



HARVARD

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Electricity Market for Distribution Networks

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Oct. 17th, 2016

Electricity Grid 1.0



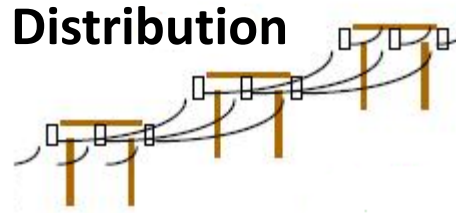
Power plant



Transmission



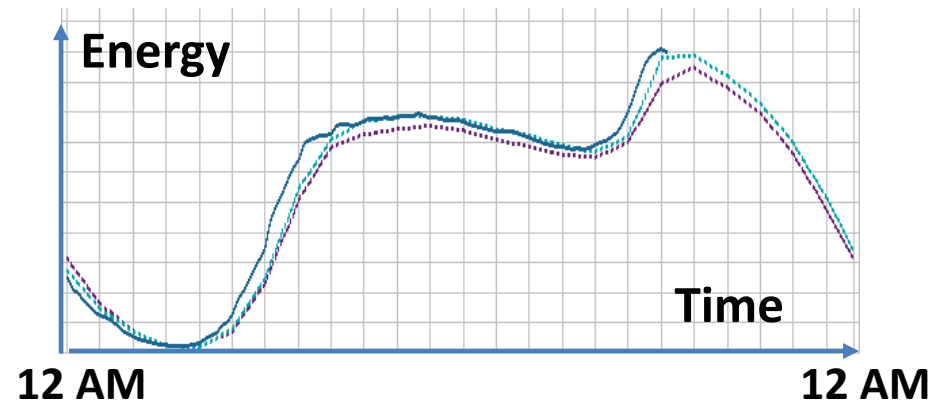
Users



Distribution

Supply = Demand

**Unresponsive
Predictable**



Electricity Grid 1.0



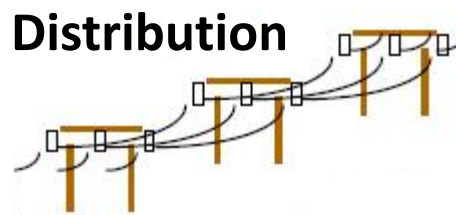
Power plant



Transmission



Users



Distribution

Supply = Demand

Controllable

Unresponsive

Predictable

Transmission market



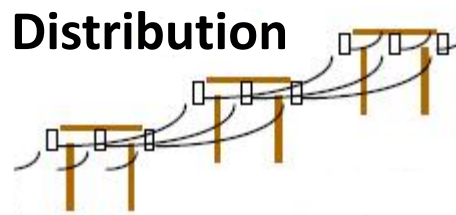
Power plant



Transmission



Users



Distribution

Supply = Demand

Trans. Market

Controllable

**Unresponsive
Predictable**

**A Monthly
Bill**

Forward Energy Market

e.g., Day-ahead market (one day forward);

Real-time Energy Market

e.g., Every five minutes in PJM;

Ancillary service market

e.g., Spinning reserve market; (short-term, unexpected changes)

Transmission market

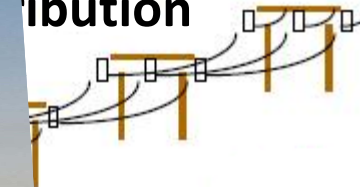


Power plant



Transmission

Distribution



Demand

Unresponsive

Predictable



Day-ahead market (one day forward);

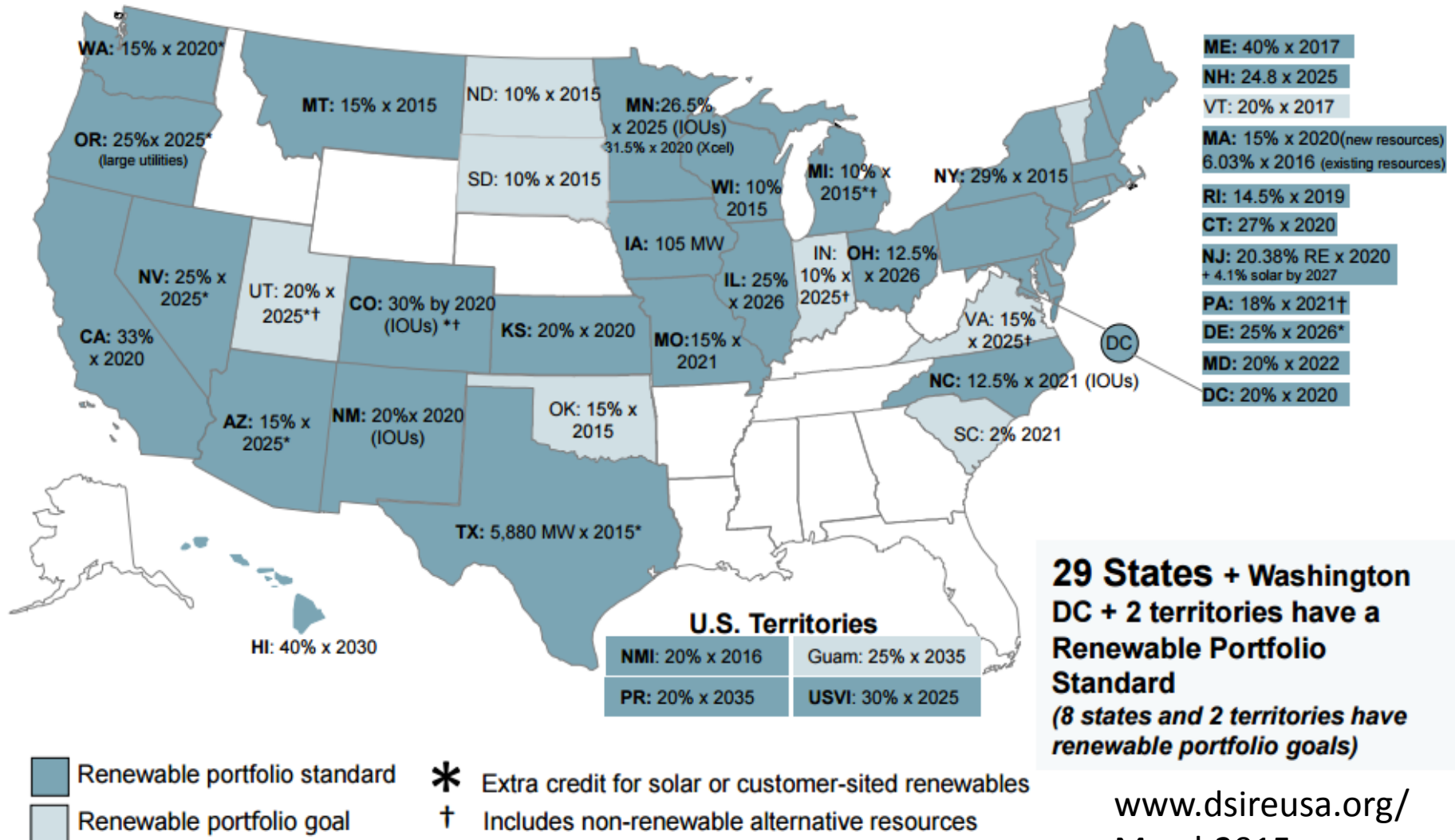
Real-time Energy Market

e.g., Every five minutes in PJM;

Ancillary service market

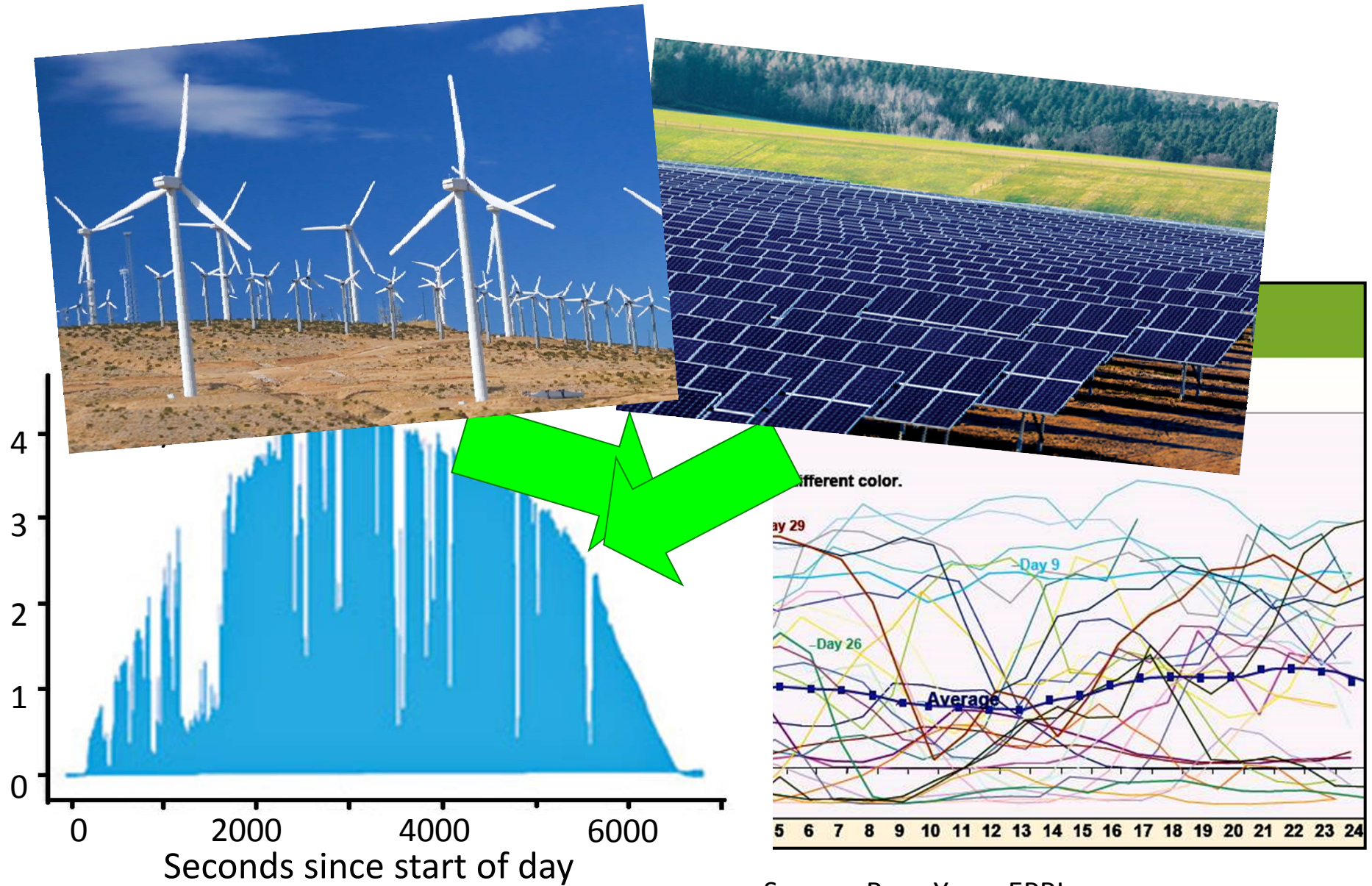
e.g., Spinning reserve market; (short-term, unexpected changes)

Renewable energy



www.dsireusa.org/
 March 2015

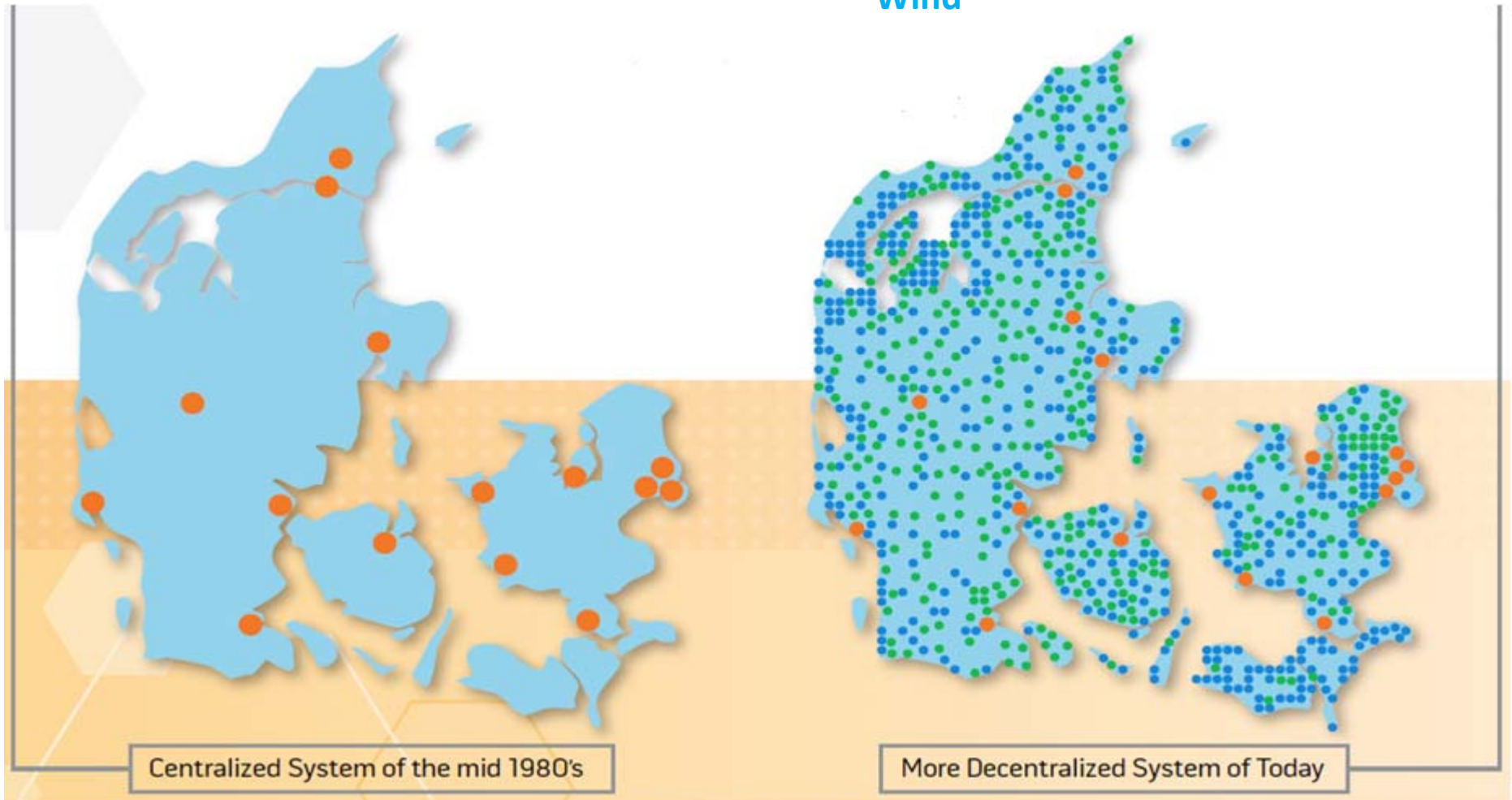
Random and intermittent



Source: Rosa Yang, EPRI

More distributed

- Small CHP (Combined Heat & Power)
- Large CHP (Combined Heat & Power)
- Wind



Denmark's progress over the past decades

Tomorrow's Grid 2.0



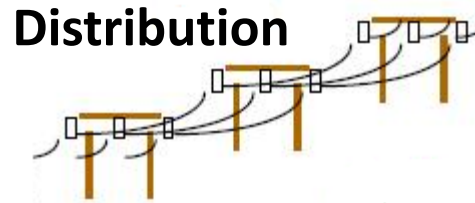
Power plant



Transmission



Users



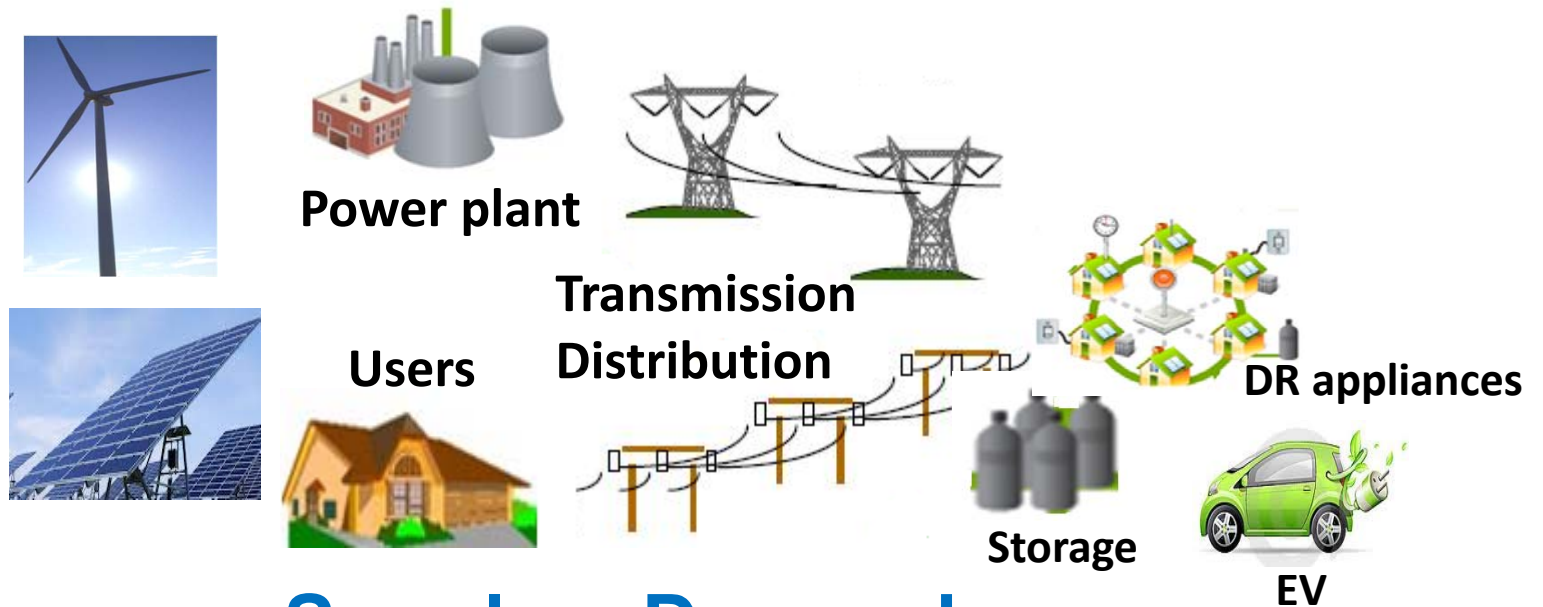
Distribution

Supply = Demand

Less controllable
Highly uncertain
Distributed
Large scale

Responsive ← Unresponsive

Tomorrow's Grid 2.0



Supply = Demand

Less controllable
Highly uncertain
Distributed

Responsive

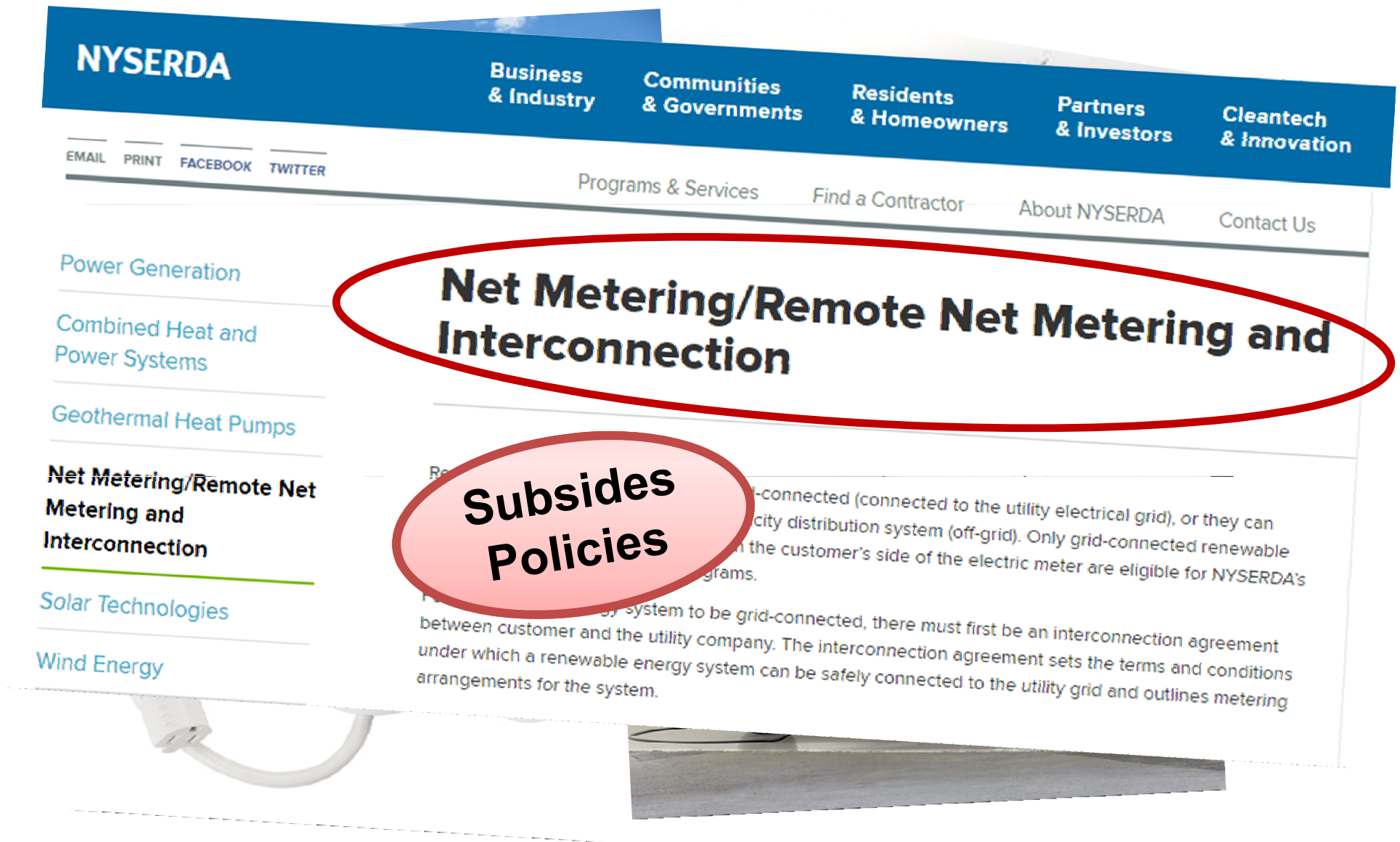
Large scale

Solar PVs
Wind turbines

**Distributed
Energy
Resources**

Smart appliances
Storage
Electric vehicles

Transforming Electricity Grid: DER



Debate over solar rates simmers in the Nevada desert

February 27, 2016



The future of home-based solar power is on the line in Nevada, as solar advocates and utility companies debate how to **regulate so-called 'net energy metering' rates** for customers using solar panels **connected to the grid**.

Sources: PBS

Electricity Market for Distribution Networks: Challenges

Power Engineering:

Power flow, system dynamics, operation constraints

Human Incentive:

Strategic behavior, self-interested, market power

Uncertainties:

Renewable energy, user's behavior, emergency

Electricity Market for Distribution Networks: Challenges

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Power flow, system dynamics, operation constraints

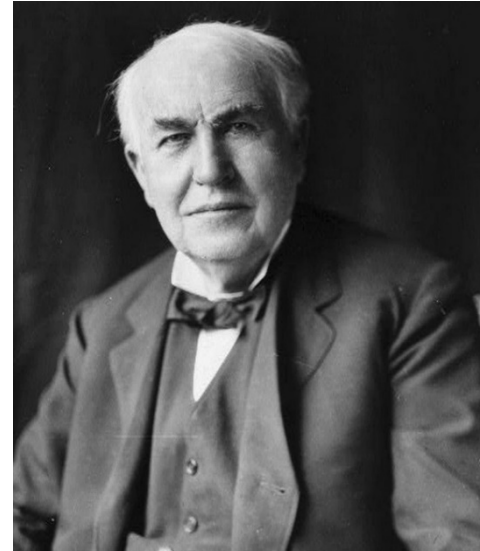
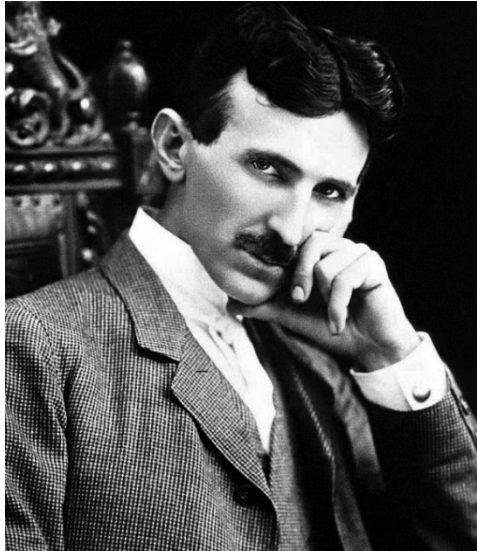
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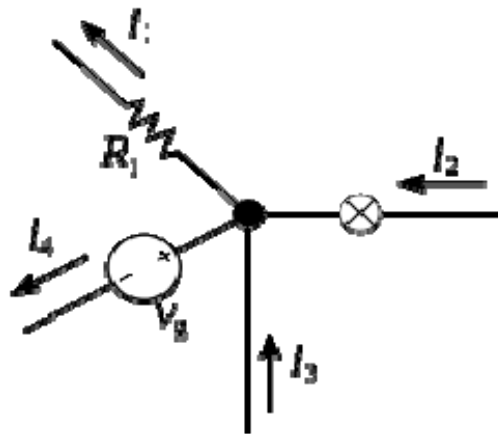
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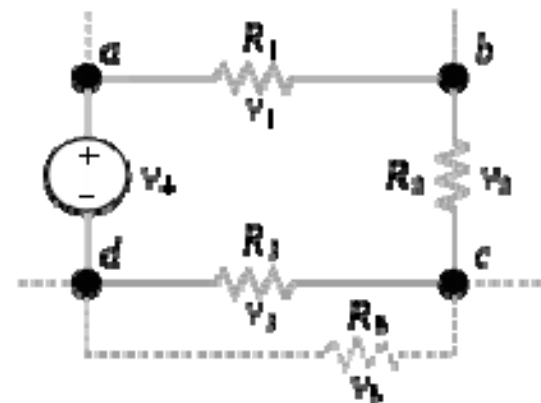
Transmit and Distribute Power



Kirchhoff's law

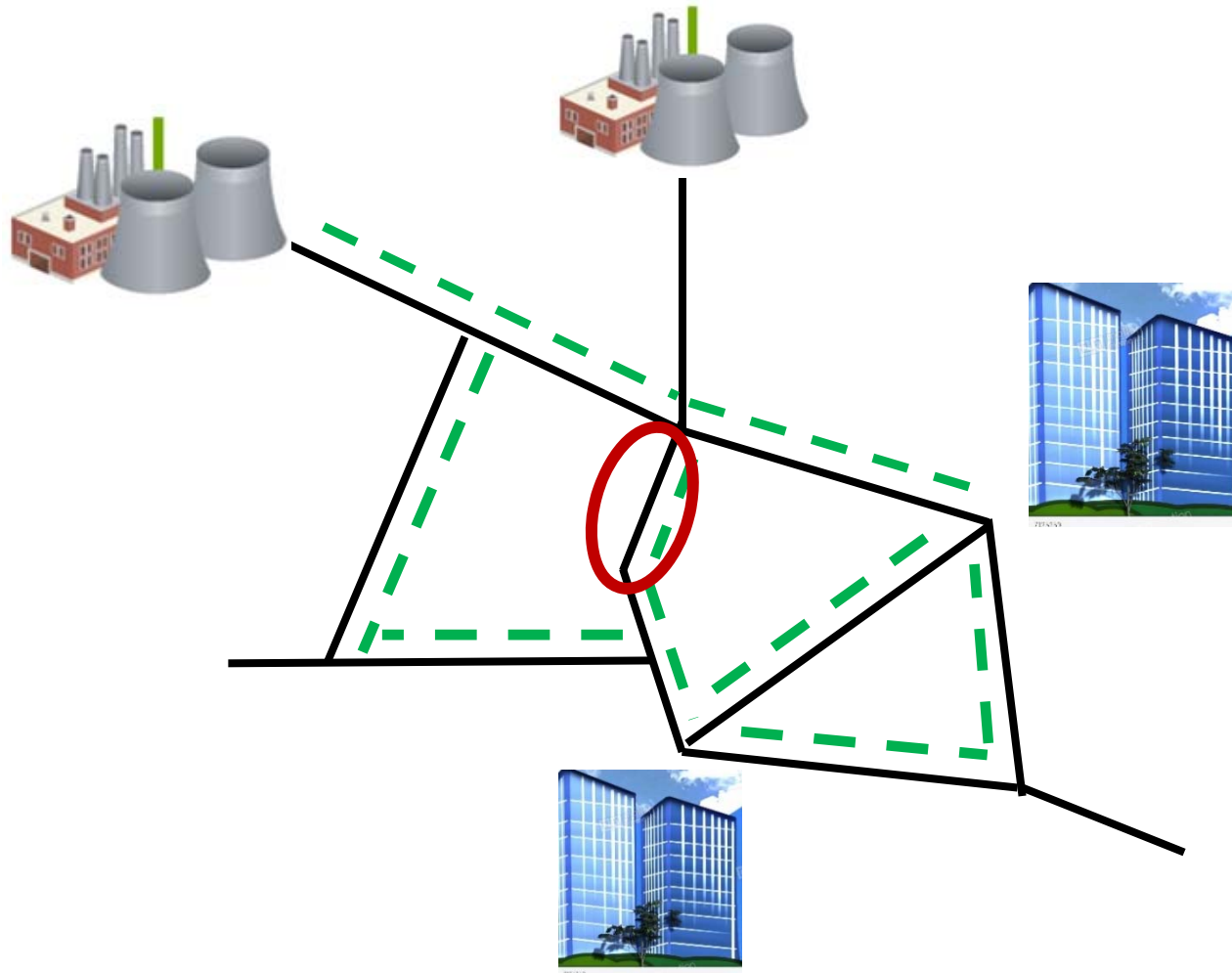


$$i_1 - i_2 - i_3 + i_4 = 0$$



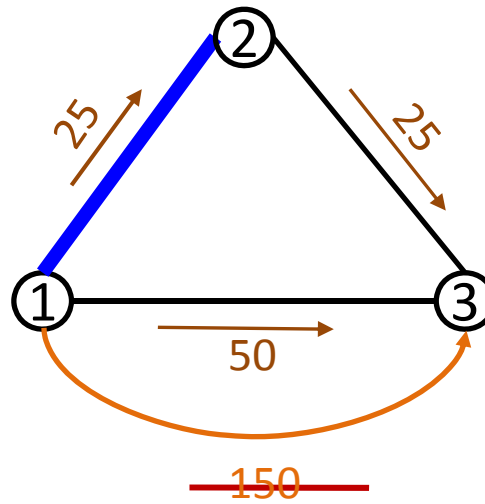
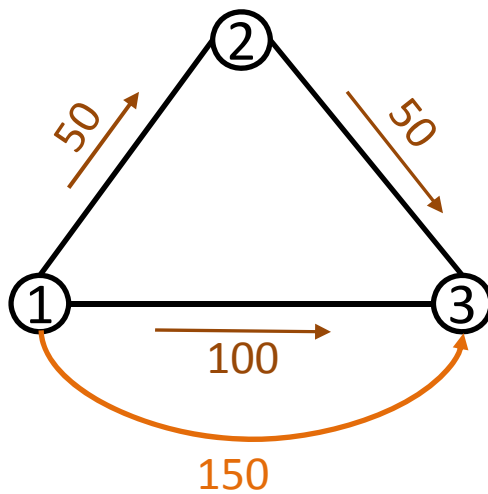
$$v_1 + v_2 + v_3 - v_4 = 0$$

Transmit and Distribute Power: Kirchhoff's Law

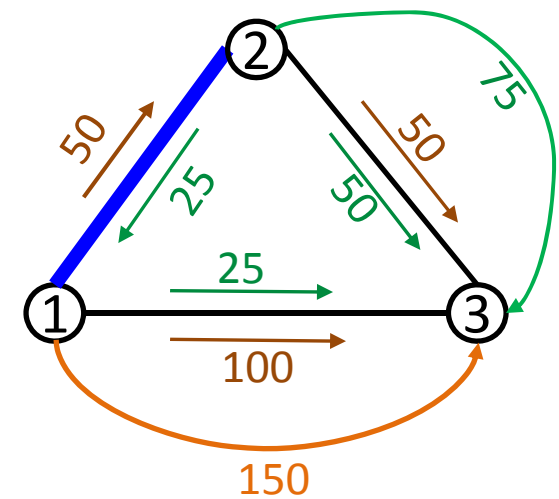


Capacity constraint on any line or node limit the entire flow

Challenges: An Example



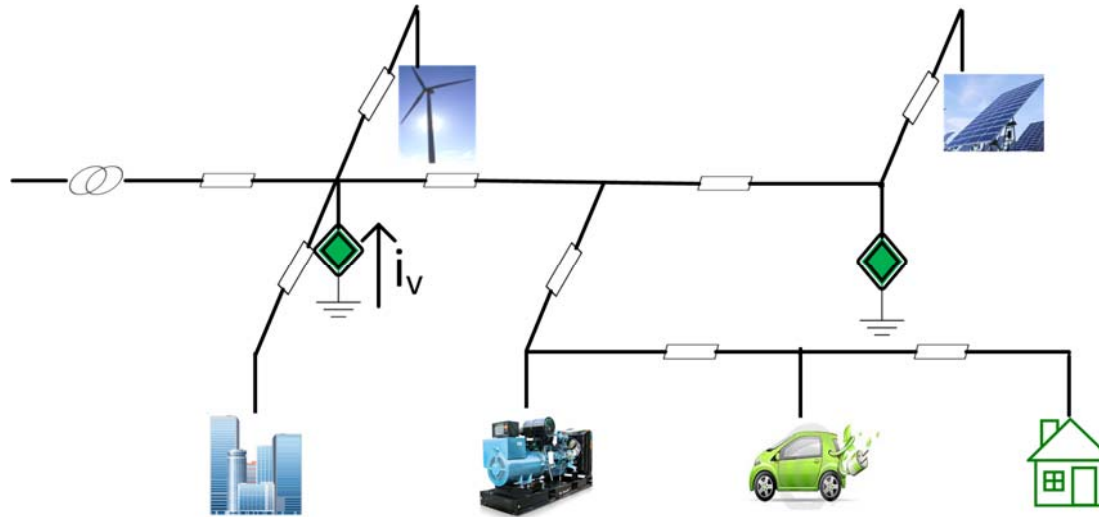
Line 1-2 capacity: 25



Transaction $2 \rightarrow 3$ alleviates
congestions on line 1-2

1, 2: generation nodes/buses; 3: load bus (two users)

How much to pay for public distribution service?



Social Welfare

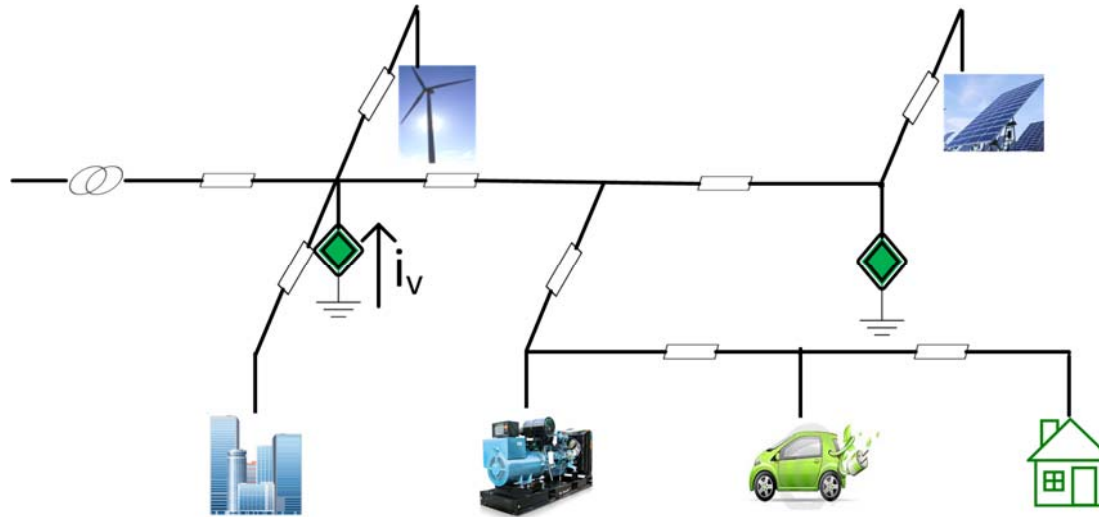
$$\begin{aligned}
 & \max_{d, g, y, u, l} && B(d) - C(g) \\
 & \text{s.t.} && d - g = y \\
 & && L(y, u) = l \\
 & && f(y, u) \leq 0
 \end{aligned}$$

Benefit Cost

Physical Constraints

d : demand;
 g : generation;
 y : net power injection
 u : other physical variables
 l : power losses;

How much to pay for public distribution service?



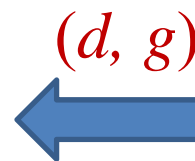
Social Welfare

$$\begin{aligned}
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 & && f(y, u) \leq 0
 \end{aligned}$$

Individual

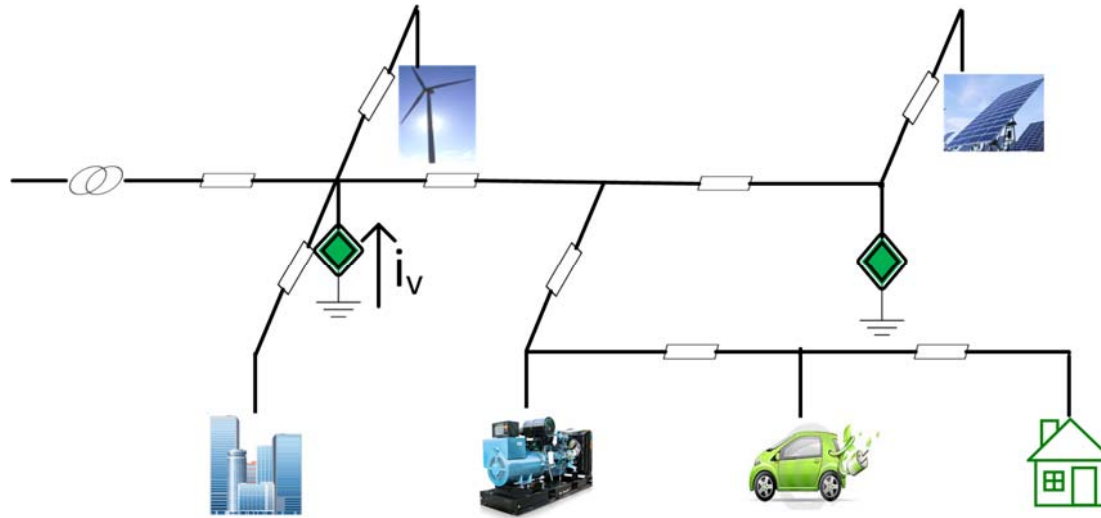
$$\begin{aligned}
 & \max_d B(d) - p \cdot d \\
 & \max_g p \cdot g - C(g)
 \end{aligned}$$

price



How to set the price?

How to choose the prices?



Social Welfare

$$\begin{array}{ll} \max_{d,g,y,u,l} & B(d) - C(g) \\ \text{s.t.} & d - g = y \\ & L(y, u) = l \\ & f(y, u) \leq 0 \end{array}$$

Given an **convex** problem,
duality of the optimization
provide efficient prices, p^*

Challenges: Nonconvexity

Nonconvex Optimal Power Flow

$$\min \quad C\left(\sum_{(0,j)} P_{0j}\right) - \sum_i U_i(p_i) + \sum_{i,j} r_{i,j} |I_{i,j}|^2$$

$$\text{over } x := (S, \ell, v, p, q)$$

$$\text{s. t. } \ell_{ij} = |S_{ij}|^2 / v_i,$$

$$v_j = v_i + 2 \operatorname{Re}(z_{ij}^* S_{ij}) - |z_{ij}|^2 \ell_{ij},$$

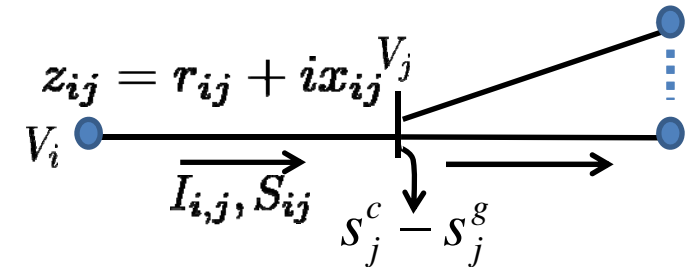
$$\sum_{i \rightarrow j} (S_{ij} - z_{ij} \ell_{ij}) - \sum_{j \rightarrow k} S_{jk} = s_j,$$

$$\underline{v}_i \leq v_i \leq \bar{v}_i,$$

$$\underline{q}_i \leq q_i \leq \bar{q}_i,$$

$$\underline{p}_i \leq p_i \leq \bar{p}_i,$$

Nonconvex



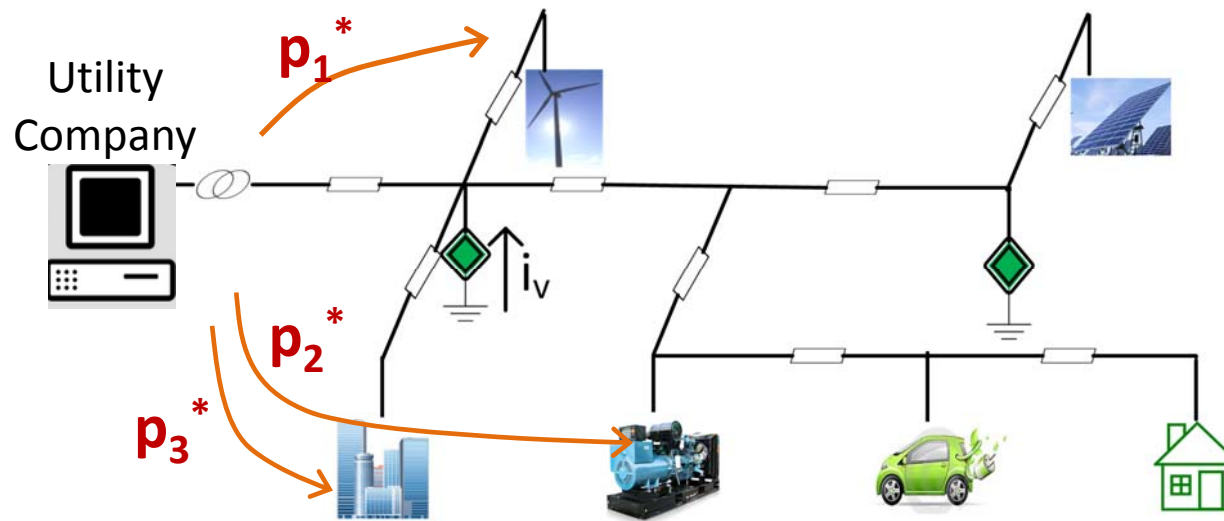
$$s_j := p_j + iq_j \quad S_j := P_j + iQ_j$$

$$\ell_{ij} := |I_{i,j}|^2 \quad v_i := |V_i|^2$$

Branch flow model

Convexification gives
exact solutions
[Lavaei 2011, Li 2012, Gan
2012, 2013]

Efficient Prices: Market Equilibrium (d^*, g^*, p^*)



Social Welfare

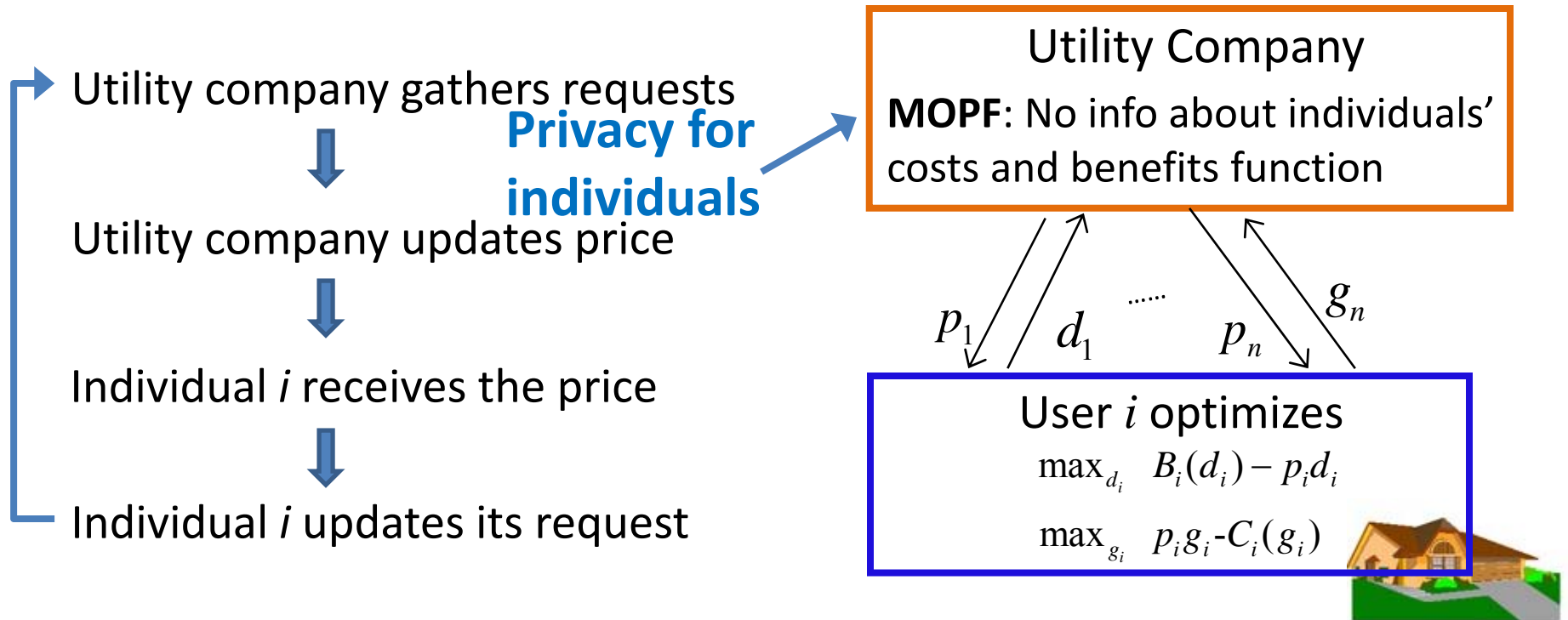
$$\begin{aligned} \max_{d, g, y, u, l} \quad & B(d) - C(g) \\ \text{s.t.} \quad & d - g = y \\ & L(y, u) = l \\ & f(y, u) \leq 0 \end{aligned}$$

(d^*, g^*)

Individual

$$\begin{aligned} \max_d \quad & B(d) - p^* \cdot d \\ \max_g \quad & p^* \cdot g - C(g) \end{aligned}$$

A Distributed Algorithm to Reach the Equilibrium

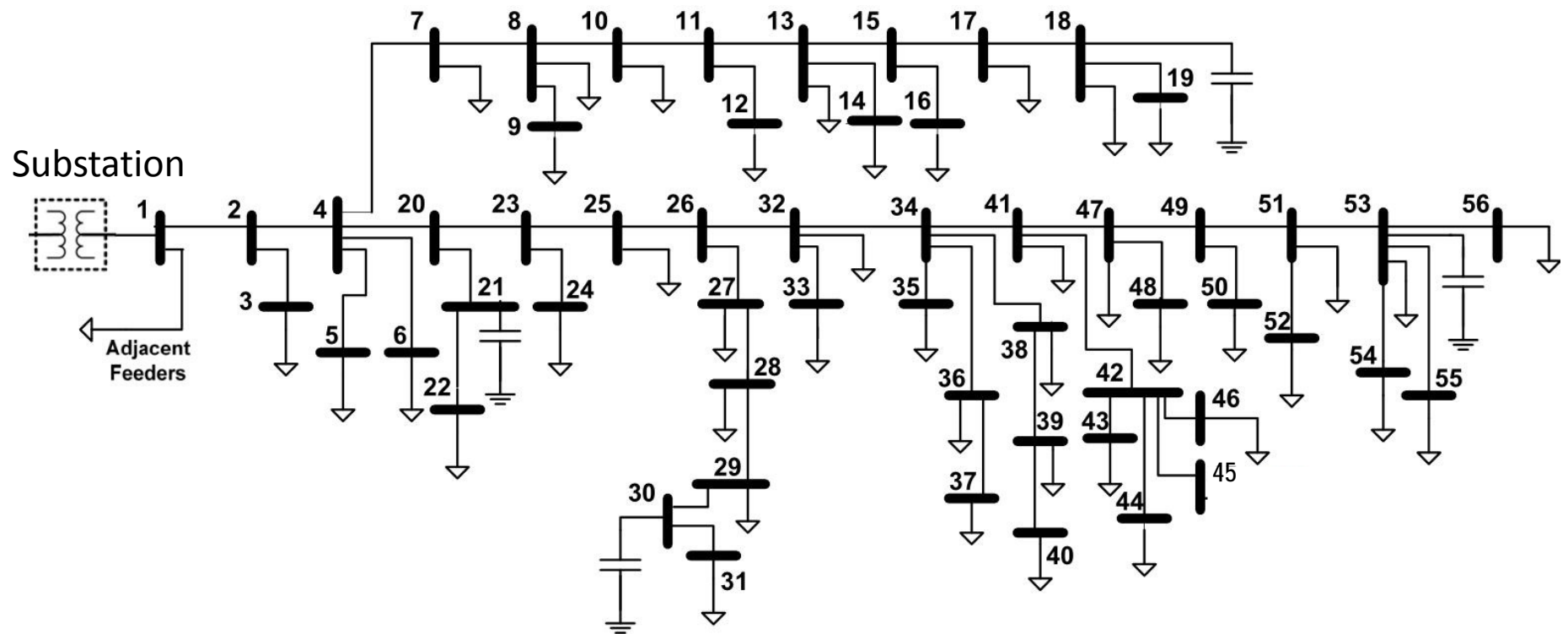


Theorem [Li et al. 2012, 2014]: The distributed algorithm converges to market equilibrium over a radial distribution network.

Recent work: Distributed algorithms with **limited communication**.

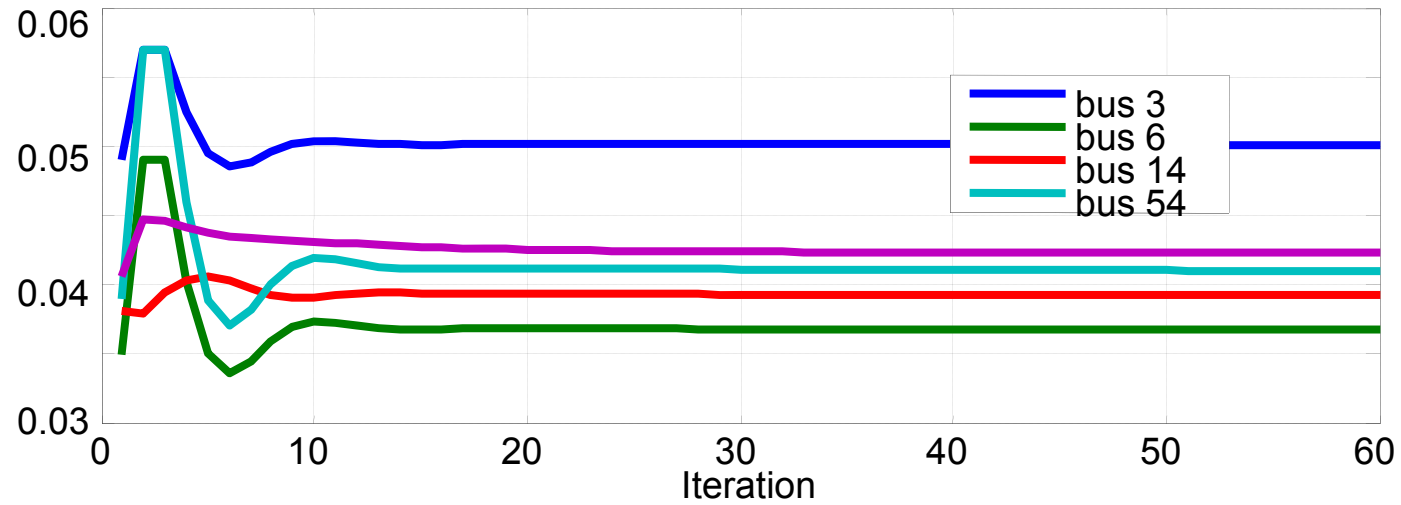
[2015, 2016]

Case studies

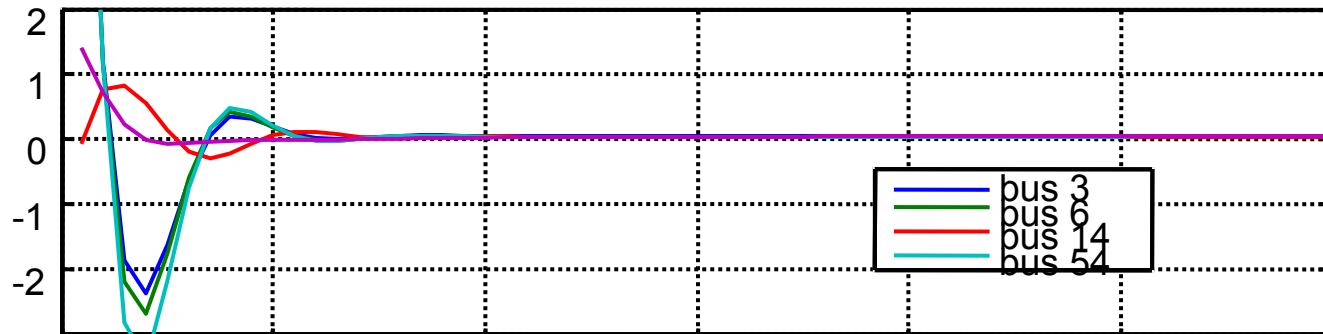


Schematic Diagram of a South California Edison distribution System

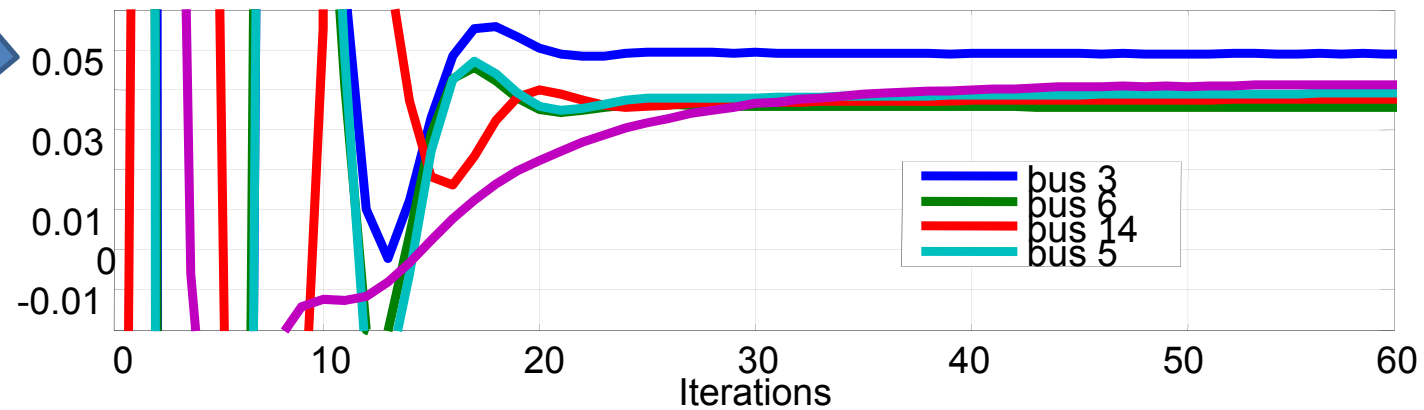
Real power calculated by the user (MW)



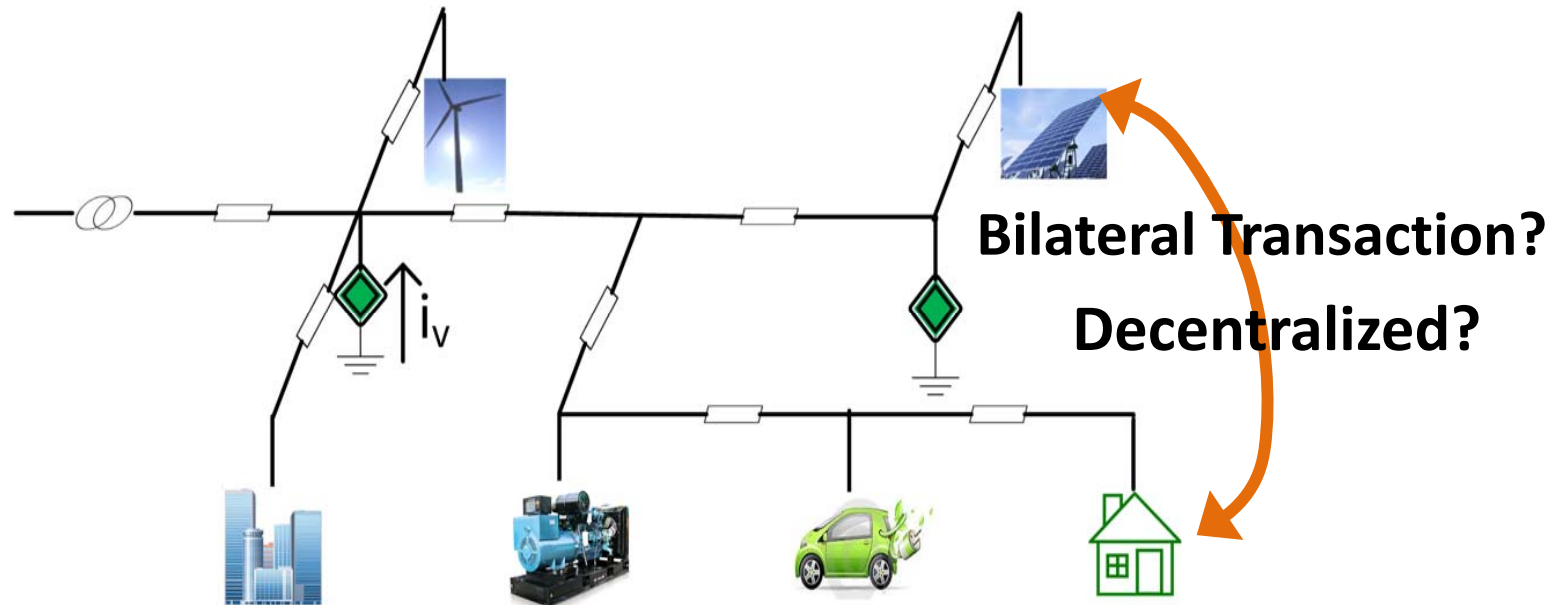
Real power calculated by the utility company (MW)



Zoom in



How about decentralized market?



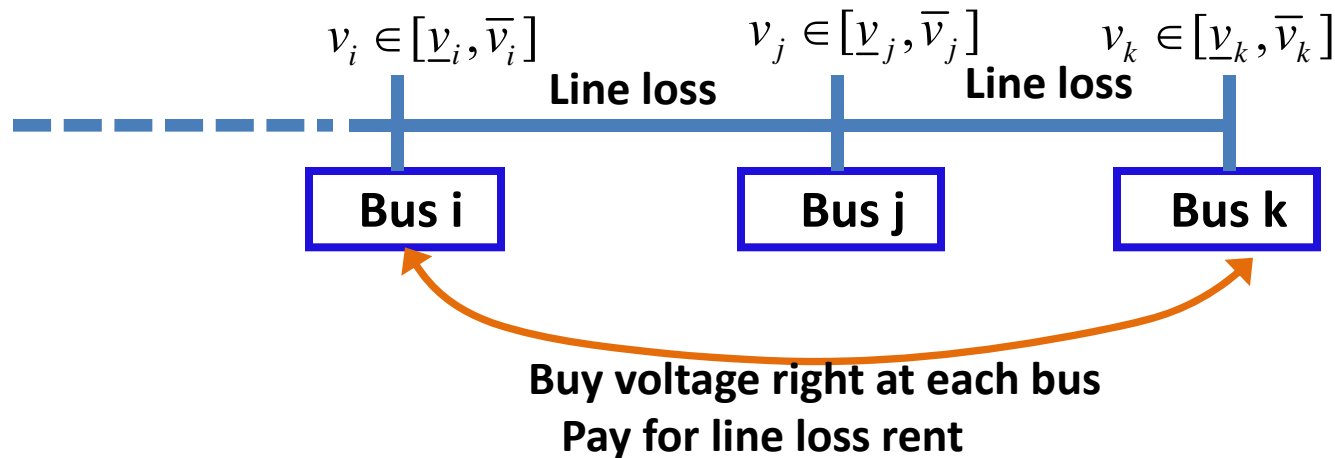
Challenge: *Externality:*

Any local change induces a (complicated) global change!

Delivery Service (in distribution networks)

- Voltage support (constraint): $\underline{v}_i \leq v_i \leq \bar{v}_i$
- Power loss

Market rule



Each Bilateral Transaction

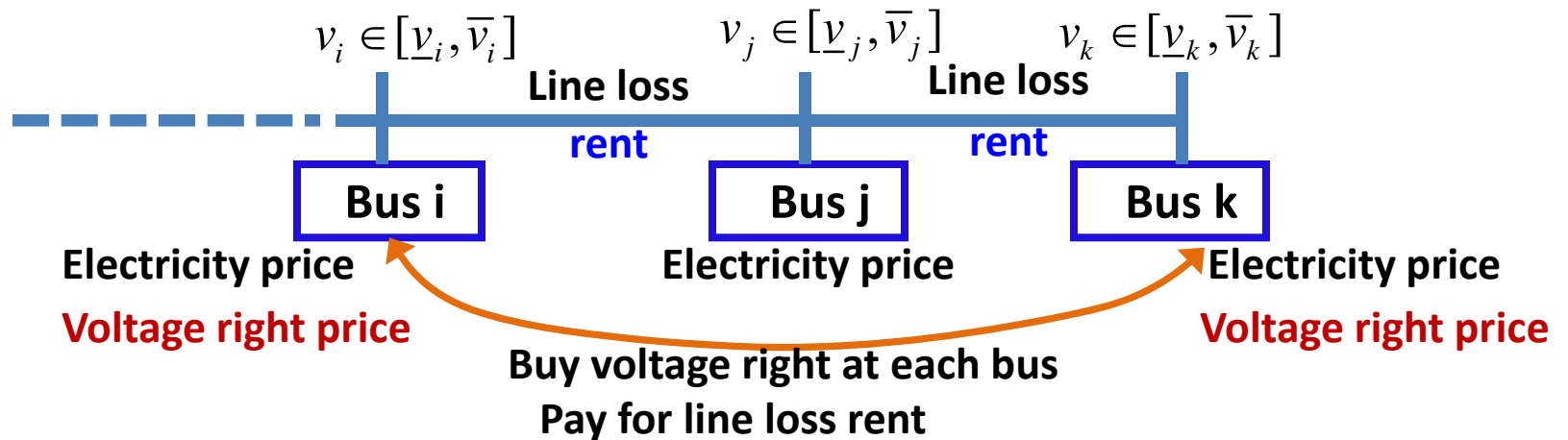
- Buy voltage right (constraint) at each bus
- Pays for line loss rent of each line

Q: Budget balance on the voltage right and also the power loss?

Voltage right at each bus = \sum_i voltage right bought by transaction i

Power losses at each line = \sum_i Losses paid by transaction i

Market Prices and Equilibrium

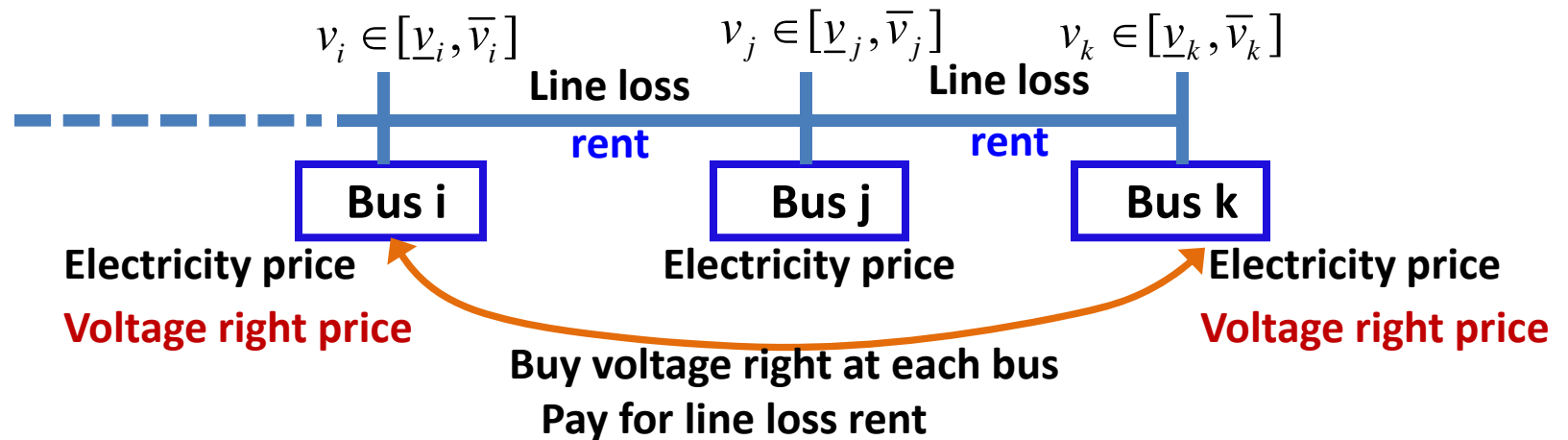


- Each user/generator maximizes net benefit/profit given elec. prices

$$\max_d B(d) - p \cdot d$$

$$\max_g p \cdot g - C(g)$$

Market Prices and Equilibrium



- Each user/generator maximizes net benefit/profit given elec. price
- For each unit transaction between any two node i and k

$$\text{Elec. Price } i = \text{Elec Price } j + \text{Sum}(\text{Voltage right price} * \text{Quantity}_1) + \text{Sum}(\text{Line loss rent} * \text{Quantity}_2)$$

- Voltage right price is 0 if there is excess voltage capacity supply

Question: How to determine Quantity_1 , Quantity_2 ?

How to determine the quantities?

Duality of the Social Welfare Maximization

$$\begin{aligned} \max_{d,g,y,u,l} \quad & B(d) - C(g) \\ \text{s.t.} \quad & d - g = y \\ & L(y, u) = l \\ & f(y, u) \leq 0 \end{aligned}$$



Budget Balance Constraints on Voltage Right and Line Losses



Quantity₁
Quantity₂
Prices

For each unit transaction between any two node i and k

$$\begin{aligned} \text{Price } i = \text{Price } j &+ \text{Sum}(\text{Voltage right price} * \text{Quantity}_1) \\ &+ \text{Sum}(\text{Line loss rent} * \text{Quantity}_2) \end{aligned}$$

How to determine the quantities?

One Allocation Rule for Voltage Right and Line Losses

Quantity₁: $\bar{\beta}_i^k = \frac{(v_k - v_k^{nom}) R_{ki}}{\sum_{j=1}^n R_{kj} p_j}; \quad \underline{\beta}_i^k = -\frac{(v_k - v_k^{nom}) R_{ki}}{\sum_{j=1}^n R_{kj} p_j}.$

Quantity₂: $\phi_i^k = \frac{L_k L_{k,i}}{\sum_{j=1}^n L_{k,j} p_j}$

R: resistance
V: voltage
p: power injection
P,Q: real/reactive power flow
L: line losses

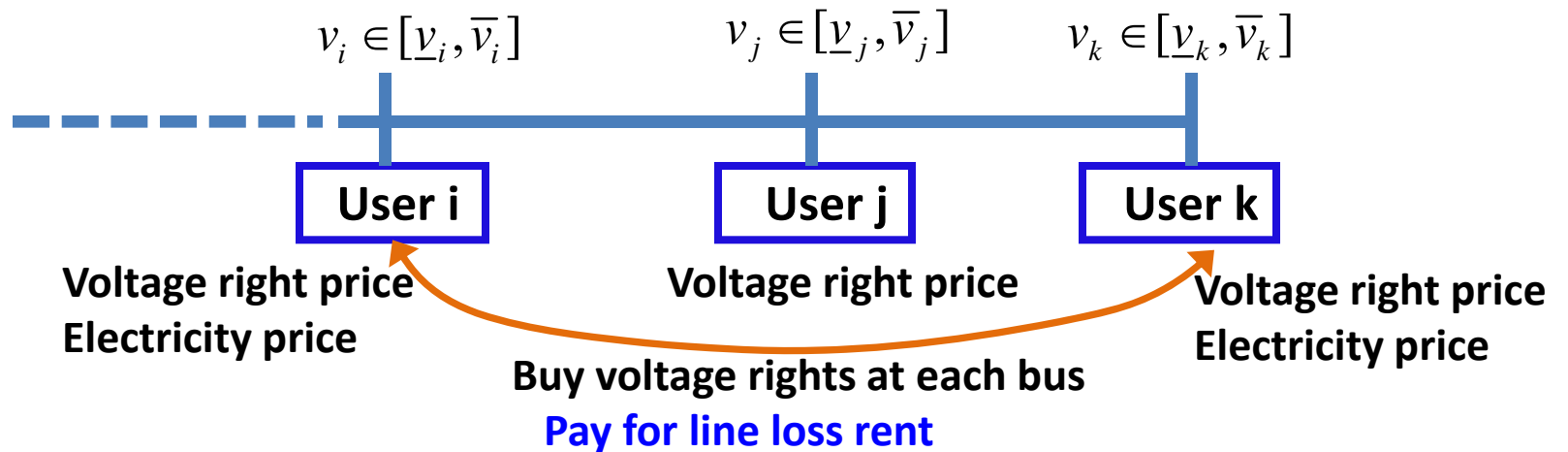
L_k denote the line losses on $(k, \pi(k))$

$$L_{k,i} = r_k \begin{cases} \frac{2P_k}{v_k} - \frac{P_k^2 + Q_k^2}{v_k^2} R_{ki}, & \text{if } k \in \mathcal{P}_i, \\ -\frac{P_k^2 + Q_k^2}{v_k^2} R_{ki}, & \text{otherwise.} \end{cases}$$

For each unit transaction between any two node i and k

$$\begin{aligned} \text{Price } i = & \text{Price } j + \text{Sum}(\text{Voltage right price} * \text{Quantity}_1) \\ & + \text{Sum}(\text{Line loss rent} * \text{Quantity}_2) \end{aligned}$$

Competitive Market Equilibrium

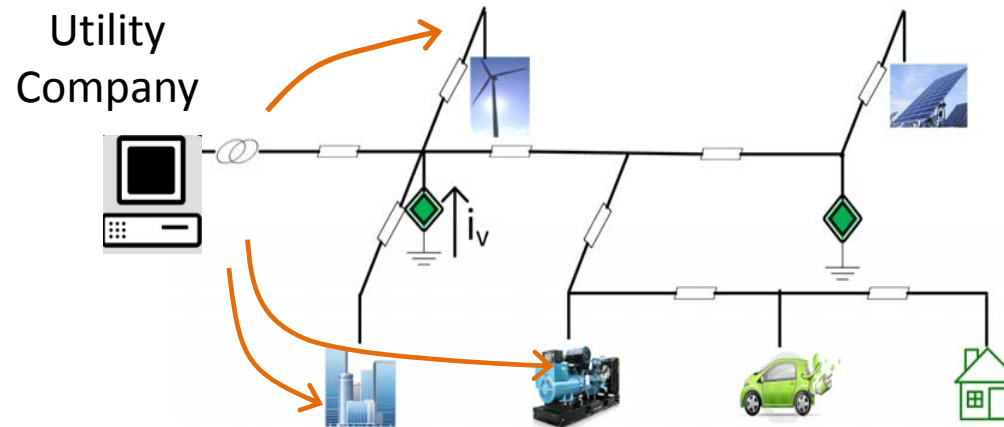


Theorem (Li 2015):

Under the designed market rule, there exists a competitive market equilibrium that is socially optimal.

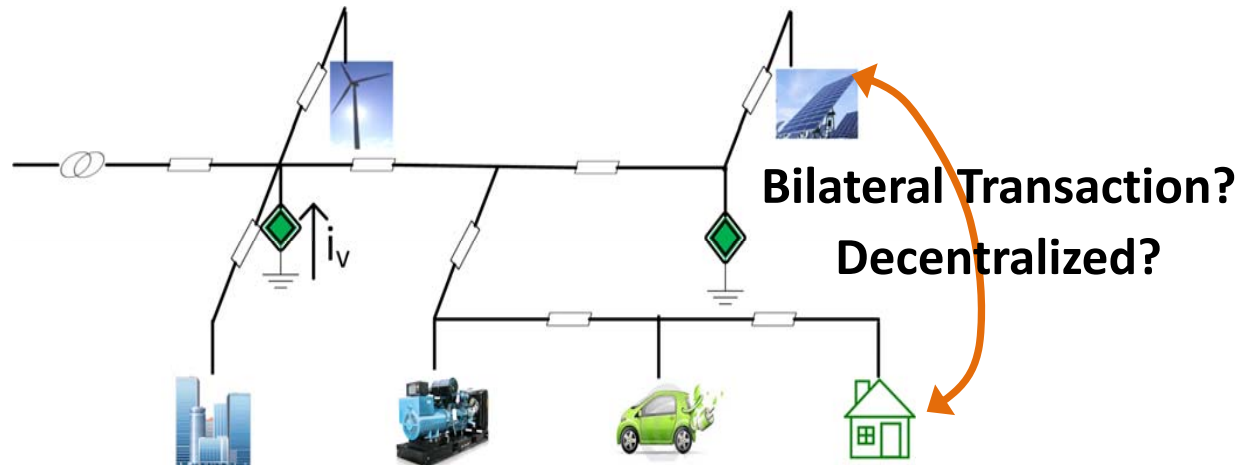
So far...

Scheme 1:



Markets are efficient

Scheme 2:



Electricity Market for Distribution Networks:

Power Engineering:

Power flow, system dynamics, operation constraints

Markets efficiently allocate delivery costs to individuals (transactions)

Human Incentive:

Strategic behavior, self-interested, market power

Uncertainties:

Renewable energy, user's behavior, emergency

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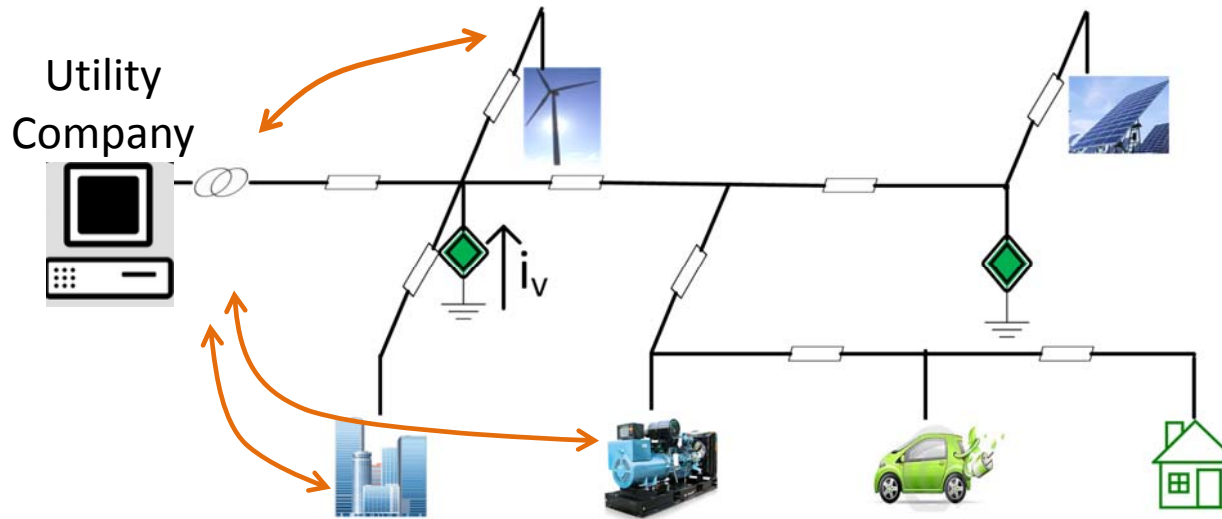
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Recall...



Social Welfare

$\max_{d,g,y,u,l}$

$$B(d) - C(g)$$

Individuals need to report info.

s.t.

$$d - g = y$$

What if they **DON'T** report **true** info.?

$$L(y, u) = l$$

$$f(y, u) \leq 0$$

Supply Function Bidding for Demand Response

- Supply deficit (or surplus) on electricity: d
weather change, unexpected events, ...
- Supply is inelastic

Problem: How to allocate the deficit among customers?
load (demand) as a resource to allocate

Supply function bidding

- Customer i load to shed: q_i
- Customer i reports a supply function (SF):

$$q_i(b_i, p) = b_i p$$

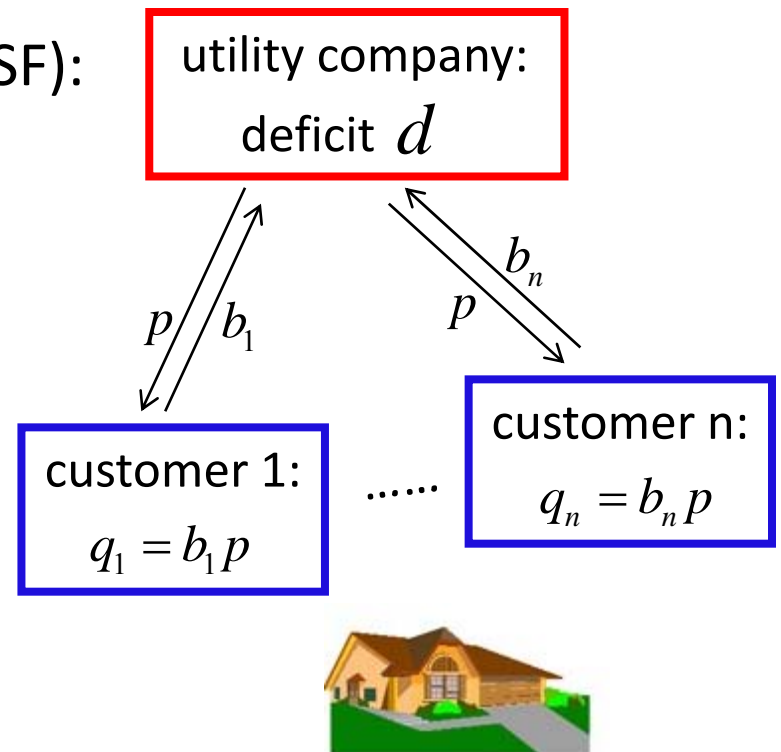
- p : price for load shedding
- b_i : price sensitivity

- Market-clearing pricing p :

$$\sum_i q_i(b_i, p) = d$$



$$p = p(b) \triangleq d / \sum_i b_i$$



Load Shedding Cost

- Customer i cost (or disutility) function:

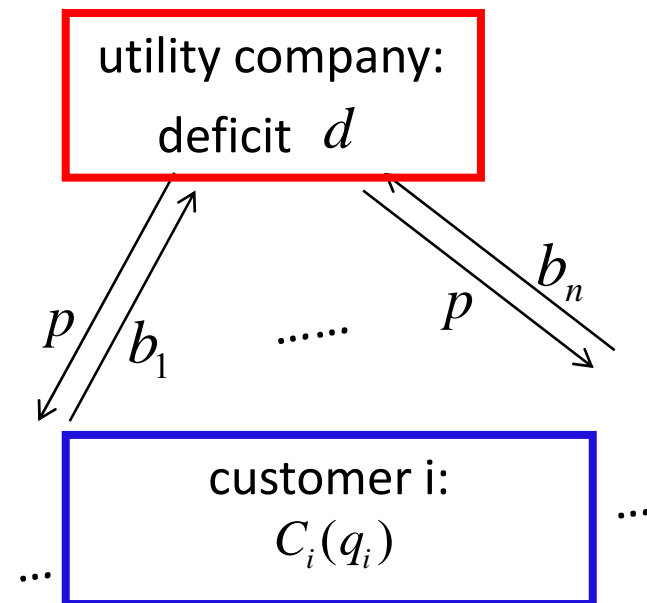
$$C_i(q_i)$$

- **Social welfare: Optimal Global Cost**

$$\begin{aligned} \min_{q_i} \quad & \sum_i C_i(q_i) \\ \text{s.t.} \quad & \sum_i q_i = d \end{aligned}$$

Question:

Can the supply function bidding achieves the optimal global cost?



Strategic demand response

- Customer i 's net revenue: $u_i = p q_i - C_i(q_i)$
- Note: Price p is a function of bidding b
- Price-anticipating, strategic customer

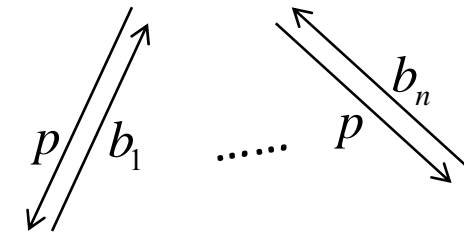
with
$$\max_{b_i} u_i(b_i, b_{-i})$$

$$u_i(b_i, b_{-i}) = p(b)q_i(b_i, p(b)) - C_i(q_i(b_i, p(b)))$$

- **Definition:** A supply function profile b^* is a **Nash equilibrium** if, for all customers i ,

$$u_i(b_i^*, b_{-i}^*) \geq u_i(b_i, b_{-i}^*), \quad \forall b_i \geq 0$$

utility company:
deficit d



... customer i :
 $\max_{b_i} u_i(b_i, b_{-i})$...



Nash equilibrium

Theorem (Li, Chen, Dahleh, 2015)

Assume $|N| \geq 3$. The demand response game has a unique Nash equilibrium. Moreover, the equilibrium solves the following convex optimization problem:

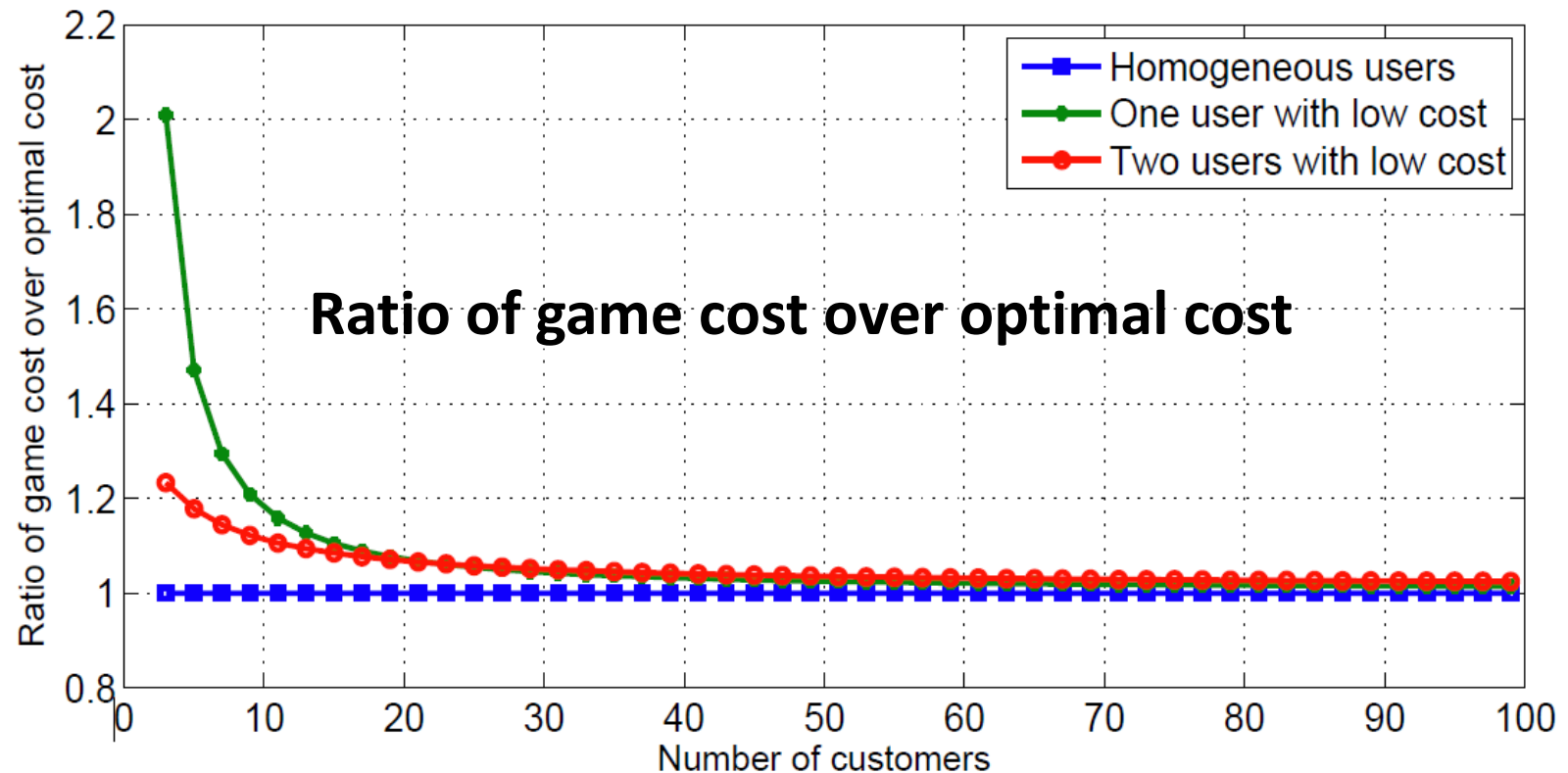
$$\begin{aligned} \min_{0 \leq q_i < d/2} \quad & \sum_i D_i(q_i) \\ \text{s.t.} \quad & \sum_i q_i = d, \end{aligned}$$

with

$$D_i(q_i) = C_i(q_i) + \Delta C_i(q_i) \leftarrow \text{False cost}$$

$$\Delta C_i(q_i) := \frac{q_i}{d-2q_i} C_i(q_i) - \int_0^{q_i} \frac{d}{(d-2x_i)^2} C_i(x_i) dx_i > 0$$

Efficiency Loss



Question:

Is there a way to make individuals report truthful information?

This Talk: Electricity Market in Distribution Networks

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Power flow, system dynamics, operation constraints

Markets efficiently allocate delivery costs to individuals (transactions)

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Supply function bidding: Efficiency loss from strategic behavior

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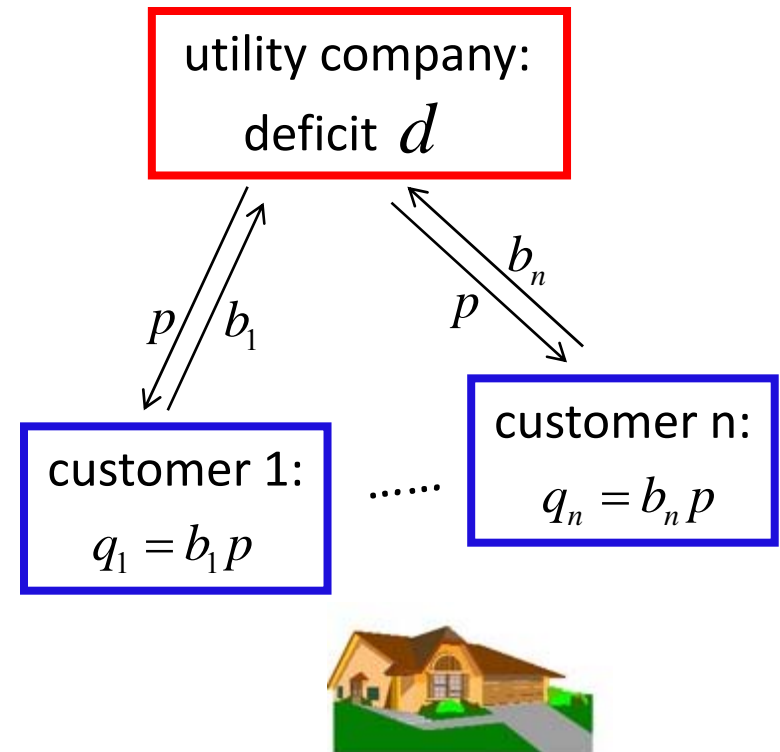
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Recall: Supply Function Bidding

- A supply deficit d
- Customer i reduced load: q_i
- Customer i reports a supply function (SF)
- Cost function of load shedding

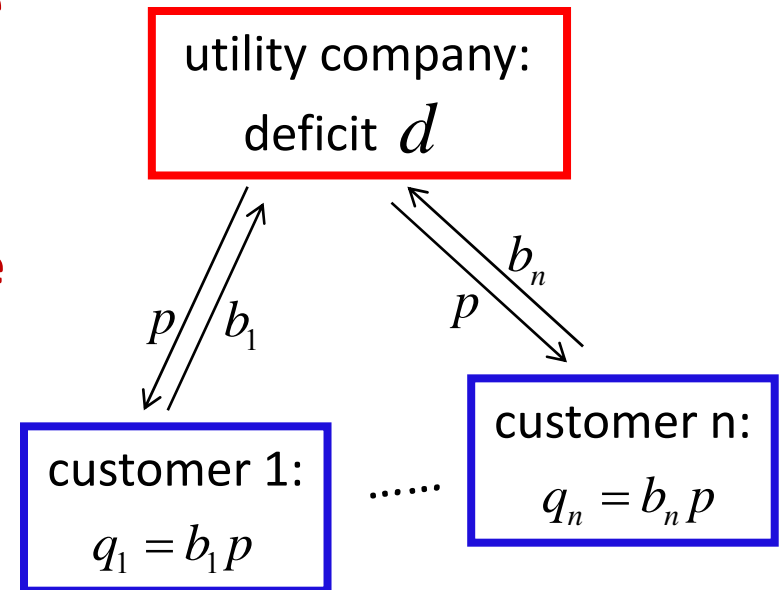
$$C_i(q_i)$$



Recall: Supply Function Bidding

- A **forecasted** supply deficit d **in the future**
- Customer i reduced load: q_i
- Customer i reports a supply function (SF)
- Cost function of load shedding **in the future**

$$C_i(q_i)$$



Caution: The information is uncertain!
Challenge: How to guarantee reliability?







Incentivizing Reliability in Demand Response

A group of customers:

- are able to reduce loads, e.g., 2-4pm in the next day

A reliability target:

- e.g. 1000 kW can be reduced with probability 99%

Challenges:

- Costly to reduce loads
- Uncertainty in the cost and ability to respond

Current practice (e.g., PJM, Con Edison, SCE, etc):

- Enlisting large number of consumers,
- Offering rewards in an order based on experience
- **Unguaranteed reliability as customers opt out in the process**

Two Period Mechanism

Time 0

Agents report
with knowledge
of type (C_i)

Uncertain, Random Cost

Mechanism selects agents
to prepare for reducing loads and
determines **rewards R_i** , **penalty Q_i**

Time 1

Agents resolve
uncertainty in
ability to respond

Agents decide
on responses,
if possible

Mechanism pays
rewards and
collects penalties

Fixed Reward R Mechanism

Direct Mechanism

- Mechanism computes agent **maximum acceptable penalty** M_i
- Select customers in **decreasing order** of M_i until reliability target is met
- Calculate **critical payment** Q_i as penalty for non-response

Indirect Mechanism

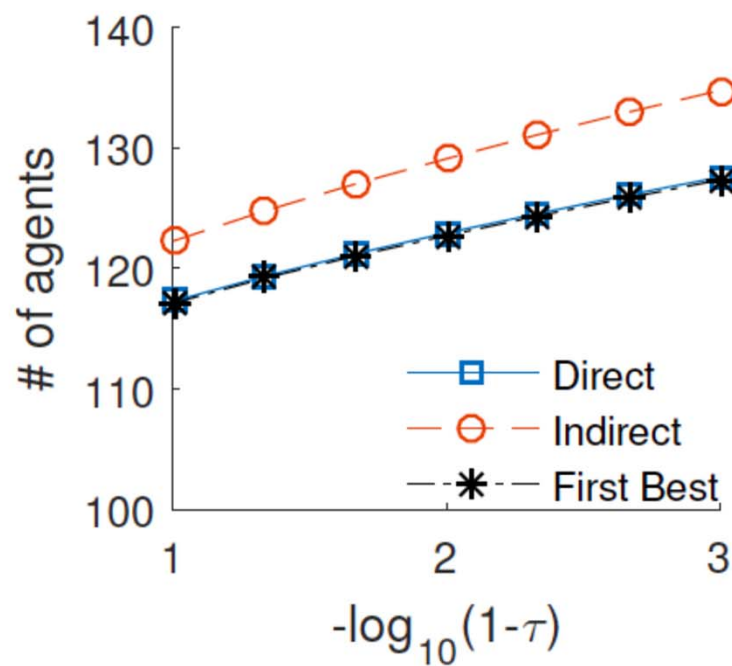
- Agents reports their maximum acceptable penalty M_i

Theorem [Ma, Robu, Li, Parkes, 2016]:

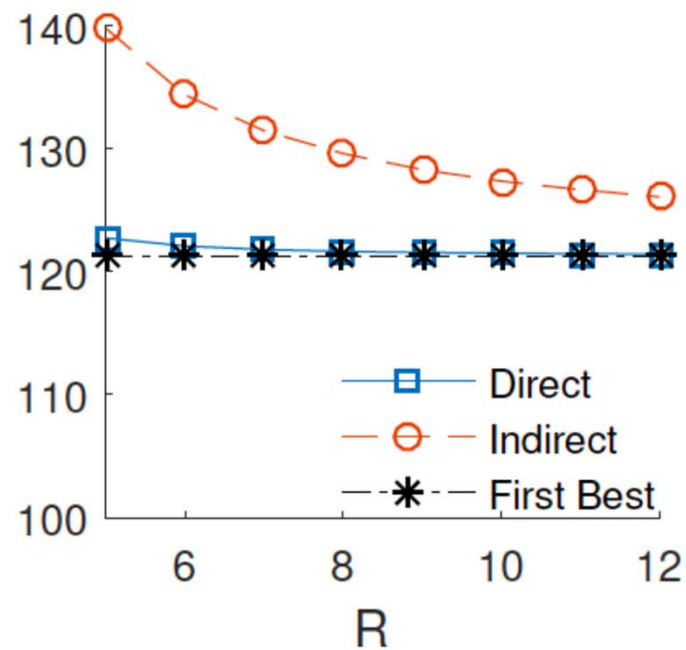
If the reward R is large enough, both direct and indirect mechanism guarantee truthful telling, individual rationality, and the reliability target.

Direct, Indirect Vs. First Best

- **First Best:** suppose individual uncertainty is available; select to optimize the reliability
- $n = 500$, $M = 100$, fix $R = 10$ or reliability $\tau = 98\%$



(a) Varying τ



(b) Varying R

Conclusion and Discussion

Power Engineering:

Power flow, System dynamics, operation constraints

Markets efficiently allocate delivery costs to individuals (transactions)

Human Incentive:

Strategic behavior, self-interested, market power

Supply function bidding: Efficiency loss from strategic behavior

Uncertainties:

Renewable energy, user's behavior, emergency

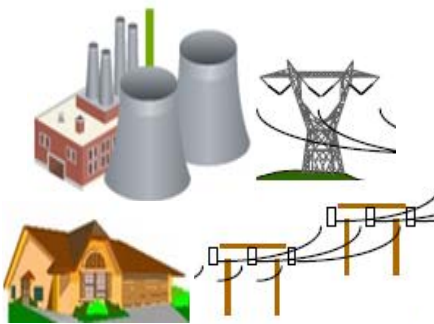
Mechanism design to ensure reliability

Challenge and future work:

A market: takes account of engineering and human factors, achieves (sub)-optimal efficiency, and ensures reliability

Research Interest

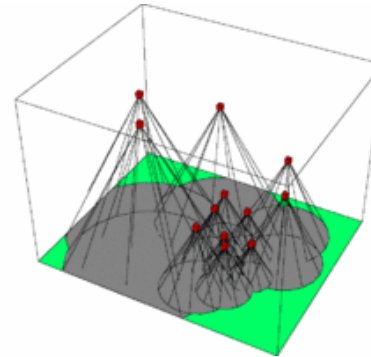
Network Optimization, Control, Economics



Power Systems



Data Center



Sensor Network

Transportation
Internet network
Parallel computing
Social network
Etc...

Design general theories and tools for:

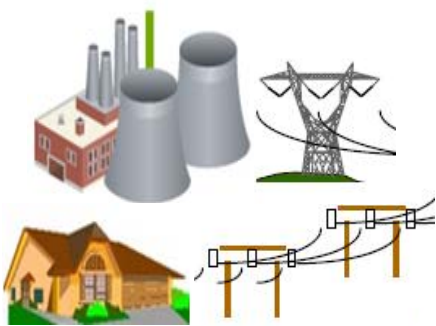
*Distributed/**Local**
Control Laws*



*Desired **Global**
System Behavior*

Research Interest

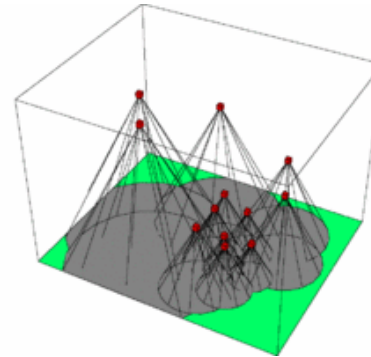
Network Optimization, Control, Economics



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Etc...

Foundational Theories

- Comm./Comp. complexity
- Tradeoff between efficiency, robustness, computation, and communication



Practical Algorithms

- Optimal first-order distributed methods
- Regularized methods
- Physical measurement-aid algorithms



Real Implementation

- Distributed power capping in data center
- Microgrid energy management

Acknowledgment :

Caltech: Steven Low

MIT: Munther Dahleh

Univ. of Colorado, Boulder: Lijun Chen

KTH: Sindri Magnusson, Carlo Fishchione

Energy Trading Analytics: Hung-po Chao

Harvard Univ: Vahid Tarokh, David Parkes, Guannan Qu, Masoud Badiei, Yingying Li, Ariana Minot, Xuan Zhang, Chinwendu Enyioha, Hongyao Ma

Funding Agencies: NSF, ARPA-E

