^\Delta(A, B) \quad \cdots \quad \Gamma_{iq}^\Delta(A, B)] = -B_{ii}^{-1}[A_{i1} \quad \cdots \quad A_{iq}]$$



$$x(k+1) = Ax(k) + Bu(k) ; x(0) = x_0,$$

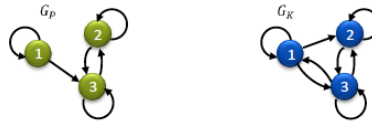
Deadbeat Control Design

Lemma: $G_K \supseteq G_P \Rightarrow r_P(\Gamma^\Delta) = 2$



Deadbeat Control Design

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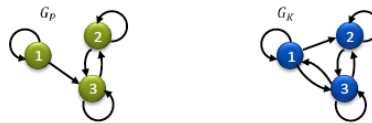


- $G_K \supseteq G_P$ means $E_K \supseteq E_P$, so more controller communications than plant interactions

Farokhi et al., 2013

Deadbeat Control Design

Lemma: $G_K \supseteq G_P \Rightarrow r_P(\Gamma^\Delta) = 2$

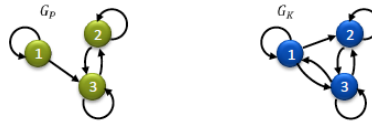


- $G_K \supseteq G_P$ means $E_K \supseteq E_P$
- $J_P(\Gamma^\Delta(A, B)) \leq 2J_P(K^*(P))$, so deadbeat never worse than twice the optimal controller

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Deadbeat Control Design

Lemma: $G_K \supseteq G_P \Rightarrow r_P(\Gamma^\Delta) = 2$

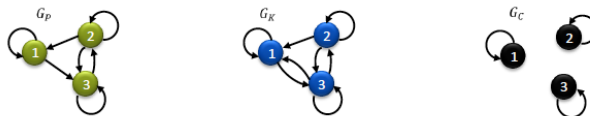


- $G_K \supseteq G_P$ means $E_K \supseteq E_P$
- $J_P(\Gamma^\Delta(A, B)) \leq 2J_P(K^*(P))$

If enough controller communication, then a simple (deadbeat) controller is quite good

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Design Strategies with Local Model Info

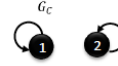
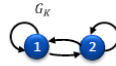


Theorem: $\left. \begin{array}{l} G_P \text{ has no sink} \\ G_K \supseteq G_P \\ G_C \text{ is fully disconnected} \end{array} \right\} \Rightarrow r_P(\Gamma) \geq r_P(\Gamma^\Delta) = 2 \quad \forall \Gamma \in \mathcal{O}$

When G_P has no sink, there is no control design strategy Γ with a better competitive ratio $r_P(\Gamma) = \sup_{P \in \mathcal{P}} J_P(\Gamma(A, B)) / J_P(K^*(P))$ than deadbeat Γ^Δ

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Example



$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix}, \quad \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

- $K^*(P) = -(I + X)^{-1}XA$ $A^T X A - A^T X (I + X)^{-1} X A + I = X$
- $\Gamma^\Delta(A, B) = -\begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix}$ $J_P(\Gamma^\Delta(A, B)) \leq 2J_P(K^*(P))$
- $\Gamma^\Theta(A, B) = -\begin{bmatrix} w a_{11} & w a_{12} \\ 0 & a_{22} \end{bmatrix}$ $J_P(\Gamma^\Theta(A, B)) \leq J_P(\Gamma^\Delta(A, B)) \leq 2J_P(K^*(P))$
 $w = \frac{a_{11}^2 - 2 + \sqrt{a_{11}^4 + 4}}{2a_{11}^2}$ and undominated

Motivating Example Revisited



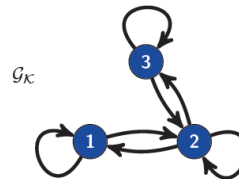
- Regulating inter-vehicle distances d_{12} and d_{23}

$$\begin{bmatrix} \dot{v}_1(t) \\ \dot{d}_{12}(t) \\ \dot{v}_2(t) \\ \dot{d}_{23}(t) \\ \dot{v}_3(t) \end{bmatrix} = \begin{bmatrix} -\varrho_1/m_1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -\varrho_2/m_2 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -\varrho_3/m_3 \end{bmatrix} \begin{bmatrix} v_1(t) \\ d_{12}(t) \\ v_2(t) \\ d_{23}(t) \\ v_3(t) \end{bmatrix} + \begin{bmatrix} b_1/m_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_2/m_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_3/m_3 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} + \begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \\ w_4(t) \\ w_5(t) \end{bmatrix}$$

$$z(t) = [d_{12}(t) \quad d_{23}(t) \quad u_1(t) \quad u_2(t) \quad u_3(t)]^\top$$

- Find a saddle point of $J(\Gamma, \alpha) = \|T_{zw}(s; \Gamma, \alpha)\|_\infty$ when $\alpha = [m_1 \ m_2 \ m_3]^\top \in [0.5, 1.0]^3$ and Γ belongs to the set of polynomials of α_i , $i = 1, 2, 3$, up to the second order.

$$\inf_{\Gamma \in \mathcal{C}} \sup_{\alpha \in \mathcal{A}} J(\Gamma, \alpha) = \inf_{\Gamma \in \mathcal{C}} \sup_{\alpha \in \mathcal{A}} \|T_{zw}(s; \Gamma, \alpha)\|_\infty$$

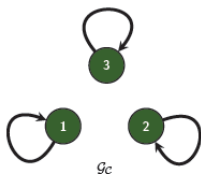


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Motivating Example Revisited

Control Design with Local Model Information

$$\max_{\alpha \in \mathcal{A}} \|T_{zw}(s; \Gamma^{\text{local}}, \alpha)\|_{\infty} = 4.7905$$

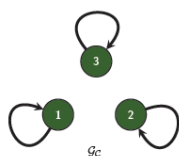


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Motivating Example Revisited

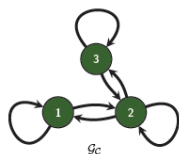
Control Design with Local Model Information

$$\max_{\alpha \in \mathcal{A}} \|T_{zw}(s; \Gamma^{\text{local}}, \alpha)\|_{\infty} = 4.7905$$



Control Design with Limited Model Information

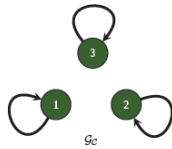
$$\max_{\alpha \in \mathcal{A}} \|T_{zw}(s; \Gamma^{\text{limited}}, \alpha)\|_{\infty} = 3.5533$$



25.8%

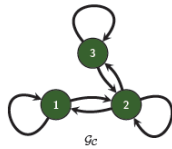
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Motivating Example Revisited



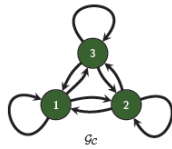
Control Design with Local Model Information

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Control Design with Limited Model Information

$$\max_{\alpha \in \mathcal{A}} \|T_{zw}(s; \Gamma^{\text{limited}}, \alpha)\|_{\infty} = 3.5533$$



Control Design with Full Model Information

$$\max_{\alpha \in \mathcal{A}} \|T_{zw}(s; \Gamma^{\text{full}}, \alpha)\|_{\infty} = 3.3596$$

25.8%

5.4%

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Outline

- Introduction
- Case study I: Goods transportation
- Case study II: Building management
- Cross-cutting scientific challenges
- Conclusions

Conclusions

- CPS architectures for large-scale control and optimization
- Applications to transportation and building management
- Influence of local plant models on global performance
- Testbed developments

<http://www.ee.kth.se/~kallej>



VW and Scania management visiting the student testbed of KTH Smart Mobility Lab



Finland's, Sweden's, and Denmark's Prime Ministers visiting the "Active House" in the Stockholm Royal Seaport