

#### Cloud Radio Access Downlink with Backhaul Constrained Oblivious Processing

Shlomo Shamai

Department of Electrical Engineering Technion–Israel Institute of Technology

Joint work with S.-H. Park, O. Simeone and O. Sahin

HyNeT Colloquium, University of Maryland October 2013







# Outline

- I. Backgrounds and motivations
- II. Basic setting
- III. State of the art
- IV. Joint precoding and multivariate compression
- V. Special cases and extensions
- VI. Numerical results
  - Wyner model and general MIMO fading
- VII. Concluding remarks





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VII. Concluding remarks

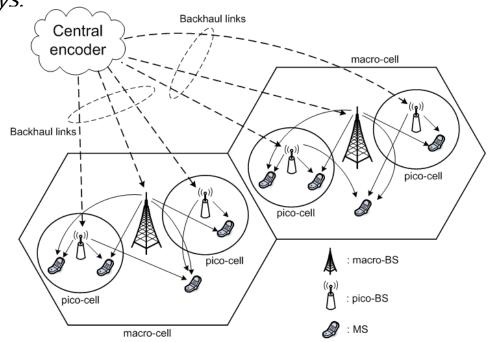


# Backgrounds

- Cloud radio access networks
  - Promoted by

Huawei [Liu et al], Intel [Intel], Alcatel-Lucent [Segel-Weldon], China Mobile [China], Texas Inst. [Flanagan], Ericsson [Ericsson]

 Base stations (BSs) (e.g., macro-BS and pico-BS) operate as soft relays.

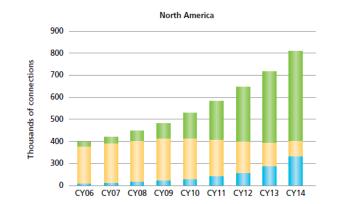


An Illustration of the downlink of cloud radio access networks



# Backgrounds

- Cloud radio access networks (ctd')
  - Low-cost deployment of BSs
    - Encoding/decoding functionalities migrated to the central unit
    - No need to consider cell association
  - Effective interference mitigation
    - Joint encoding/decoding at the central unit
  - But, the backhaul links have limited capacity
    - Macro BSs: increasingly fiber cables [Segel-Weldon]
    - Dedicated relays: wireless [Maric et al][Su-Chang]
    - Home BSs: last-mile connections



The distribution of backhaul connections for macro BSs (green: fiber, orange: copper, blue: air) [Segel-Weldon].



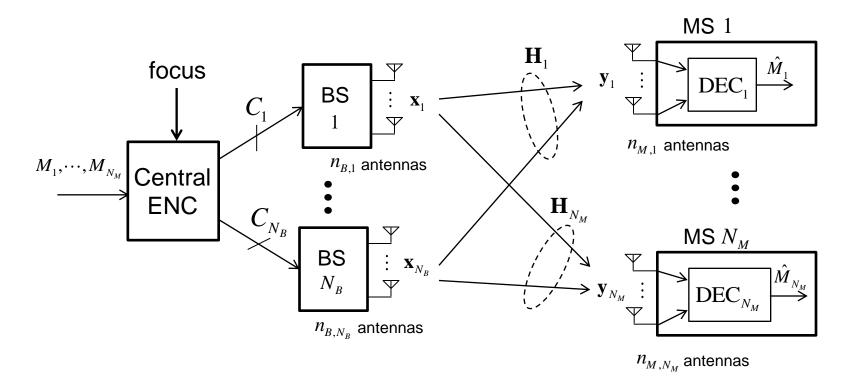
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# **Basic Setting**

• We focus on the downlink



- Notation:  $\mathcal{N}_{\mathcal{B}} = \{1, \dots, N_B\}, \ \mathcal{N}_{\mathcal{M}} = \{1, \dots, N_M\}$ 



#### **Basic Setting**

 Assuming flat-fading channel, the received signal at MS k is given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{z}_k, \quad k \in \mathscr{N}_{\mathscr{M}}$$

where 
$$\mathbf{H}_{k} = \begin{bmatrix} \mathbf{H}_{k,1}, \cdots, \mathbf{H}_{k,N_{B}} \end{bmatrix}$$
,  
 $\mathbf{x} = \begin{bmatrix} \mathbf{x}_{1}^{H}, \cdots, \mathbf{x}_{N_{B}}^{H} \end{bmatrix}^{H}$ ,  $\mathbf{z}_{k} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ 

• Per-BS power constraints

$$\mathbf{E}\left\|\mathbf{x}_{i}\right\|^{2} \leq P_{i}, \quad i \in \mathcal{N}_{\mathscr{B}}$$

- The results of this work can be extended to more general power constraints:

$$\mathbf{E}\left[\mathbf{x}^{H}\mathbf{\Theta}_{l}\mathbf{x}\right] \leq \delta_{i}, \quad l = 1, \cdots, L.$$



# **Basic Setting**

- Backhaul constraints
  - Each BS i is connected to the central encoder via a backhaul link of capacity  $C_i$  bits per channel use (c.u.).
- Oblivious BSs
  - The codebooks of the MSs are not known to the BSs.
    - As assumed in cloud radio access networks, e.g., [Liu et al]-[Ericsson].
  - Systems with informed BSs treated in [Ng et al][Sohn et al][Zakhour-Gesbert][Simeone et al: 12].



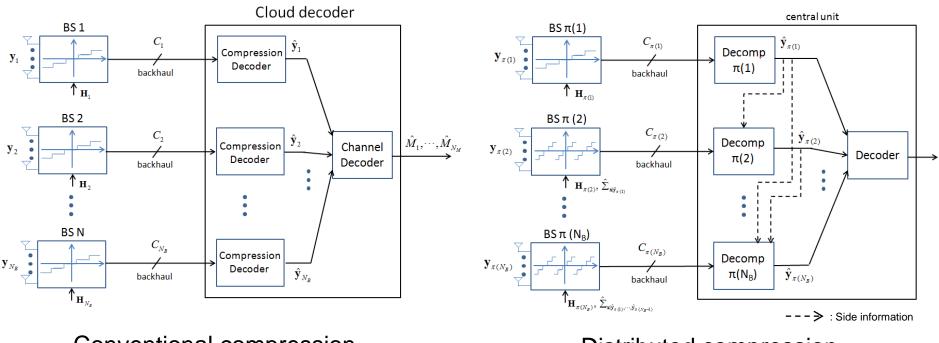


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# Previous Work: Uplink

- Distributed compression
  - Received signals at different BSs are statistically correlated.
  - This correlation can be utilized to improve the achievable rates [Sanderovich et al][dCoso-Simoens][Park et al:TVT][Zhou et al].



Conventional compression

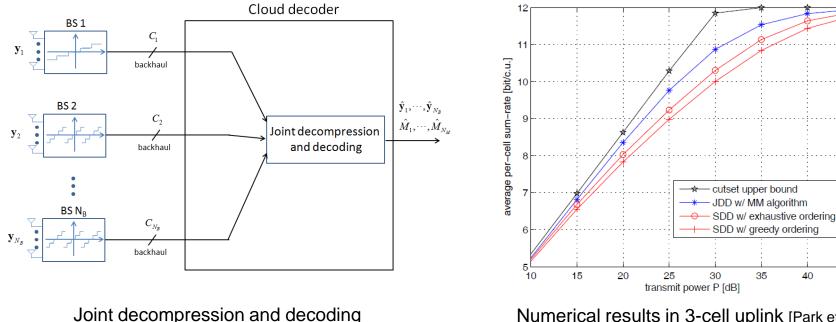
Distributed compression



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# Previous Work: Uplink

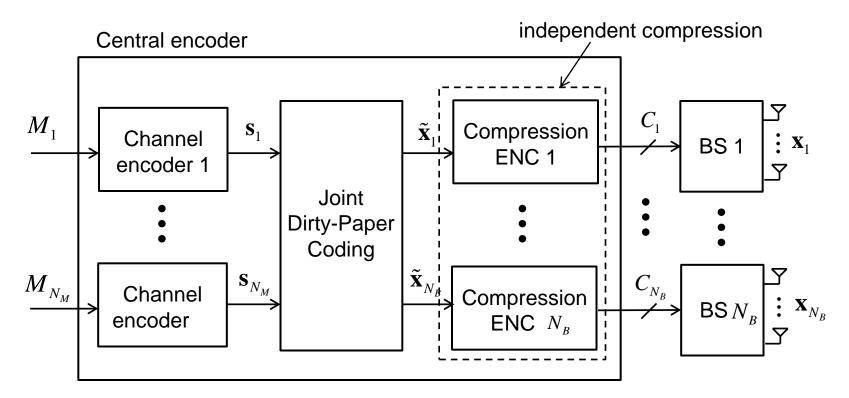
- Joint decompression and decoding [Sanderovich et al][Yassaee-Aref][Lim et al]
  - Potentially larger rates can be achieved with joint decompression and decoding (JDD) at the central unit [Sanderovich et al].
  - Optimization of the Gaussian test channels with JDD [Park et al:SPL].



Numerical results in 3-cell uplink [Park et al:SPL] (SDD: separate decompression and decoding)

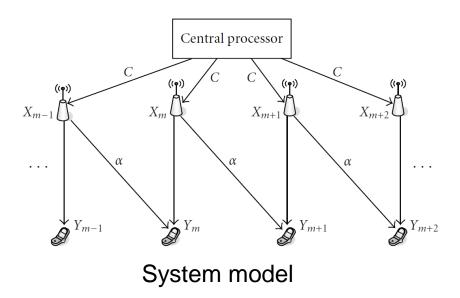


- Compressed dirty-paper coding (CDPC) [Simeone et al:09]
  - Joint dirty-paper coding [Costa] for all MSs
    - A simpler scheme based on zero-forcing DPC [Caire-Shamai] was studied in [Mohiuddin et al:13].
  - Followed by independent compression
    - DPC output signals for different BSs are compressed independently.





• Compressed dirty-paper coding [Simeone et al:09] (ctd')



Quantization is performed at the central unit using the forward test channel

 $X_m = \tilde{X}_m + Q_m,$ 

where  $\tilde{X}_m$ : DPC precoding output,

 $Q_m$ : quantization noise with  $Q_m \sim \mathcal{CN}(0, P/2^c)$ ,

*m*: cell-index, thus  $Q_m$  is independent over the index *m*.

- With constrained backhaul links, we obtain a modified BC with the added quantization noises.
- Per-cell sum-rate

$$R_{\text{per-cell}} = \log\left(\frac{1 + (1 + \alpha^2)\tilde{P} + \sqrt{1 + 2(1 + \alpha^2)\tilde{P} + (1 - \alpha^2)^2\tilde{P}^2}}{2}\right)$$

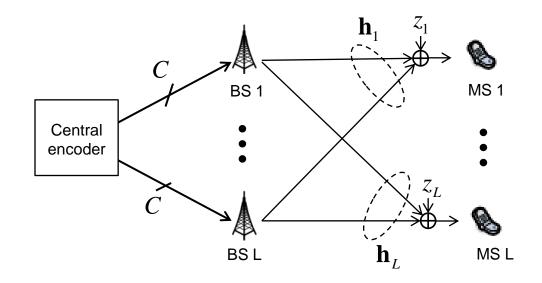
where  $\tilde{P}$  is the effective SNR at the MSs decreased from P to

$$\tilde{P} = \frac{P}{\left(1 + (1 + \alpha^2)P\right) / (2^C - 1) + 1}$$



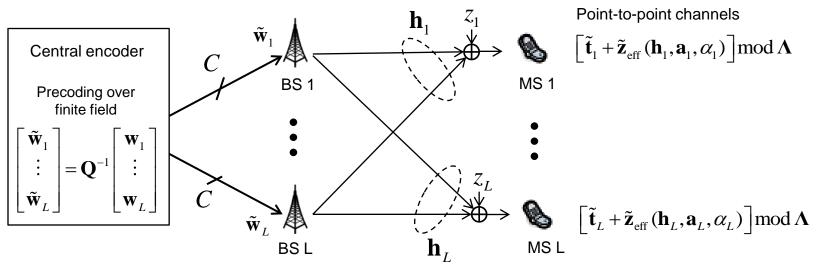
- Reverse compute-and-forward (RCoF) [Hong-Caire]
  - Downlink counterpart of the compute-and-forward (CoF) scheme proposed for the uplink in [Nazer et al].
    - Exchange the role of BSs and MSs and use CoF in reverse direction.
  - System model

• 
$$N_B = N_M = L$$
,  $C_i = C$  for all  $i \in \mathcal{L} = \{1, \dots, L\}$ .





• Reverse compute-and-forward (RCoF) [Hong-Caire] (ctd')



- The same lattice code is used by each BS.
- Each MS *k* estimates a function  $\hat{\mathbf{w}}_k = \sum_{j=1}^{L} a_{k,j} \tilde{\mathbf{w}}_j$  by decoding on the lattice code.
- Achievable rate per MS is given by

$$R_{\text{per-MS}} = \min\left\{C, \min_{l \in \mathcal{L}} R(\mathbf{h}_l, \mathbf{a}_l, \text{SNR})\right\} \text{ where } R(\mathbf{h}, \mathbf{a}, \text{SNR}) = \max\left\{\log\left(\frac{\text{SNR}}{\mathbf{a}^H \left(\text{SNR}^{-1}\mathbf{I} + \mathbf{h}\mathbf{h}^H\right)^{-1}\mathbf{a}}\right), 0\right\}$$



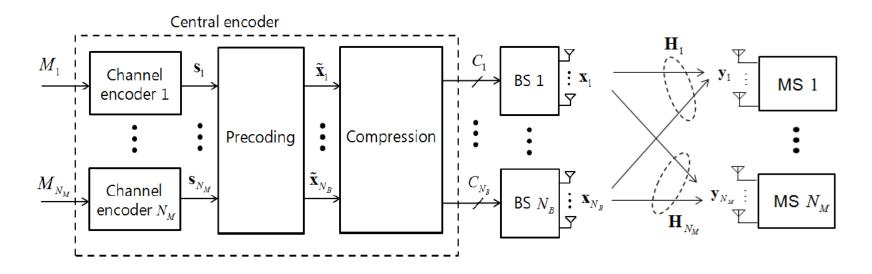
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# Central Encoder

- Structure of the central encoder
  - Precoding: interference mitigation
  - Compression: backhaul communication



– Achievable rate for MS k (single-user detection)

$$R_k = I\left(\mathbf{s}_k; \mathbf{y}_k\right)$$



# Channel Encoding

- Channel encoding for MS k
  - Assume Gaussian codewords

$$\mathbf{s}_k = \mathrm{ENC}_k(M_k) \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$$

where 
$$M_k \in \{1, \dots, 2^{nR_k}\},\$$
  
 $R_k$ : rate for MS  $k$ ,  
 $n$ : coding block length.



# Precoding

• Linear precoding

$$\tilde{\mathbf{x}} = \mathbf{A}\mathbf{s} \implies \begin{bmatrix} \tilde{\mathbf{x}}_1 \\ \vdots \\ \tilde{\mathbf{x}}_{N_B} \end{bmatrix} = \begin{bmatrix} \mathbf{E}_1^H \mathbf{A}\mathbf{s} \\ \vdots \\ \mathbf{E}_{N_B}^H \mathbf{A}\mathbf{s} \end{bmatrix}$$

where 
$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{1}, \dots, \mathbf{A}_{N_{M}} \end{bmatrix}, \mathbf{s} = \begin{bmatrix} \mathbf{s}_{1}^{H}, \dots, \mathbf{s}_{N_{M}}^{H} \end{bmatrix}^{H},$$
  
 $\mathbf{E}_{i} = \begin{bmatrix} \mathbf{0}_{\overline{n}_{B,i-1} \times n_{B,i}} \\ \mathbf{I}_{n_{B,i}} \\ \mathbf{0}_{(n_{B} - \overline{n}_{B,i}) \times n_{B,i}} \end{bmatrix}, \overline{n}_{B,j} = \sum_{l=1}^{j} n_{B,l}, n_{B} = \sum_{l=1}^{N_{B}} n_{B,l}$ 

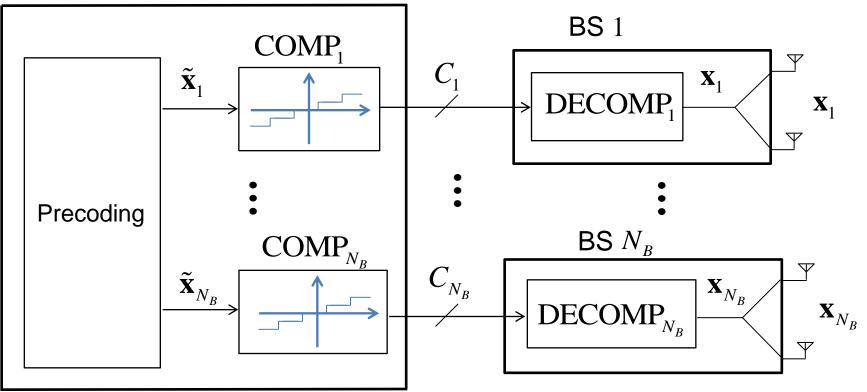
- Remark: Non-linear dirty-paper coding [Costa] can be also considered.
  - All kind of pre-processing can be accommodated as long as the message to compress is treated as a Gaussian vector.



# Conventional Compression

Conventional Compression

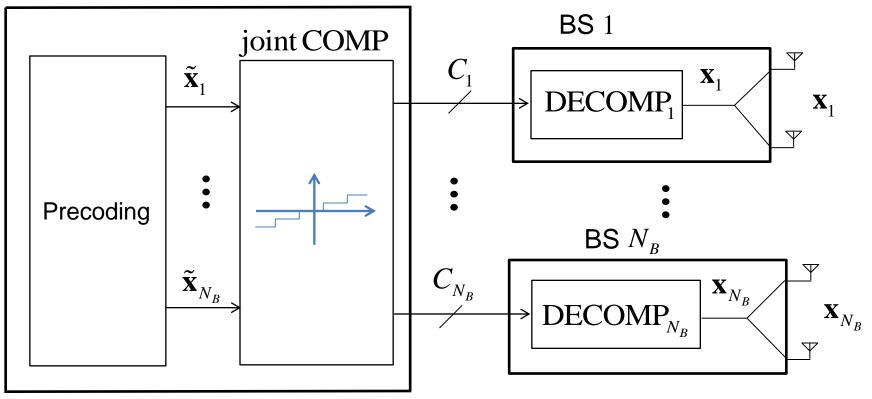






• Multivariate Compression

Central encoder





- Multivariate Compression (ctd')
  - Gaussian test channel [dCoso-Simoens][Simeone et al:09]

$$\mathbf{x}_i = \tilde{\mathbf{x}}_i + \mathbf{q}_i, \quad \mathbf{q}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Omega}_{i,i}), \ i \in \mathcal{N}_{\mathscr{B}}$$

- Overall, the compressed signal  $\mathbf{x} = \begin{bmatrix} \mathbf{x}_1^H, \dots, \mathbf{x}_{N_B}^H \end{bmatrix}^H$  is given as

$$\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{q}, \qquad \cdots (1)$$

with the compression noise  $\mathbf{q} = \left[\mathbf{q}_{1}^{H}, \cdots, \mathbf{q}_{N_{B}}^{H}\right]^{H} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Omega})$  where

$$\mathbf{\Omega} = \begin{bmatrix} \mathbf{\Omega}_{1,1} & \mathbf{\Omega}_{1,2} & \cdots & \mathbf{\Omega}_{1,N_B} \\ \mathbf{\Omega}_{2,1} & \mathbf{\Omega}_{2,2} & \cdots & \mathbf{\Omega}_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Omega}_{N_B,1} & \mathbf{\Omega}_{N_B,2} & \cdots & \mathbf{\Omega}_{N_B,N_B} \end{bmatrix},$$

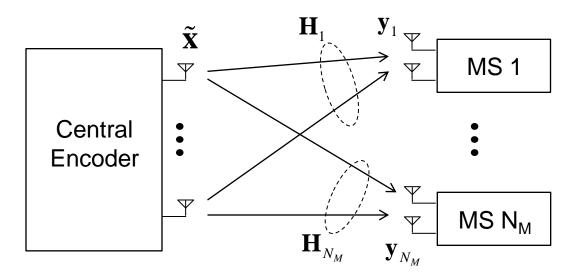
and  $\mathbf{\Omega}_{i,j} = E\left[\mathbf{q}_i\mathbf{q}_j^H\right]$ .



).

# Multivariate Compression

- Multivariate Compression (ctd')
  - For a precoder  $\mathbf{A}$  and a compression correlation  $\mathbf{\Omega}$ , we have the following modified BC.

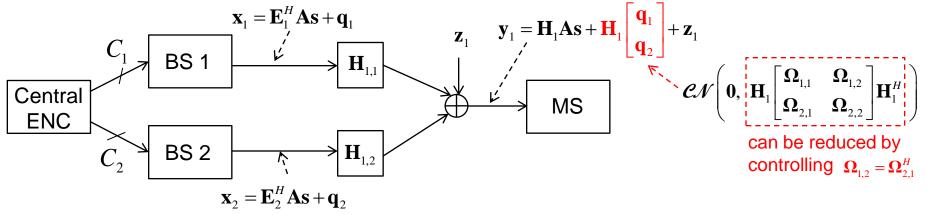


– Received signal at MS k

$$\mathbf{y}_{k} = \mathbf{H}_{k} \tilde{\mathbf{x}} + \tilde{\mathbf{z}}_{k} \quad \text{where} \quad \tilde{\mathbf{z}}_{k} = \mathbf{z}_{k} + \mathbf{H}_{k} \mathbf{q}$$
$$\sim \mathcal{CN}(\mathbf{0}, \mathbf{I} + \mathbf{H}_{k} \mathbf{\Omega} \mathbf{H}_{k}^{H})$$



- Multivariate Compression (ctd')
  - Leverages correlated compression in order to better control the effect of the additive quantization noises at the MSs.
    - Ex: Consider the case with single-MS and two-BSs.



• The conventional independent compression [Simeone et al:09] is a special case of multivariate compression by setting

$$\boldsymbol{\Omega}_{i,j} = \mathrm{E}\left[\mathbf{q}_{i}\mathbf{q}_{j}^{H}\right] = \mathbf{0}, \quad i \neq j.$$

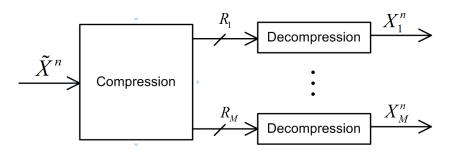


- Multivariate Compression (ctd')
  - Lemma 1 [ElGamal-Kim, Ch. 9]

Consider an i.i.d. sequence  $\tilde{X}^n$  and n large enough. Then, there exist codebooks  $\mathcal{C}_1, \ldots, \mathcal{C}_M$  with rates  $R_1, \ldots, R_M$ , that have at least one tuple of codewords  $(X_1^n, \ldots, X_M^n) \in \mathcal{C}_1 \times \ldots \times \mathcal{C}_M$  jointly typical with  $X^n$  with respect to the given joint distribution  $p(\tilde{x}, x_1, \ldots, x_M) = p(\tilde{x}) p(x_1, \ldots, x_M | \tilde{x})$  with probability arbitrarily close to one, if the inequlities

$$\sum_{i \in \mathcal{S}} h(X_i) - h(X_{\mathcal{S}} | \tilde{X}) \le \sum_{i \in \mathcal{S}} R_i, \text{ for all } \mathcal{S} \subseteq \{1, \dots, M\}$$

are satisfied.

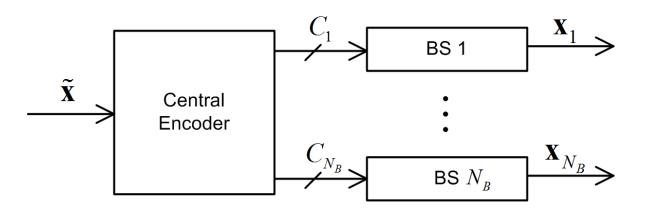




- Multivariate Compression (ctd')
  - Lemma 2 (Lemma 1 applied to our setting) [Park et al:13] The signals  $\mathbf{X}_1, \dots, \mathbf{X}_{N_B}$  obtained via (1) can be reliably transferred to the BSs on the backhaul links if the condition

$$g_{\mathcal{S}}(\mathbf{A}, \mathbf{\Omega}) = \sum_{i \in \mathcal{S}} h(\mathbf{x}_{i}) - h(\mathbf{x}_{\mathcal{S}} \mid \tilde{\mathbf{x}})$$
$$= \sum_{i \in \mathcal{S}} \log \det \left( \mathbf{E}_{i}^{H} \mathbf{A} \mathbf{A}^{H} \mathbf{E}_{i} + \mathbf{\Omega}_{i,i} \right) - \log \det \left( \mathbf{E}_{\mathcal{S}}^{H} \mathbf{\Omega} \mathbf{E}_{\mathcal{S}} \right) \leq \sum_{i \in \mathcal{S}} C_{i}$$

is satisfied for all subsets  $\mathcal{S} \subseteq \mathscr{N}_{B}$ .





- Multivariate Compression (ctd')
  - Consider the case with two BSs.
    - The multivariate constraints in Lemma 2 become

$$h(\mathbf{x}_1) - h(\mathbf{x}_1 | \tilde{\mathbf{x}}) = I(\tilde{\mathbf{x}}_1; \mathbf{x}_1) \le C_1,$$
(A1)

$$h(\mathbf{x}_2) - h(\mathbf{x}_2 \mid \tilde{\mathbf{x}}) = I(\tilde{\mathbf{x}}_2; \mathbf{x}_2) \le C_2, \qquad (A2)$$

$$h(\mathbf{x}_1) + h(\mathbf{x}_2) - h(\mathbf{x}_1, \mathbf{x}_2 | \tilde{\mathbf{x}}) = I(\tilde{\mathbf{x}}; \mathbf{x}_1, \mathbf{x}_2) + I(\mathbf{x}_1; \mathbf{x}_2) \le C_1 + C_2.$$
(A3)

- With independent compression  $\,\Omega_{_{1,2}}=0$  , the constraints (A) reduce to

$$I\left(\tilde{\mathbf{x}}_{1};\mathbf{x}_{1}\right) \leq C_{1}, \qquad (B1)$$
$$I\left(\tilde{\mathbf{x}}_{2};\mathbf{x}_{2}\right) \leq C_{2}. \qquad (B2)$$

- Constraints (A) are stricter than (B).
  - The introduction of correlation among the quantization noises for different BSs leads to additional constraints on the backhaul link capacities.



# Problem Definition

• Weighted sum-rate maximization

$$\underset{\mathbf{A},\mathbf{\Omega}\geq\mathbf{0}}{\text{maximize}} \sum_{k=1}^{N_{M}} w_{k} f_{k} (\mathbf{A},\mathbf{\Omega})$$
(2*a*)

s.t. 
$$g_{\mathcal{S}}(\mathbf{A}, \mathbf{\Omega}) \leq \sum_{i \in \mathcal{S}} C_i$$
, for all  $\mathcal{S} \subseteq \mathcal{N}_B$ , (2b)

$$\operatorname{tr}\left(\mathbf{E}_{i}^{H}\mathbf{A}\mathbf{A}\mathbf{E}_{i}+\mathbf{\Omega}_{i,i}\right) \leq P_{i}, \text{ for all } i \in \mathcal{N}_{B}.$$
 (2*c*)

where 
$$f_k(\mathbf{A}, \mathbf{\Omega}) = I(\mathbf{s}_k; \mathbf{y}_k)$$
  
 $= \log \det (\mathbf{I} + \mathbf{H}_k(\mathbf{A}\mathbf{A}^H + \mathbf{\Omega})\mathbf{H}_k^H) - \log \det (\mathbf{I} + \mathbf{H}_k (\sum_{l \neq k} \mathbf{A}_l \mathbf{A}_l^H + \mathbf{\Omega})\mathbf{H}_k^H),$   
 $g_s(\mathbf{A}, \mathbf{\Omega}) = \sum_{i \in S} h(\mathbf{x}_i) - h(\mathbf{x}_s | \tilde{\mathbf{x}})$   
 $= \sum_{i \in S} \log \det (\mathbf{E}_i^H \mathbf{A}\mathbf{A}^H \mathbf{E}_i + \mathbf{\Omega}_{i,i}) - \log \det (\mathbf{E}_s^H \mathbf{\Omega}\mathbf{E}_s) \leq \sum_{i \in S} C_i.$ 

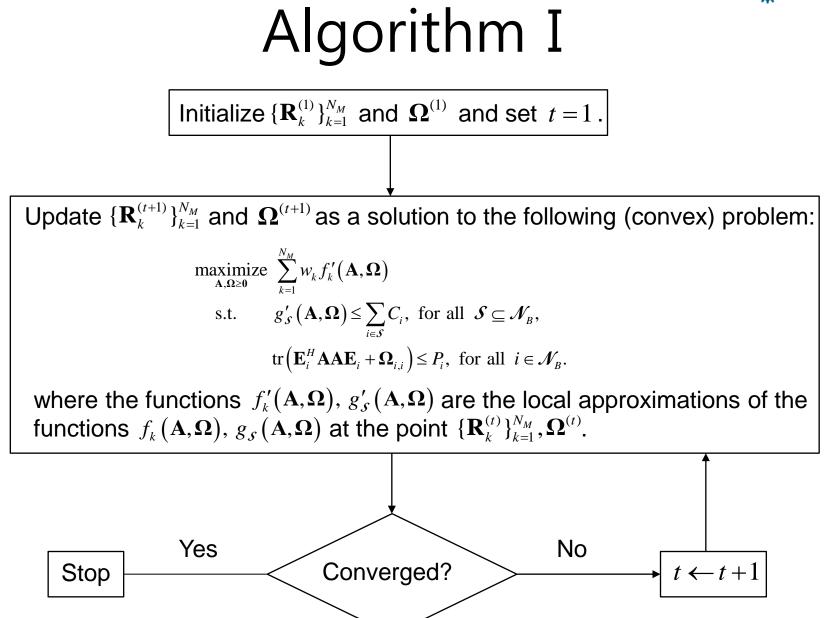
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# MM Algorithm

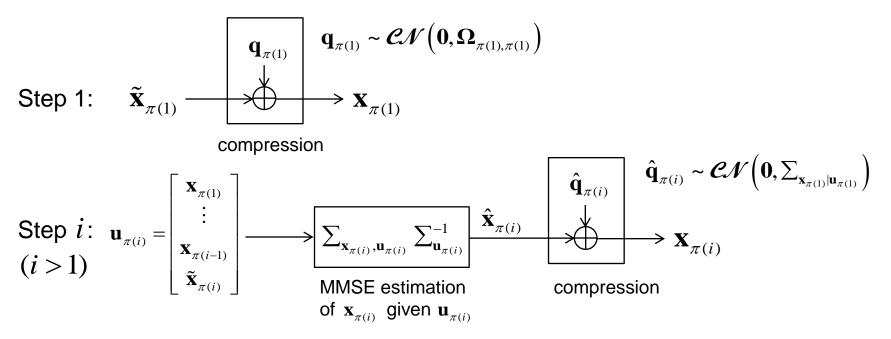
- If we define R<sub>k</sub> = A<sub>k</sub>A<sub>k</sub><sup>H</sup> for k ∈ N<sub>M</sub>, the problem (2) falls in the class of difference-of-convex problem [Beck-Teboulle] with respect to the variables {R<sub>k</sub>}<sub>k∈NM</sub>, Ω.
  - We can use a Majorization Minimization (MM) algorithm [Beck-Teboulle] to find a stationary point of the problem.
    - At each iteration, linearize non-convex parts.
    - The algorithm is detailed in **Algorithm I** in the next slide.







- For given variables  $(\mathbf{A}, \mathbf{\Omega})$ , the implementation of the joint compression is relatively complex.
- A successive architecture with a given permutation  $\pi: \mathcal{N}_{\mathcal{B}} \to \mathcal{N}_{\mathcal{B}}$ .



- Compression rate at step i (i > 1):  $I(\hat{\mathbf{x}}_{\pi(i)}; \mathbf{x}_{\pi(i)})$ 



• **Lemma 3:** The region of the backhaul capacity tuples  $(C_1, \dots, C_{N_B})$ satisfying the constraints (2b) is a *contrapolymatroid* [Tse-Hanly, Def. 3.1]. Therefore, it has a corner point for each permutation  $\pi$  of the BS indices  $\mathcal{N}_{\mathscr{B}}$ , and each such corner point is given by the tuple  $(C_{\pi(1)}, \dots, C_{\pi(N_B)})$  with

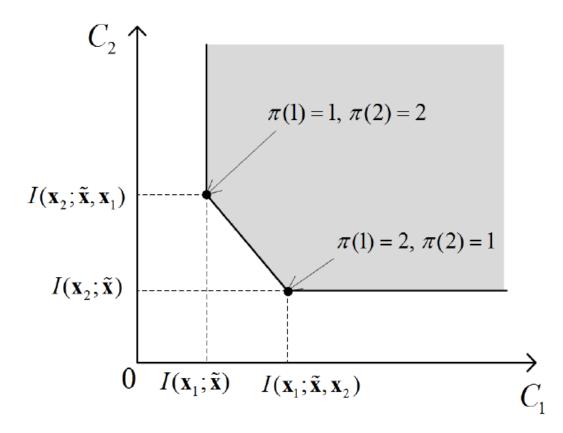
$$C_{\pi(i)} = I\left(\mathbf{x}_{\pi(i)}; \mathbf{\tilde{x}}, \mathbf{x}_{\pi(1)}, \cdots, \mathbf{x}_{\pi(i-1)}\right) \quad \text{for } i \in \mathcal{N}_{\mathcal{B}}$$
$$= I\left(\mathbf{x}_{\pi(i)}; \mathbf{\hat{x}}_{\pi(i)}\right)$$

where  $\hat{\mathbf{x}}_{\pi(i)}$ : MMSE estimate of  $\mathbf{x}_{\pi(i)}$  given  $\tilde{\mathbf{x}}, \mathbf{x}_{\pi(1)}, \dots, \mathbf{x}_{\pi(i-1)}$ .

- Thus, we have 
$$\mathbf{x}_{\pi(i)} - \hat{\mathbf{x}}_{\pi(i)} - (\tilde{\mathbf{x}}, \mathbf{x}_{\pi(1)}, \cdots, \mathbf{x}_{\pi(i-1)})$$
.

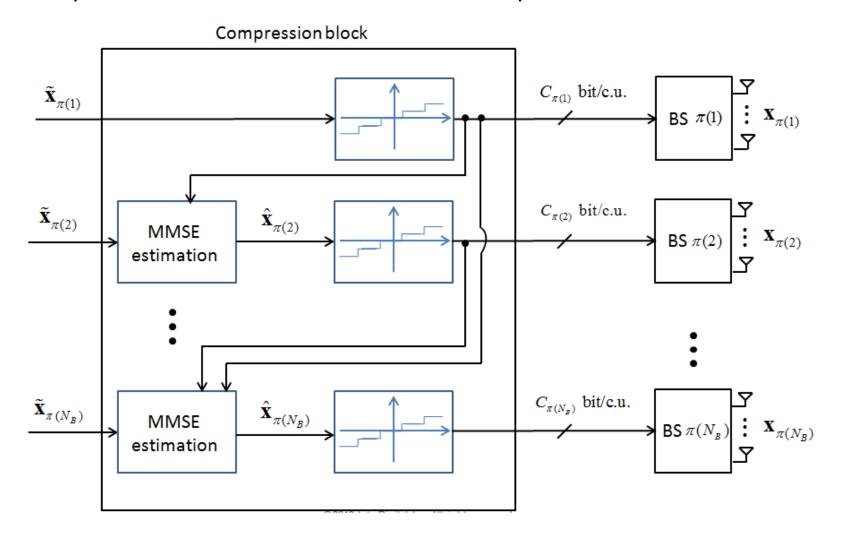


• Example of the backhaul capacity region for  $N_B = 2$ 





• Proposed successive estimation-compression architecture







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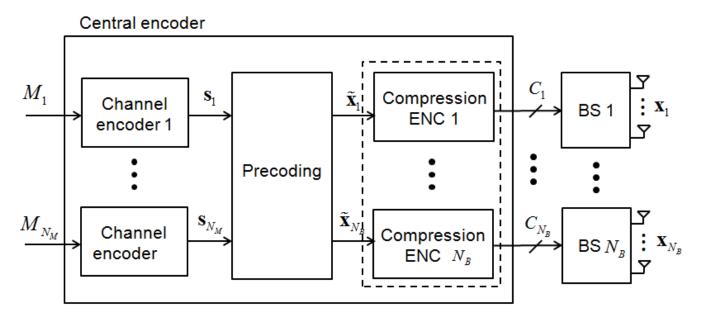


#### Independent Quantization

• For reference, consider independent quantization [Simeone et al:09], i.e.,

$$\Omega_{i,j} = 0$$
, for all  $i \neq j \in \mathcal{N}_{\mathscr{B}}$ .

• Since the above constraint is affine, the MM algorithm is still applicable.





#### Separate Design

- For reference, consider the separate design of precoding and compression.
  - Selection of the precoding matrix A
    - $\mathbf{A}$  is first selected according to some standard criterion
      - e.g., zero-forcing [RZhang], MMSE [Hong et al], sum-rate max. [Ng-Huang]
    - Assume a reduced power constraint  $\gamma_i P_i$  with  $\gamma_i < 1$  since

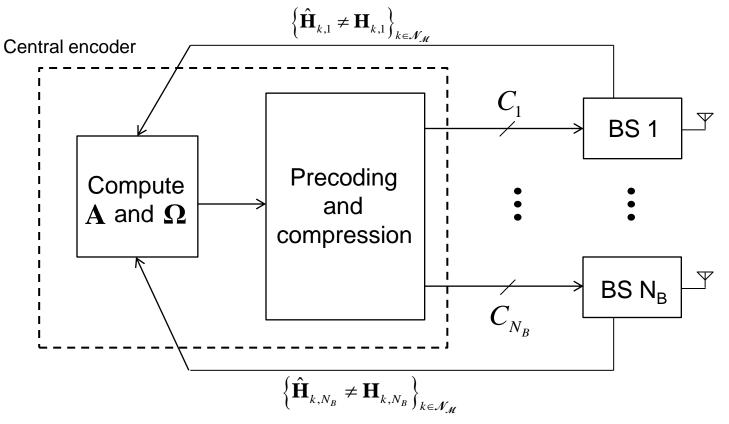
$$\mathbf{x}_i = \widetilde{\mathbf{x}}_i + \mathbf{q}_i$$

precoded quantization noise

- Optimization of the compression covariance  $\Omega$ 
  - Having fixed A, the problem then reduces to solving (2) only with respect to  $\Omega$ .



• Assuming perfect channel state information (CSI) at the central encoder might be unrealistic.





- Singular value uncertainty model [Loyka-Charalambous:Sec. II-A]
  - The actual CSI  $\mathbf{H}_k$  is modeled as

$$\mathbf{H}_{k}=\hat{\mathbf{H}}_{k}\left(\mathbf{I}+\boldsymbol{\Delta}_{k}\right),$$

where  $\hat{\mathbf{H}}_k$ : the CSI known at the central encoder,

 $\Delta_k$ : the multiplicative uncertainty with  $\sigma_{\max}(\Delta_k) \leq \varepsilon_k < 1$ .

- Worst-case optimization problem

$$\underset{\mathbf{A},\boldsymbol{\Omega}\geq\mathbf{0}}{\operatorname{maximize}} \quad \underset{\{\boldsymbol{\Delta}_{k}:\,\sigma_{\max}\,(\boldsymbol{\Delta}_{k})\leq\varepsilon_{k}\}_{k\in\mathcal{N},\mathcal{M}}}{\min} \sum_{k=1}^{N_{M}} w_{k}f_{k}\left(\mathbf{A},\boldsymbol{\Omega}\right) \tag{3a}$$

s.t. 
$$g_{\mathcal{S}}(\mathbf{A}, \mathbf{\Omega}) \leq \sum_{i \in \mathcal{S}} C_i$$
, for all  $\mathcal{S} \subseteq \mathcal{N}_B$ , (3b)

$$\operatorname{tr}\left(\mathbf{E}_{i}^{H}\mathbf{A}\mathbf{A}\mathbf{E}_{i}+\mathbf{\Omega}_{i,i}\right) \leq P_{i}, \text{ for all } i \in \mathcal{N}_{B}.$$

$$(3c)$$



- Singular value uncertainty model (ctd')
  - **Lemma.** The problem (3) is equivalent to the original weighted sum-rate maximization problem with  $\mathbf{H}_k = (1 \varepsilon_k) \hat{\mathbf{H}}_k$  for  $k \in \mathcal{N}_{\mathcal{M}}$ , i.e.,

$$\begin{array}{ll} \underset{\mathbf{A}, \boldsymbol{\Omega} \geq \mathbf{0}}{\text{maximize}} & \sum_{k=1}^{N_{M}} w_{k} \tilde{f}_{k} \left( \mathbf{A}, \boldsymbol{\Omega} \right) \\ \text{s.t.} & g_{\mathcal{S}} \left( \mathbf{A}, \boldsymbol{\Omega} \right) \leq \sum_{i \in \mathcal{S}} C_{i}, \text{ for all } \mathcal{S} \subseteq \mathcal{N}_{B}, \\ & \operatorname{tr} \left( \mathbf{E}_{i}^{H} \mathbf{A} \mathbf{A} \mathbf{E}_{i} + \boldsymbol{\Omega}_{i,i} \right) \leq P_{i}, \text{ for all } i \in \mathcal{N}_{B} \end{array}$$

where 
$$\tilde{f}_k(\mathbf{A}, \mathbf{\Omega}) = \log \det \left( \mathbf{I} + (1 - \varepsilon_k)^2 \hat{\mathbf{H}}_k (\mathbf{A}\mathbf{A}^H + \mathbf{\Omega}) \hat{\mathbf{H}}_k^H \right)$$
  
$$-\log \det \left( \mathbf{I} + (1 - \varepsilon_k)^2 \hat{\mathbf{H}}_k \left( \sum_{l \neq k} \mathbf{A}_l \mathbf{A}_l^H + \mathbf{\Omega} \right) \hat{\mathbf{H}}_k^H \right).$$



- Ellipsoidal uncertainty model [Shen et al][Bjornson-Jorswieck]
  - Consider MISO case such that  $\mathbf{H}_{k} = \mathbf{h}_{k}^{H}$ ,  $k \in \mathcal{N}_{\mathcal{M}}$ .
  - The actual channel  $\mathbf{h}_k$  is modeled as

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k$$

where  $\hat{\mathbf{h}}_k$ : the CSI known at the central encoder,

 $\mathbf{e}_k$ : the error vector bounded with  $\mathbf{e}_k^H \mathbf{C}_k \mathbf{e}_k \leq 1$ ,

 $(\mathbf{C}_k \succ \mathbf{0}$  specifies the size and shape of the ellipsoid.)



- Ellipsoidal uncertainty model (ctd')
  - The "dual" problem of power minimization under SINR constraints for all MSs, i.e.,

$$\underset{\{\mathbf{R}_{k} \geq \mathbf{0}\}_{k \in \mathcal{N}_{\mathcal{M}}}, \Omega \geq \mathbf{0} }{\text{minimize}} \sum_{i=1}^{N_{B}} \mu_{i} \cdot \operatorname{tr} \left( \sum_{k=1}^{N_{M}} \mathbf{E}_{i}^{H} \mathbf{R}_{k} \mathbf{E}_{i} + \Omega_{i,i} \right)$$
(4*a*)  
s.t. 
$$\frac{\mathbf{h}_{k}^{H} \mathbf{R}_{k} \mathbf{h}_{k}}{\sum_{j \in \mathcal{N}_{\mathcal{M}} \setminus \{k\}} \mathbf{h}_{k}^{H} \mathbf{R}_{j} \mathbf{h}_{k} + \mathbf{h}_{k}^{H} \Omega \mathbf{h}_{k} + 1} \geq \Gamma_{k},$$
(4*b*)  
for all  $\mathbf{e}_{k}$  with  $\mathbf{e}_{k}^{H} \mathbf{C}_{k} \mathbf{e}_{k} \leq 1$  and  $k \in \mathcal{N}_{\mathcal{M}},$   
 $g_{\mathcal{S}}(\mathbf{A}, \mathbf{\Omega}) \leq \sum_{i \in \mathcal{S}} C_{i},$  for all  $\mathcal{S} \subseteq \mathcal{N}_{B}.$ (4*c*)

where  $\mathbf{R}_{k} \triangleq \mathbf{A}_{k} \mathbf{A}_{k}^{H}, \ k \in \mathcal{N}_{\mathcal{M}}.$ 



- Ellipsoidal uncertainty model (ctd')
  - **Lemma.** Constraint (4b) holds if and only if there exist constants  $\{\beta_k\}_{k \in \mathcal{N}_{\mathcal{M}}}$  such that the condition

$$\begin{bmatrix} \mathbf{\Xi}_{k} & \mathbf{\Xi}_{k} \hat{\mathbf{h}}_{k} \\ \hat{\mathbf{h}}_{k}^{H} \mathbf{\Xi}_{k} & \hat{\mathbf{h}}_{k}^{H} \mathbf{\Xi}_{k} \hat{\mathbf{h}}_{k} - \Gamma_{k} \end{bmatrix} - \beta_{k} \begin{bmatrix} \mathbf{C}_{k} & \mathbf{0} \\ \mathbf{0} & -1 \end{bmatrix} \ge \mathbf{0}$$

is satisfied for all  $k \in \mathcal{N}_{\mathcal{M}}$  where we have defined

$$\boldsymbol{\Xi}_{k} = \mathbf{R}_{k} - \boldsymbol{\Gamma}_{k} \sum_{j \in \mathcal{N}_{\mathcal{M}} \setminus \{k\}} \mathbf{R}_{j} - \boldsymbol{\Gamma}_{k} \boldsymbol{\Omega}, \quad \text{for } k \in \mathcal{N}_{\mathcal{M}}.$$

pf: Follows by applying the S-procedure [Boyd-Vandenberghe, Appendix B-2].





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# Wyner Model

- Three-cell SISO circular Wyner model [Gesbert et al]
  - The channel coefficients given by

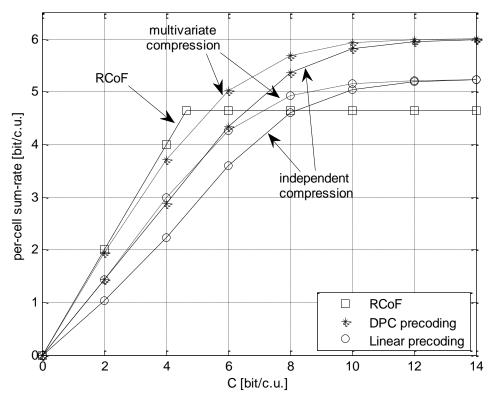
$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & g & g \\ g & 1 & g \\ g & g & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

- Compare the following schemes
  - Reverse Compute-and-Forward (RCoF) [Hong-Caire]
    - Structured codes, but sensitive to the channel coefficients.
  - Dirty-paper coding with
    - Multivariate compression
    - Independent quantization (this case corresponds to the *compressed DPC* in [Simeone et al:09])
  - Linear precoding with
    - Multivariate compression
    - Independent quantization (this case corresponds to *quantized network MIMO* in [Zakhour-Gesbert, Sec. IV-A])



#### Wyner Model

- Three-cell SISO Wyner model [Gesbert et al] (ctd')
  - Per-cell sum-rate versus C when P = 20 dB and g = 0.5.



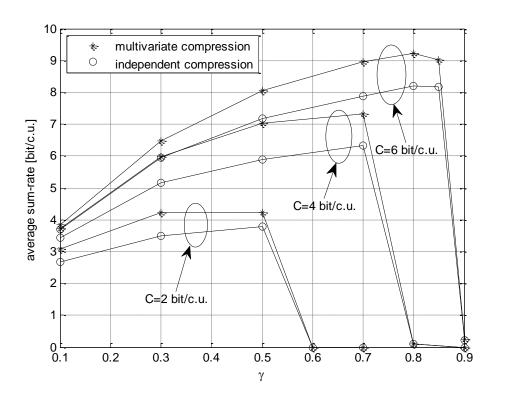
- Multivariate compression is significantly advantageous for both linear and DPC precoding.
- RCoF in [Hong-Caire] remains the most effective approach in the regime of moderate backhaul *C*, although multivariate compression allows to compensate for most of the rate loss of standard DPC precoding in the lowbackhaul regime.
- The curve of RCoF flattens before the others do, since it is limited by the integer approximation penalty when the backhaul capacity is large enough.



- More general MIMO fading model
  - There are three BSs and three MSs, i.e.,  $N_B = N = 3$ .
  - Each BS uses two antennas while each MS uses a single antenna.
  - The elements of  $\mathbf{H}_{k,i}$  between MS k and BS i are i.i.d. with  $\mathcal{CN}(0, \alpha^{|i-k|})$ .
    - We call  $\alpha$  the inter-cell channel gain.
  - In the separate design,
    - The precoding matrix  ${\bf A}$  is obtained via the sum-rate maximization scheme in  $_{\rm [Ng-Huang]}$ .
      - Under the power constraint  $\gamma P$  for each BS with  $\gamma$  selected so that the compression problem be feasible.



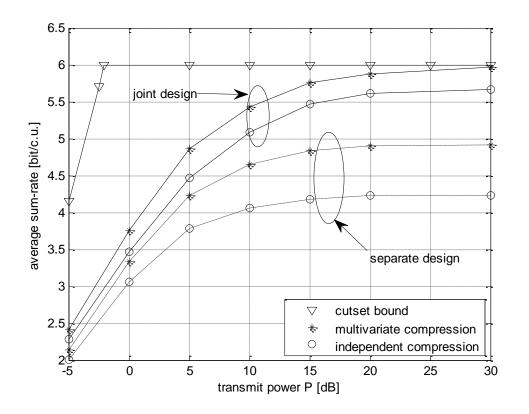
- More general MIMO fading model (ctd')
  - Sum-rate versus  $\gamma$  for the separate design of linear precoding and compression with P = 5 dB and  $\alpha = 0 \text{ dB}$



- Increasing  $\ensuremath{\mathcal{I}}$  generally results in a better sum-rate.
- However, if  $\gamma$  exceeds some threshold value, the problem of optimizing the correlation  $\Omega$  given the precoder A is more likely to be infeasible.
- This threshold value grows with the backhaul capacity, since a larger backhaul capacity allows for a smaller power of the quantization noises.



- More general MIMO fading model (ctd')
  - Sum-rate versus P for linear precoding with C = 2 and  $\alpha = 0$  dB

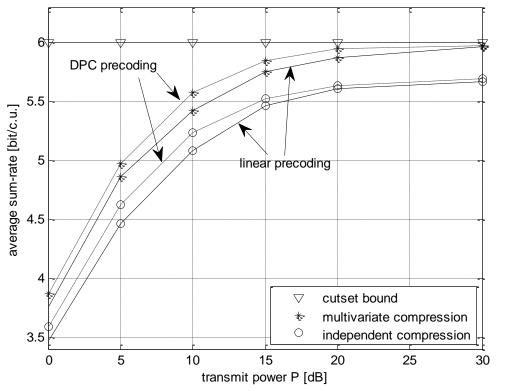


- The gain of multivariate compression is more pronounced when each BS uses a larger power.

- As the received SNR increases, more efficient compression strategies are called for.
- Multivariate compression is effective in partly compensating for the suboptimality of the separate design.
- Only the proposed joint design with multivariate compression approaches the cutset bound as the transmit power increases.



- More general MIMO fading model (ctd')
  - Sum-rate versus P for the joint design with C = 2 and  $\alpha = 0$  dB

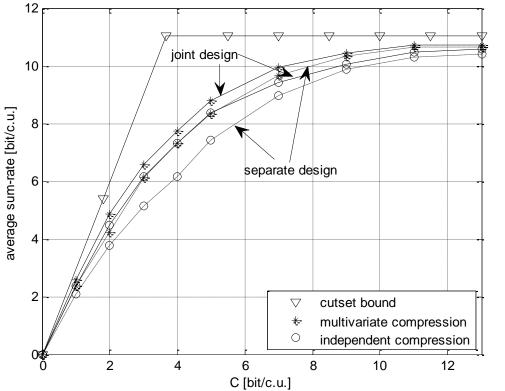


- DPC is advantageous only in the regime of intermediate P due to the limited-capacity backhaul links.

- Unlike the conventional BC channels with perfect backhaul links where there exists constant sum-rate gap between DPC and linear precoding at high SNR (see, e.g., [Lee-Jindal]).
- The overall performance is determined by the compression strategy rather than precoding method when the backhaul capacity is limited at high SNR.



- More general MIMO fading model (ctd')
  - Sum-rate versus C for linear precoding with P = 5 dB and  $\alpha = 0 \text{ dB}$

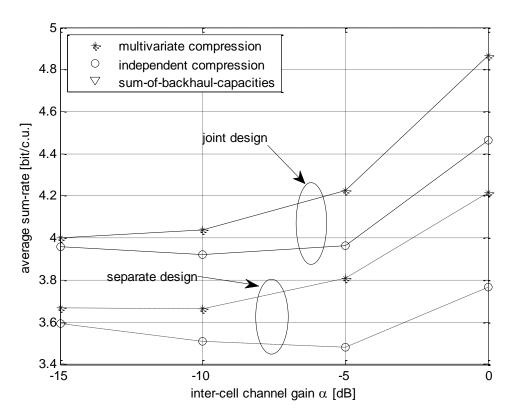


- When the backhaul links have enough capacity, the benefits of multivariate compression or joint design of precoding and compression become negligible.

- since the overall performance becomes limited by the sum-capacity achievable when the BSs are able to fully cooperate with each other.
- The separate design with multivariate compression outperforms the joint design with independent quantization for backhaul capacities larger than 5 bit/c.u.



- More general MIMO fading model (ctd')
  - Sum-rate versus the inter-cell channel gain  $\alpha$  for linear precoding with C = 2 and P = 5 dB



- The multi-cell system under consideration approaches the system consisting of  $N_B$  parallel single-cell networks as the inter-cell channel gain  $\alpha$  decreases.
- The advantage of multivariate compression is not significant for small values of  $\alpha$ , since introducing correlation of the quantization noises across BSs is helpful only when each MS suffers from a superposition of quantization noises emitted from multiple BSs.





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# Concluding Remarks

- We have studied the design of joint precoding and compression strategies for the donwlink of cloud radio access networks.
  - The BSs are connected to the central encoder via finite-capacity backhaul links.
- We have proposed to exploit multivariate compression of the signals of different BSs.
  - In order to control the effect of the additive quantization noises at the MSs.
- The problem of maximizing the weighted sum-rate subject to power and backhaul constraints was formulated.
  - An iterative MM algorithm was proposed that achieves a stationary point.



# Concluding Remarks

- Moreover, we have proposed a novel way of implementing multivariate compression.
  - based on successive per-BS estimation-compression steps.
- Via numerical results, it was confirmed that
  - The proposed approach based on multivariate compression and on joint precoding and compression strategy outperforms the conventional approaches based on independent compression and separate design of precoding and compression strategies.
    - Especially when the transmit power or the inter-cell channel gain are large, and when the limitation imposed by the finite-capacity backhaul link is significant.



# Concluding Remarks

- Interesting open problems
  - Impact of CSI quality
    - The central unit has a different (worse) CSI quality than the distributed BSs.
    - Some related works found in [Park et al:13, Sec. V][Marsch-Fettweis][Hoydis et al].
  - Broadcast approach [Shamai-Steiner][Verdu-Shamai]
    - The overall system can be regarded as a broadcast channel with different fading states among the MSs.
  - Combination of structured codes [Nazer et al][Hong-Caire], partial decoding [Sanderovich et al][dCoso-Ibars] and multivariate processing [Park et al:13].
  - Multi-hop backhaul links
    - The BSs may communicate with the central unit through multi-hop backhaul links.
    - Related works can be found in [Yassaee-Aref][Goela-Gastpar].

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#### Abstract

#### Cloud Radio Access Downlink with Backhaul Constrained Oblivious Processing

The talk considers the downlink of cloud radio access networks, in which a central encoder is connected to multiple multi-antenna base stations (BSs) via finite-capacity backhaul links. The processing is done at the central encoder, while the distributed BSs employ only oblivious (robust) processing. We first review current state-of-the-art approaches, where the signals intended for different BSs are compressed independently, or alternatively the recently introduced structured coding ideas (Reverse Compute-and-Forward) are employed. We propose to leverage joint compression, also referred to as multivariate compression, of the signals of different BSs in order to better control the effect of the additive quantization noises at the mobile stations. We address the maximization of a weighted sumrate. For joint compression this is associated with the optimization of the precoding matrix and the joint correlation matrix of the quantization noises, subject to power and backhaul capacity constraints. An iterative algorithm is described that achieves a stationary point of the problem, and a practically appealing architecture is proposed based on successive steps of minimum mean-squared error estimation and per-BS compression. We conclude by comparison of different processing techniques, discussing a robust design concerning the available accuracy of the channel state information and overviewing some aspects for future research.

Joint work with S.-H. Park, O. Simeone (NJIT), and O. Sahin (InterDigital)