Generalized Model Predictive Control

(Discretely Generalized MPC)

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ISR @ UMD College Park, February 24, 2016

Part

Opening

Model Predictive Control of the Day Before Yesterday

Model Predictive Control Synthesis

Model Predictive Control Lower–Synthesis

Model Predictive Control Upper–Synthesis

Model Predictive Control Generalized Synthesis

Closing

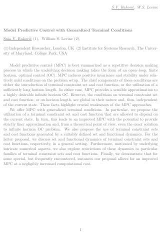
Details

Abstract

A framework, developed in collaboration with William S. Levine and Behçet Açikmeşe, for generalized MPC is outlined.

W. S. Levine

B. Açikmeşe


Model Predictive Control with Generalized Terminal Conditions

S.V. Raković, W.S. Levine

ORCOS VW 2015.


Discretely Generalized Model Predictive Control Synthesis

Saša V. Raković, William S. Levine and Behçet Açikmeşe

ACC 2016.


Continuously Generalized Model Predictive Control

Saša V. Raković, William S. Levine and Behçet Açikmeşe

CDC 2016. In Progress.


Generalized Model Predictive Control

Saša V. Raković, William S. Levine and Behçet Açikmeşe

Journal. In Progress.

Part

MPC Analogy

Jean Piaget (1896 – 1980)

Cognitive Psychology

Children learning and environment controlling

1. Image

2. Aim

3. Action

4. Collation

MPC Analogy

Jacques Richalet (1936 –)

Predictive Functional Control

Credits for Brilliant Analogy

1. Image

1. Model

2. Aim

2. Reference

3. Action

3. Control

4. Collation

4. Feedback

MPC Paradigm

Goals:

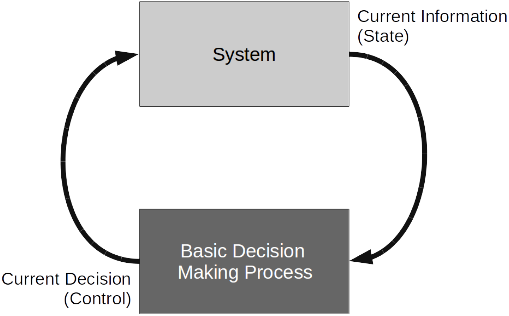
Constraint satisfaction,

Stability, and

Optimized performance.

Tool:

Model predictive control.



Model predictive control (MPC):

Repetitive decision making process (DMP).

Basic DMP is finite horizon optimal control.

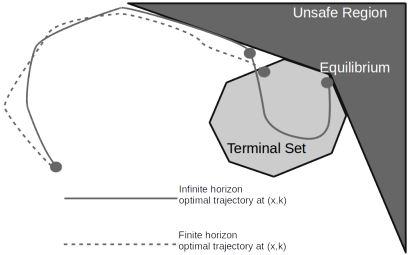
Basic DMP (Finite Horizon Optimal Control)

Given an integer $N \in \mathfrak{N}$ and a state $x \in \mathbb{X}$ select predicted sequences of

control actions $\mathbf{u}_{N-1} := \{u_0, u_1, \ldots, u_{N-1}\}$, and

controlled states $\mathbf{x}_N := \{x_0, x_1, \ldots, x_{N-1}, x_N\}$,

which, for each $k \in \{0, 1, \ldots, N-1\}$, satisfy



$x_{k+1} = f(x_k, u_k)$ with $x_0 = x$,

$x_k \in \mathbb{X}$,

$u_k \in \mathbb{U}$, and

$x_N \in \mathbb{X}_f$,

and which minimize $V_N(\mathbf{x}_N, \mathbf{u}_{N-1}) := \sum_{k=0}^{N-1} \ell(x_k, u_k) + V_f(x_N)$.

(Hereafter, $\mathbb{X}_f$ and $V_f(\cdot)$ are terminal constraint set and cost function.)

Key Facts

Main properties:

- MPC law $u_0^0(\cdot)$ is feedback implicitly evaluated at current state.
- Predictions and optimized predictions are, however, open–loop.
- Consistently improving and stabilizing (under mild assumptions).

Theoretical implementation:

- Mathematical (nonlinear) programming in general case.
- Strictly convex programming in most frequent cases.

Practical implementation:

- Online optimization.
- Offline parameteric optimization and online look–up tables.
- Combinations of the online and offline parameteric optimization.

Setting

The system: $x^+ = f(x, u)$
$f(\cdot, \cdot)$ the state transition map,
x the state variable, and
u the control variable.

The constraints: $x \in \mathbb{X}$ and $u \in \mathbb{U}$
$\mathbb{X}$ the state constraint set, and
$\mathbb{U}$ the control constraint set.

The cost: the (accumulated) sum of the stage costs.
The stage cost: $\ell(\cdot, \cdot)$.

Synthesis tool: Model Predictive Control (MPC).
MPC: Repetitive Decision Making Process (DMP).
Basic DMP: Open–Loop Optimal Control (OLOC).

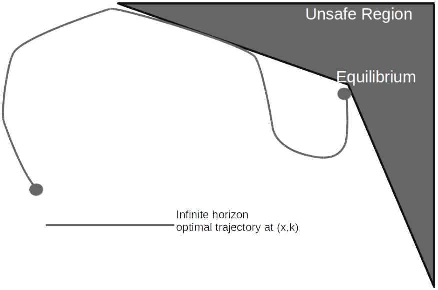
(Perfect) Synthesis via Infinite Horizon OLOC

Given a state $x \in \mathbb{X}$ select infinite sequences of

control actions $\mathbf{u}_\infty := \{u_0, u_1, \ldots, u_{N-1}, \ldots\}$, and

controlled states $\mathbf{x}_\infty := \{x_0, x_1, \ldots, x_{N-1}, x_N, \ldots\}$,

which, for each $k \in \{0, 1, \ldots, N-1, \ldots\}$, satisfy



$x_{k+1} = f(x_k, u_k)$ with $x_0 = x$,

$x_k \in \mathbb{X}$, and

$u_k \in \mathbb{U}$,

and which minimize $V_\infty(\mathbf{x}_\infty, \mathbf{u}_\infty) := \sum_{k=0}^{\infty} \ell(x_k, u_k)$.

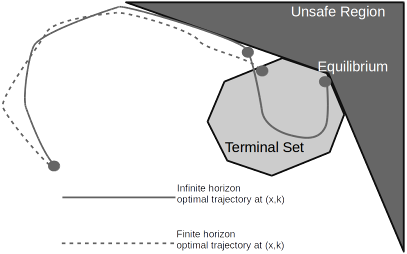
(Actual) Synthesis via Modified Finite Horizon OLOC

Given an integer $N \in \mathfrak{N}$ and a state $x \in \mathbb{X}$ select finite sequences of

control actions $\mathbf{u}_{N-1} := \{u_0, u_1, \ldots, u_{N-1}\}$, and

controlled states $\mathbf{x}_N := \{x_0, x_1, \ldots, x_{N-1}, x_N\}$,

which, for each $k \in \{0, 1, \ldots, N-1\}$, satisfy



$x_{k+1} = f(x_k, u_k)$ with $x_0 = x$,

$x_k \in \mathbb{X}$,

$u_k \in \mathbb{U}$, and

$x_N \in \mathbb{X}_f$,

and which minimize $V_N(\mathbf{x}_N, \mathbf{u}_{N-1}) := \sum_{k=0}^{N-1} \ell(x_k, u_k) + V_f(x_N)$.

Model Predictive Control Principal Components

- Control law $u_0^0(\cdot)$.
  ([Possibly set–valued] Feedback implicitly evaluated at current state.)
- Closed–loop controlled dynamics $x^+ = f(x, u_0^0(x))$.
  ([Possibly set–valued] Implicitly evaluated at encountered states.)
- Value function $V_N^0(\cdot)$.
  (Lyapunov certificate for closed–loop controlled dynamics.)
- Controllability set, the domain of the value function, $\mathcal{X}_N$.
  (Positively invariant set for closed–loop controlled dynamics.)

Synthesis Properties

Under relatively mild assumptions on problem setting
(e.g., regular continuous–compact-ls–continuous setting) design process is:

- Well–posed.
- Consistently improving.
- Positive invariance–inducing.
- Stabilizing.
- Optimizing.

However, the principal components and associated properties depend strongly on the terminal conditions!
(Terminal constraint set $\mathbb{X}_f$ and terminal cost function $V_f(\cdot)$.)

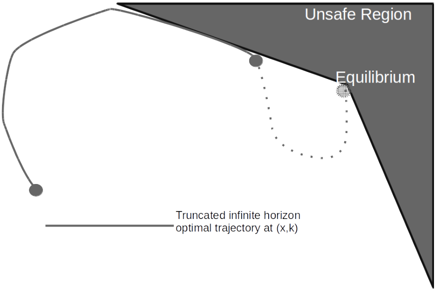
Part

Given an integer $N \in \mathfrak{N}$ and a state $x \in \mathbb{X}$ select finite sequences of

control actions $\mathbf{u}_{N-1} := \{u_0, u_1, \ldots, u_{N-1}\}$, and

controlled states $\mathbf{x}_N := \{x_0, x_1, \ldots, x_{N-1}, x_N\}$,

which, for each $k \in \{0, 1, \ldots, N-1\}$, satisfy



$x_{k+1} = f(x_k, u_k)$ with $x_0 = x$,

$x_k \in \mathbb{X}$,

$u_k \in \mathbb{U}$, and

$x_N \in \mathbb{X}$,

and which minimize $\underline{V}_N(\mathbf{x}_N, \mathbf{u}_{N-1}) := \sum_{k=0}^{N-1} \ell(x_k, u_k)$.

Terminal Ingredients and Assumptions

Constant (induced) terminal
constraint set $\mathbb{X}_f = \mathbb{X}$ and cost function $V_f(\cdot) \equiv 0$

Assumptions (key parts only):

- $\mathbb{X}_f = \mathbb{X}$ is control invariant.
  (Very strong assumption.)
- Prediction horizon N is sufficiently large.
  (Controllability through $\ell(\cdot, \cdot)$ assumption.)

Part

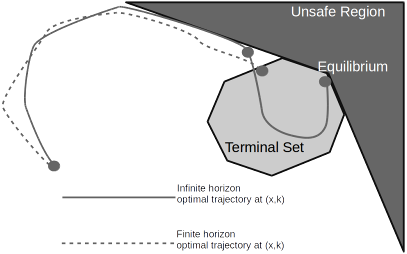
Upper–Synthesis via Modified Finite Horizon OLOC

Given an integer $N \in \mathfrak{N}$ and a state $x \in \mathbb{X}$ select finite sequences of

control actions $\mathbf{u}_{N-1} := \{u_0, u_1, \ldots, u_{N-1}\}$, and

controlled states $\mathbf{x}_N := \{x_0, x_1, \ldots, x_{N-1}, x_N\}$,

which, for each $k \in \{0, 1, \ldots, N-1\}$, satisfy



$x_{k+1} = f(x_k, u_k)$ with $x_0 = x$,

$x_k \in \mathbb{X}$,

$u_k \in \mathbb{U}$, and

$x_N \in \mathbb{X}_f$,

and which minimize $\overline{V}_N(\mathbf{x}_N, \mathbf{u}_{N-1}) := \sum_{k=0}^{N-1} \ell(x_k, u_k) + V_f(x_N)$.

Terminal Ingredients and Assumptions

Constant (designed) terminal constraint set $\mathbb{X}_f \subseteq \mathbb{X}$ and cost function $V_f(\cdot) \succeq 0$

Assumptions (key parts only):

- Local positive invariance of $\mathbb{X}_f$:
  $\mathbb{X}_f \subseteq \mathbb{X},\ \forall x \in \mathbb{X}_f,\ \kappa_f(x) \in \mathbb{U}$ and $f(x, \kappa_f(x)) \in \mathbb{X}_f$
- Local Lyapunov stability with certificate $V_f(\cdot)$:
  $\forall x \in \mathbb{X}_f,\ V_f(f(x, \kappa_f(x))) \leq V_f(x) - \ell(x, \kappa_f(x))$

Lower– and Upper–Syntheses Bottle-neck

Lower–synthesis:

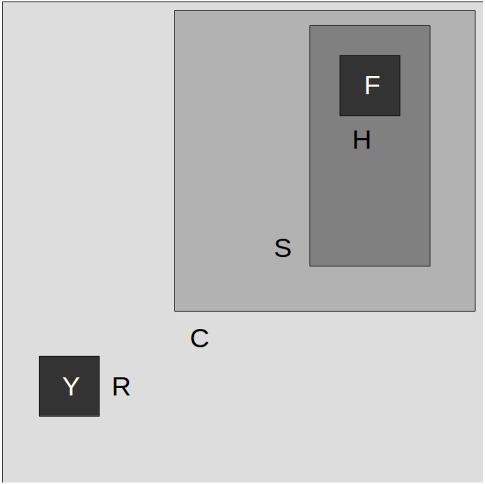
Prediction horizon N	Large enough.
Terminal ingredients $\mathbb{X}_f$ and $V_f(\cdot)$	Induced (state independent).
The estimate for large enough N	Global (state independent).

Upper–synthesis:

Prediction horizon N	Any (non–negative).
Terminal ingredients $\mathbb{X}_f$ and $V_f(\cdot)$	Global (state independent).
The estimate for large enough N	Not needed.

There is no reason for above induced hypothesis!

Visiting a Friend and MPC



1. Can go from R to H.
2. Can go from R to C, then from C to S, then from S to H.
3. Can go from R to S, from S to H.
4. Can do many other things. (e.g., fail to visit a friend :-(.)

How to make some maths of this for MPC (and increase its value)?

Revisiting MPC Synthesis Process

Take an infinite horizon OC process

0	1	2	...	$N-1$	N	$N+1$	...
x_0^0	x_1^0	x_2^0	...	x_{N-1}^0	x_N^0	x_{N+1}^0	...
u_0^0	u_1^0	u_2^0	...	u_{N-1}^0	u_N^0	u_{N+1}^0	...

and rewrite it via finite horizon OC processes as shown on right.

Reconsider and revise traditionally employed terminal conditions.

A/P	0	1	2	...	$N-1$	N
0	x_0^0	x_1^0	x_2^0	...	x_{N-1}^0	x_N^0
	u_0^0	u_1^0	u_2^0	...	u_{N-1}^0	
1	x_1^0	x_2^0	x_3^0	...	x_N^0	x_{N+1}^0
	u_1^0	u_2^0	u_3^0	...	u_N^0	
2	x_2^0	x_3^0	x_4^0	...	x_{N+1}^0	x_{N+2}^0
	u_2^0	u_3^0	u_4^0	...	u_{N+1}^0	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$
k	x_k^0	x_{1+k}^0	x_{2+k}^0	...	x_{N+k-1}^0	x_{N+k}^0
	u_k^0	u_{1+k}^0	u_{2+k}^0	...	u_{N+k-1}^0	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$

Revisiting MPC Synthesis Process

A/P	0	1	2	...	$N-1$	N
	x_0^0	x_1^0	x_2^0	...	x_{N-1}^0	x_N^0
0	u_0^0	u_1^0	u_2^0	...	u_{N-1}^0	
	x_1^0	x_2^0	x_3^0	...	x_N^0	x_{N+1}^0
1	u_1^0	u_2^0	u_3^0	...	u_N^0	
	x_2^0	x_3^0	x_4^0	...	x_{N+1}^0	x_{N+2}^0
2	u_2^0	u_3^0	u_4^0	...	u_{N+1}^0	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$
	x_k^0	x_{1+k}^0	x_{2+k}^0	...	x_{N+k-1}^0	x_{N+k}^0
k	u_k^0	u_{1+k}^0	u_{2+k}^0	...	u_{N+k-1}^0	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	$\vdots$

Simple (semi–group like) observation:

$$x_{N+k}^0(x_0^0) = x_N^0(x_k^0).$$

Key steps:

1. Allow terminal constraint set to be state–dependent $\mathbb{X}_f(\cdot)$.
2. Allow terminal cost function to be state–dependent $V_f(\cdot,\cdot)$.
3. Rework the usual terminal conditions.

Generalized Terminal Conditions: Approach

Key idea:

Employ a set $\mathcal{T}_f$ of triplets $\mathbb{T}_f$ that are composed of terminal constraint sets $\mathbb{X}_f$, control laws $\kappa_f(\cdot)$ and cost functions $V_f(\cdot)$.

Discrete setting in this talk for simplicity and practicality.

Generalized Terminal Conditions: Discrete Setting

A discrete set $\mathcal{T}_f$ of $\mathbb{T}_f := (\mathbb{X}_f, \kappa_f(\cdot), V_f(\cdot))$ triplets:

$$\mathcal{T}_f := \{\mathbb{T}_{f\,i} = (\mathbb{X}_{f\,i}, \kappa_{f\,i}(\cdot), V_{f\,i}(\cdot)) \ : \ i \in \mathcal{I}\}$$

Generalized conditions (Strong variant; Key points):

State and control constraints admissibility:

$\forall i \in \mathcal{I},$ $\quad \mathbb{X}_{f\,i} \subseteq \mathbb{X}.$

$\forall x \in \mathbb{X}_{f\,i},\ \kappa_{f\,i}(x) \in \mathbb{U}.$

Positive invariance and stability requirements:

$\forall i \in \mathcal{I},\ \exists j \in \mathcal{I},$ $\quad \forall x \in \mathbb{X}_{f\,i},\ f(x, \kappa_{f\,i}(x)) \in \mathbb{X}_{f\,j}.$

$\forall x \in \mathbb{X}_{f\,i},\ V_{f\,j}(f(x, \kappa_{f\,i}(x))) \leq V_{f\,i}(x) - \ell(x, \kappa_{f\,i}(x)).$

(Note: For a weak variant, replace $\forall i \in \mathcal{I},\ \exists j \in \mathcal{I},\ \forall x \in \mathbb{X}_{f\,i}$ with $\forall i \in \mathcal{I},\ \forall x \in \mathbb{X}_{f\,i},\ \exists j \in \mathcal{I}$, i.e. allow j to depend on both i and x.)

Generalized Synthesis: Discrete Setting

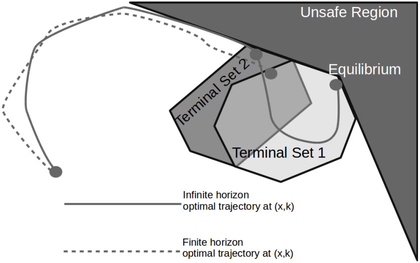
Given an integer $N \in \mathfrak{N}$ and a state $x \in \mathbb{X}$ select

an index i and finite sequences of

control actions $\mathbf{u}_{N-1} := \{u_0, u_1, \ldots, u_{N-1}\}$, and

controlled states $\mathbf{x}_N := \{x_0, x_1, \ldots, x_{N-1}, x_N\}$,

which, for each $k \in \{0, 1, \ldots, N-1\}$, satisfy



$x_{k+1} = f(x_k, u_k)$ with $x_0 = x$,

$x_k \in \mathbb{X}$,

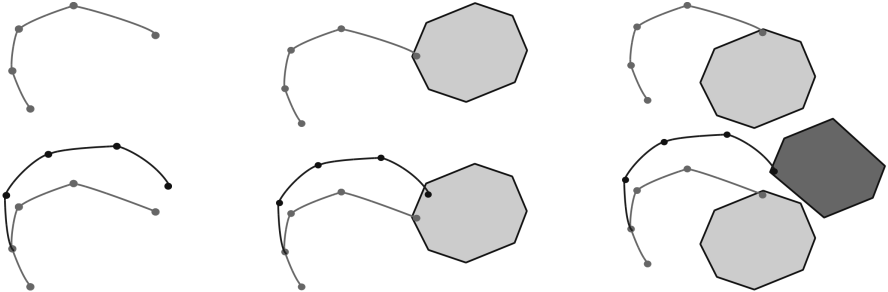
$u_k \in \mathbb{U}$,

$x_N \in \mathbb{X}_{f\,i}$, and $i \in \mathcal{I}$,

and which minimize $V_N(\mathbf{x}_N, \mathbf{u}_{N-1}, i) := \sum_{k=0}^{N-1} \ell(x_k, u_k) + V_{f\,i}(x_N)$.

Lower–, Upper– and Generalized Syntheses: Illustration

Syntheses time transitions (from k to $k+1$)



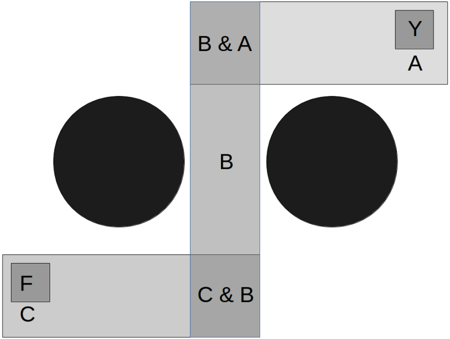
Lower–Synthesis. ($\mathbb{X}_f = \mathbb{X}$ fixed.)

Upper–Synthesis. ($\mathbb{X}_f \subseteq \mathbb{X}$ fixed.)

Generalized Synthesis. ($\mathbb{X}_{f\,i} \subseteq \mathbb{X}$ variable.)

Fact: Generalized synthesis relaxes both the lower– and upper– syntheses.

GMPC + Reachability = Smart Autonomous Behaviour



1. Can go from Y to F.
2. Can go from Y to B & A, then from B & A to C & B, then from C & B to F.
3. Can go from Y to C & B, from C & B to F.
4. Can do many other things. (e.g., hit obstacles :-(.)

GMPC uses dynamically consistent covers that can be easily constructed using (backward) reachability analysis!

Generalized Synthesis: Summary

Payoff: Obvious improvements of all properties.

Price: Increased complexity and need for computable parametrizations.

	Lower	Upper	Generalized
N	Long enough	All	All
$(\mathbb{X}_f, \kappa_f(\cdot), V_f(\cdot))$	Constant	Constant	Variable
estimate of N	Global	–	–

Word of caution: The dynamics of terminal constraint sets and cost functions are not necessarily "stabilized".

Generalization: Employment of generalized stage (and overall) cost penalizing additionally the deviation of terminal triplets $(\mathbb{X}_f, \kappa_f(\cdot), V_f(\cdot))$ from the "equilibrium" terminal triplet $(\mathbb{X}_f^*, \kappa_f^*(\cdot), V_f^*(\cdot))$.

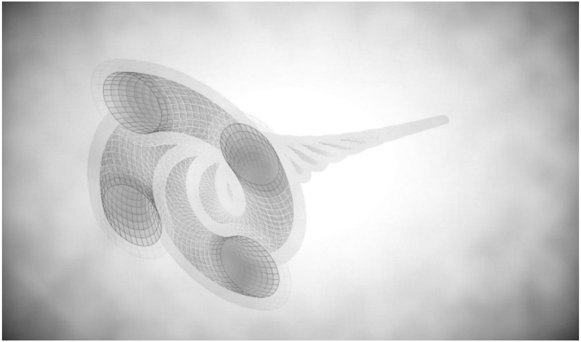
Part

A Big Picture in MPC

- Integration of identification and MPC
  (e.g., Adaptive MPC).
- Integration of uncertainty modelling and MPC
  (e.g., Flexible MPC under uncertainty).
- Integration of estimation and MPC
  (e.g., Output feedback MPC).
- Integration of fault tolerance and MPC
  (e.g., Reconfigurable and actively fault tolerant MPC).
- Integration of MPC's general components and optimization
  (i.e., Integrated MPC synthesis).

Make sure that the sum of parts is equal to the whole!

Thanks for the attention!



Questions are, as always, welcome!

ACC 2016 Events

- Double Invited Session
  "MPC, Quo Vadis?".
  - with W. S. Levine, B. Açikmeșe and I. V. Kolmanovsky
  - 12 papers by well–known contributors in MPC.
- Workshop
  "MPC Under Uncertainty: Theory, Computations and Applications".
  - with W. S. Levine, B. Açikmeșe and I. V. Kolmanovsky
  - Concise and unifying tutorial to MPC under uncertainty.